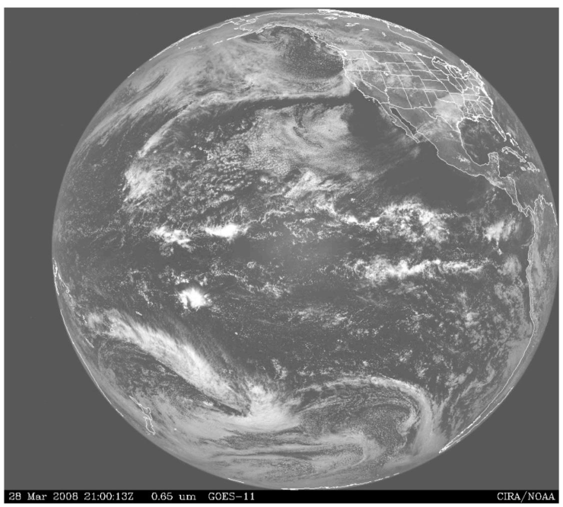
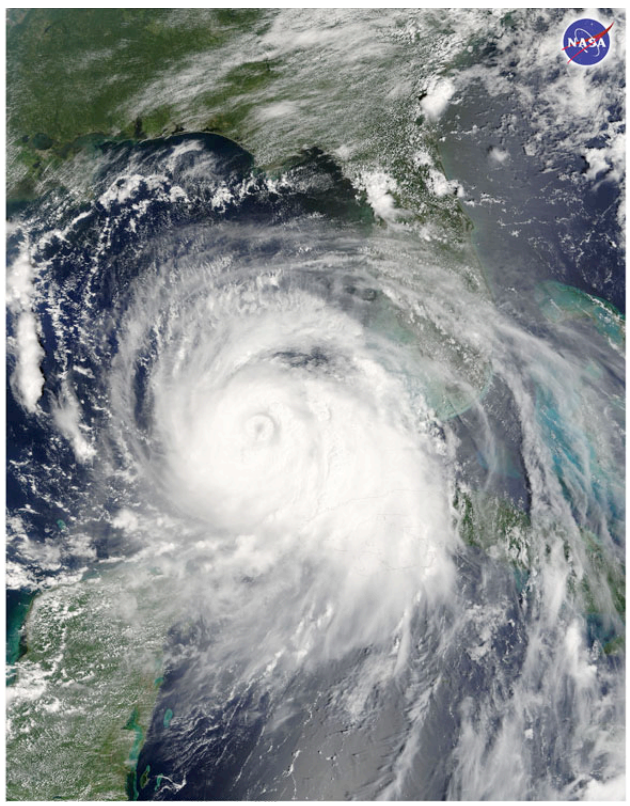
Kinetic Energy Spectra and Model Filters

Bill Skamarock NCAR/MMM





(Courtesy of Morris Weisman)

Applications?

Existing and future applications require meso-scale and cloud-scale resolution in a global model.

<u>Why use higher resolution?</u>

- Explicitly simulate convective systems:
  - Capture system evolution (growth, decay, propagation).
  - Resolve moisture redistribution, cloud systems.
  - Remove need for deep convection parameterization (with sufficient resolution - Δx < a few km).

- Explicitly simulate gravity waves, wave breaking:
  - Remove the need for gravity-wave drag parameterization.

- Better resolution of external forcing:
  - topography, land-use, etc.

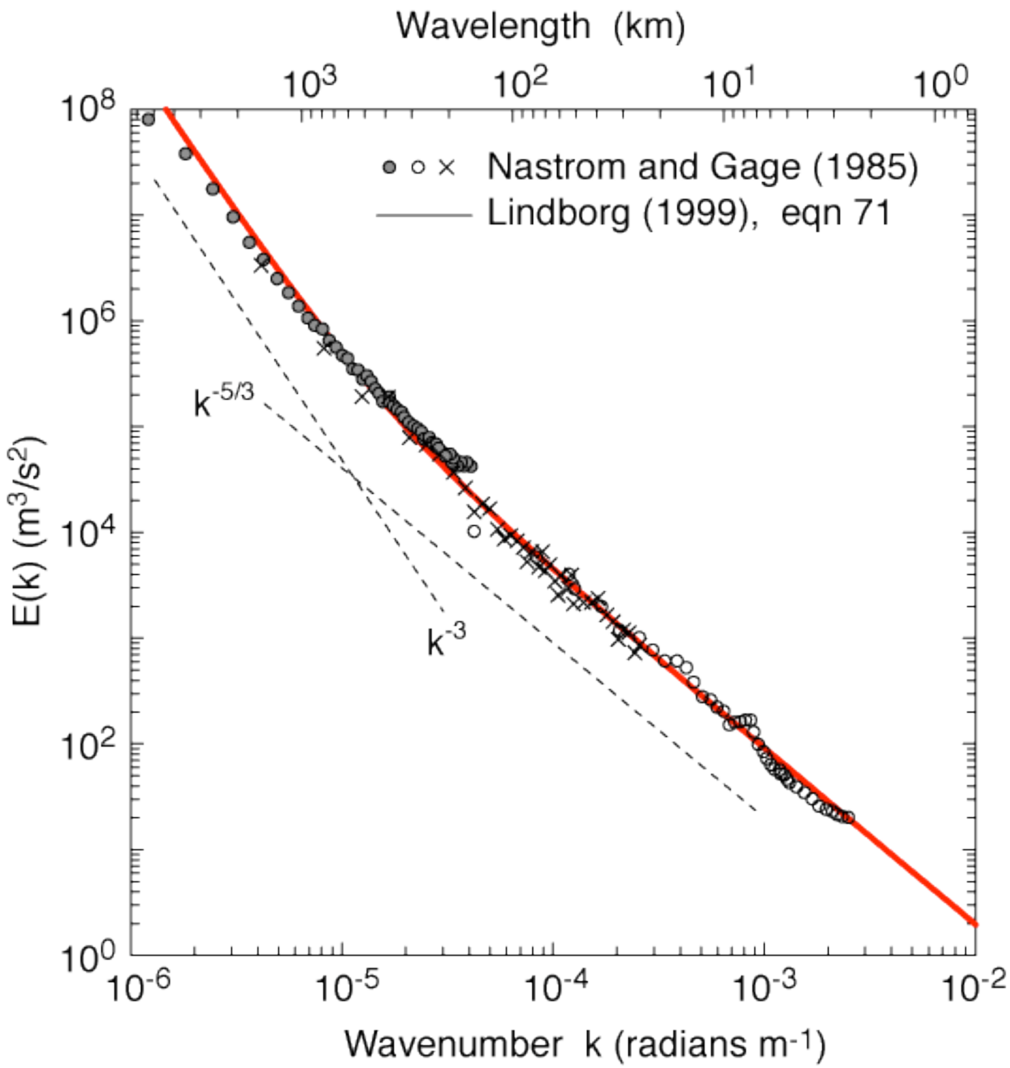
What is Wrong With Our Existing Global Models?

1. They do not scale to 10^4 - 10^5 processors. (e.g. lat-long grid models)
2. They were not constructed for mesoscale/cloudscale application.

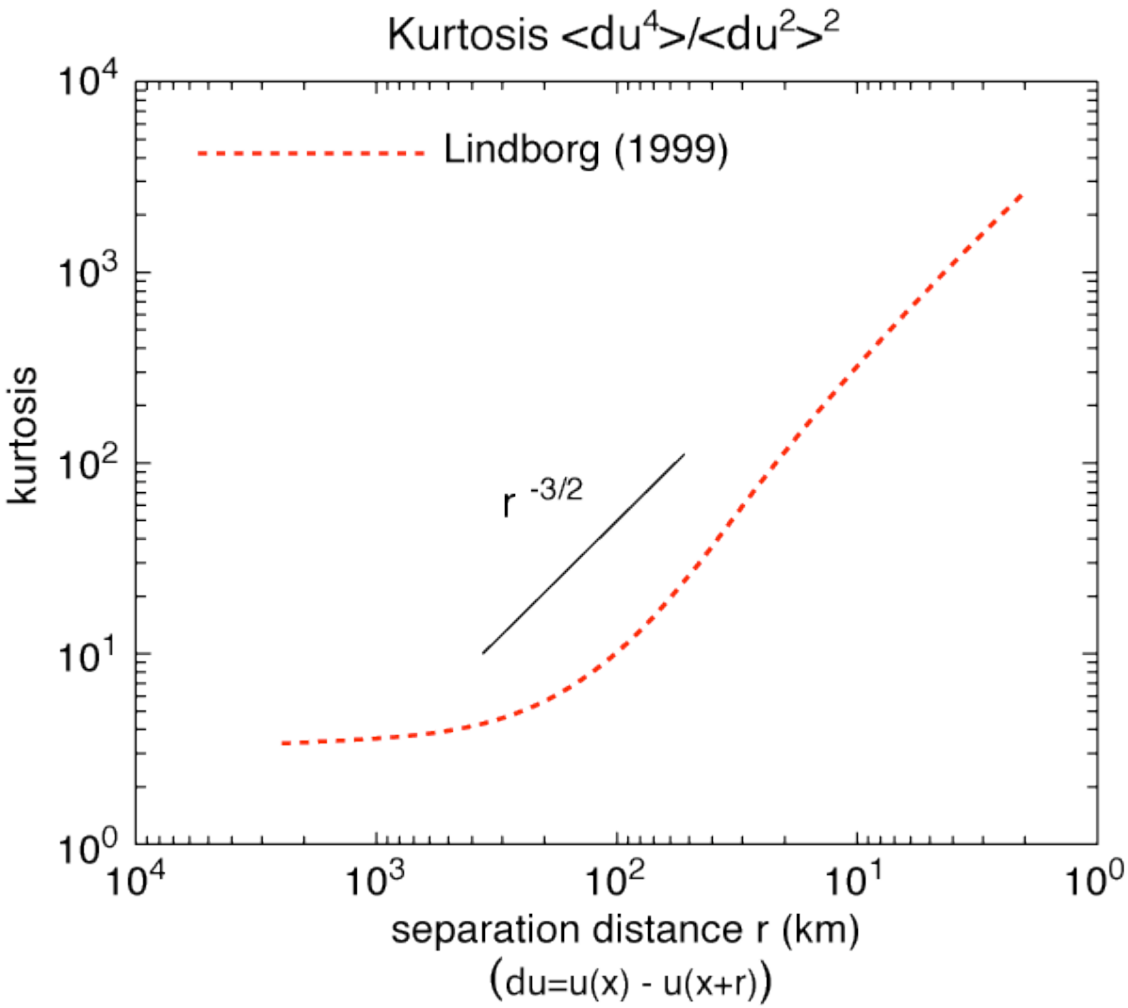
Kinetic Energy Spectra

Nastrom and Gage (1985)
Spectra computed from
GASP observations
(commercial aircraft)

Lindborg (1999) functional
fit from MOZAIC
observations (aircraft)



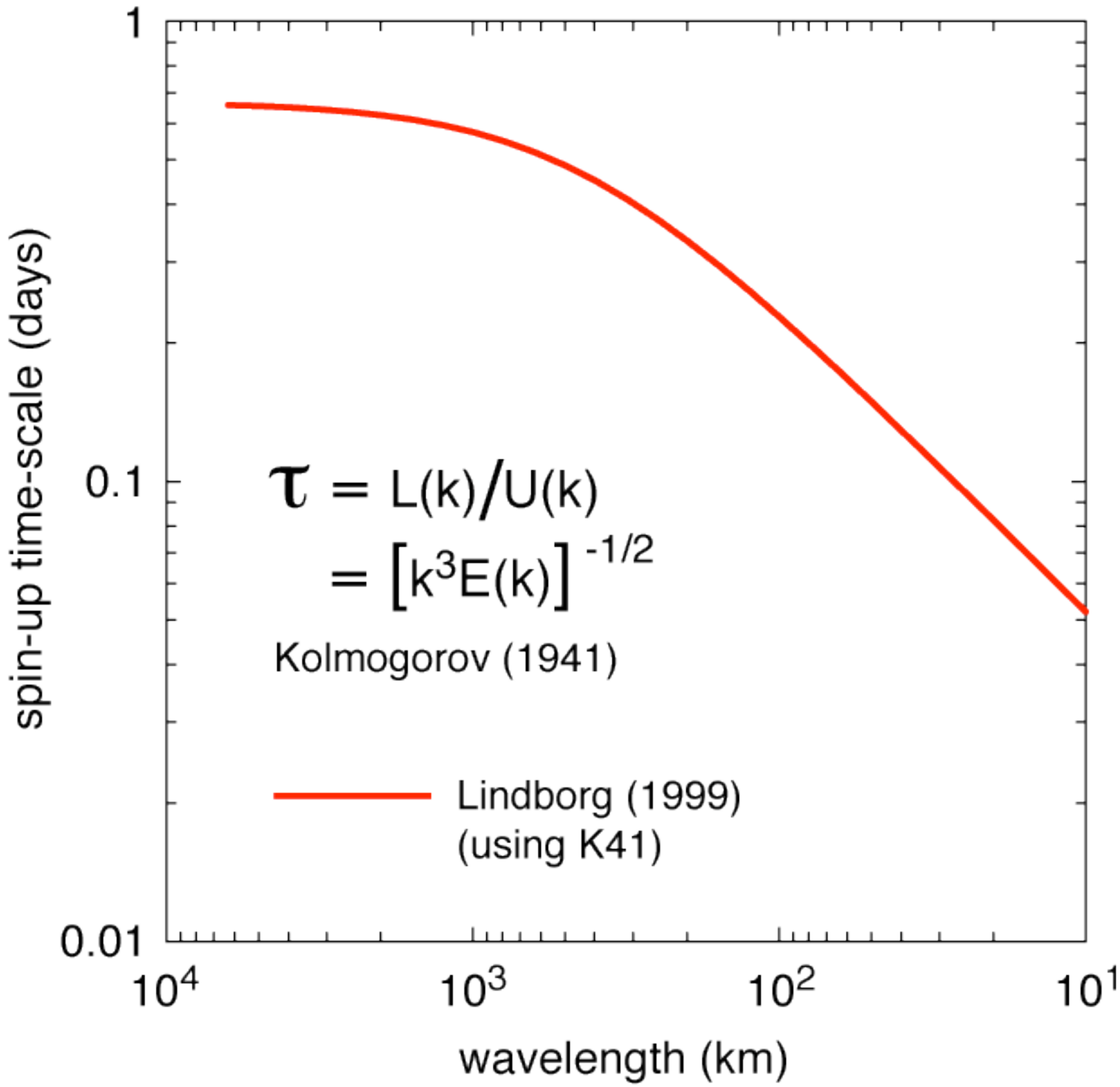
Structure Functions: Kurtosis (flatness factor)



Kurtosis = 3
for a Gaussian PDF

Strong growth of
the kurtosis
at small scales
indicates significant
intermittency.

Spectra Spin-up Time: Theory



WRF Decomposed Spectra
Spring Experiment 2005 Forecast

$$V = V_\psi + V_\chi + V_{def}$$

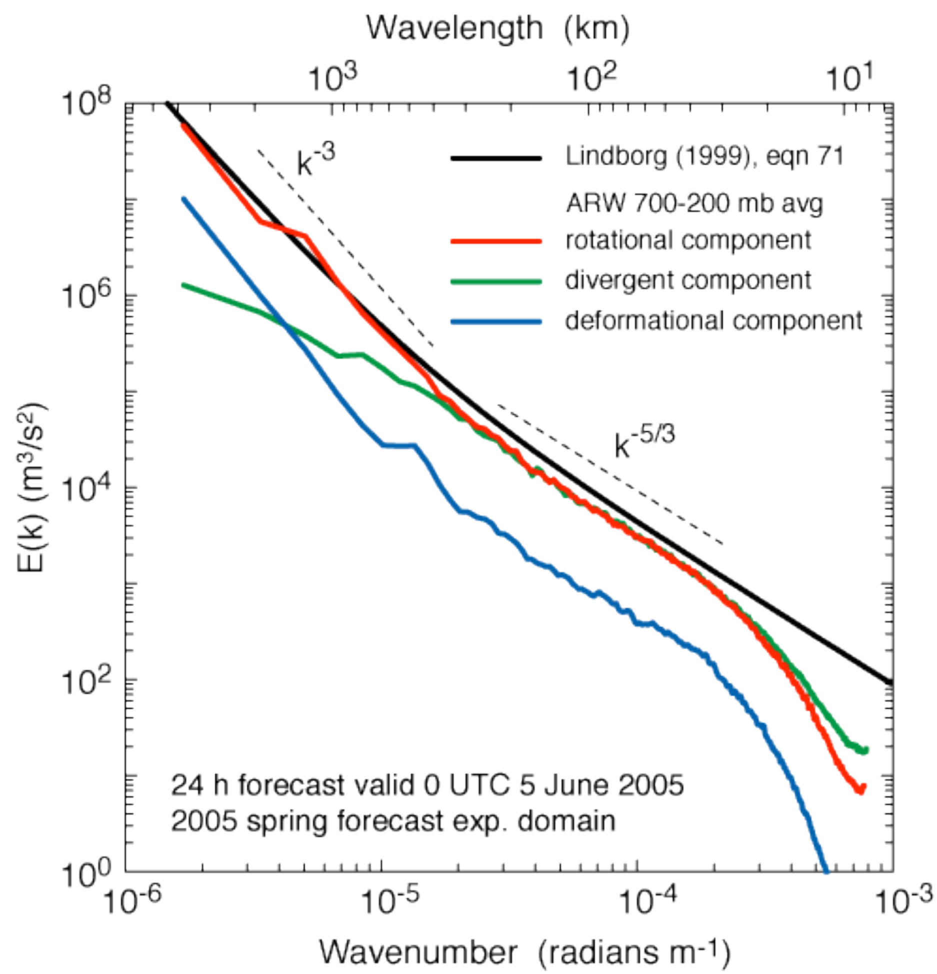
V_ψ rotational component

V_χ divergent component

V_{def} deformational component

$$V_\psi = k \times \nabla\psi$$
$$V_\chi = \nabla\chi$$
$$V_{def} = V - V_\psi - V_\chi$$

$$\nabla^2\psi = \zeta, \quad \zeta = k \cdot \nabla \times V$$
$$\nabla^2\chi = D, \quad D = \nabla \cdot V$$



Recap: Atmospheric Dynamics

1. KE spectra: Transition from k^{-3} (large scales) to $k^{-5/3}$ (meso- and smaller scales).
2. Kurtosis: Strong intermittency at meso- and smaller scales.
3. Turbulence theory + observed spectrum: Shorter timescale for spinup and predictability.
4. Model results (and some observations): KE spectrum is rotational at large scale, divergent at small scales.

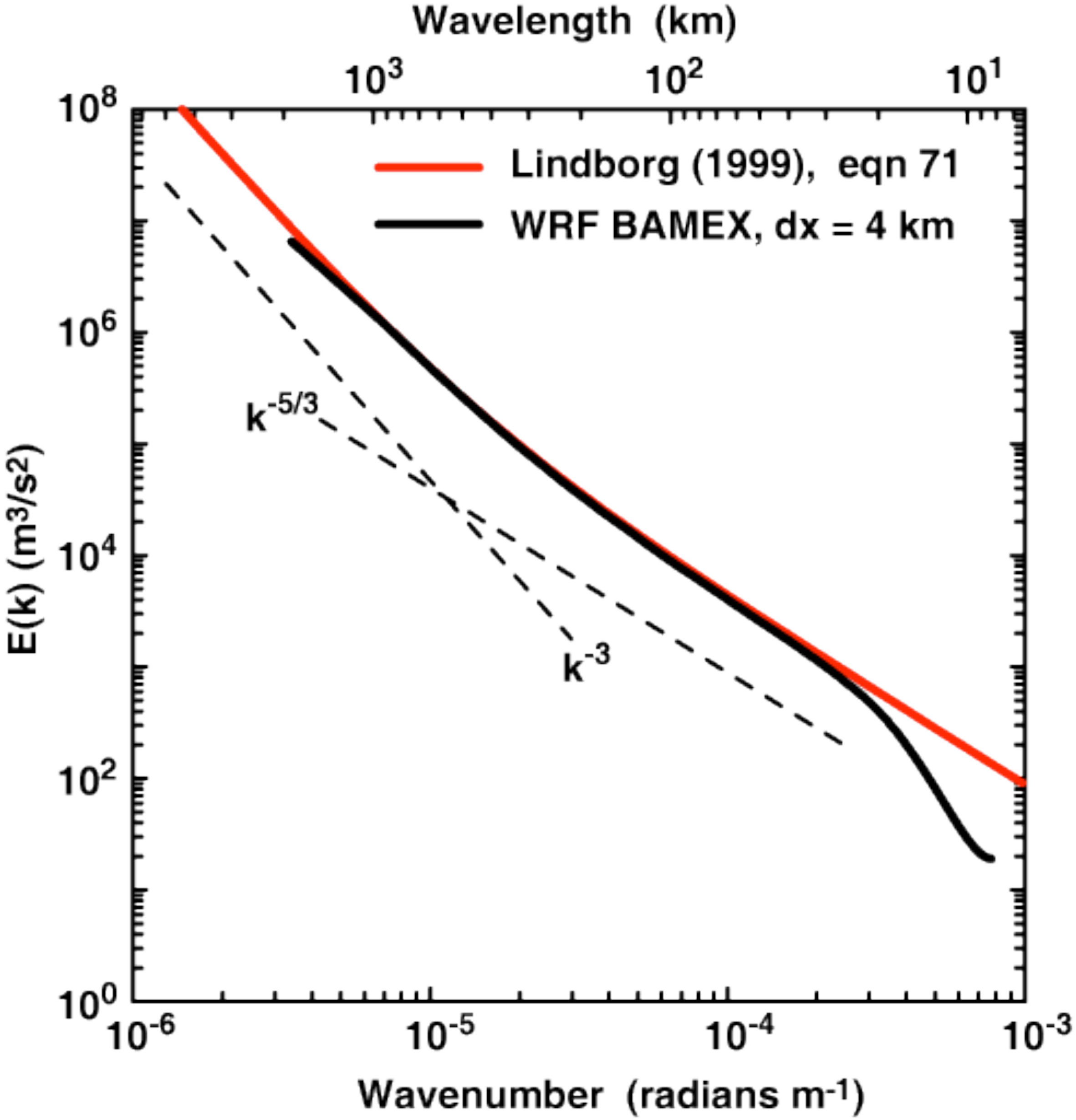
How well do atmospheric models reproduce these statistics?

1. Some models do well.
2. Some models do not do well.
3. For many models, we do not know.

Spectra for WRF-ARW BAMEX Forecasts, 5 May – 14 July 2003

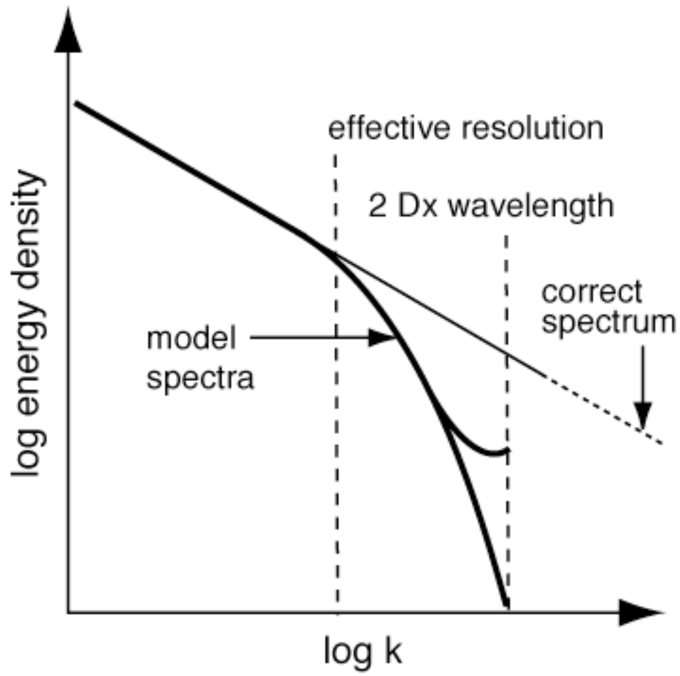
Average over approx. 4 – 9 km height, on model surfaces.

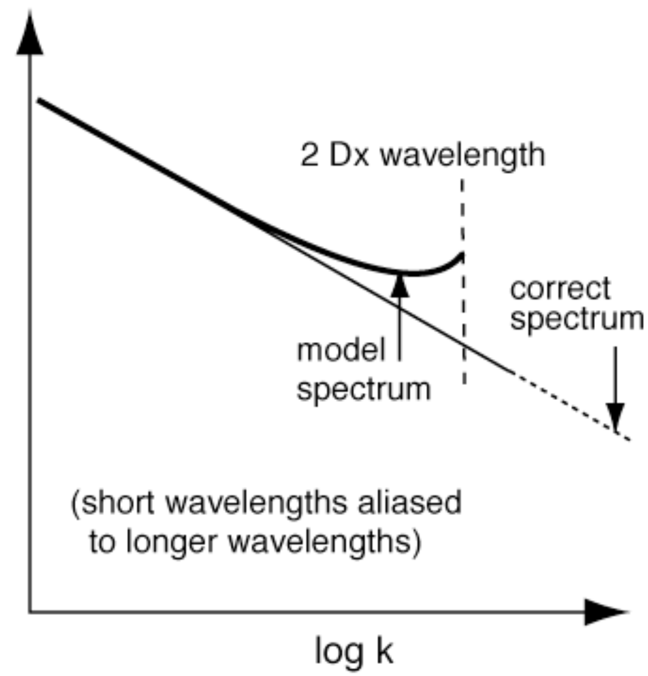
4 km WRF-ARW: 12 - 36 h forecast avg.



Spectral Characteristics and Effective Resolution

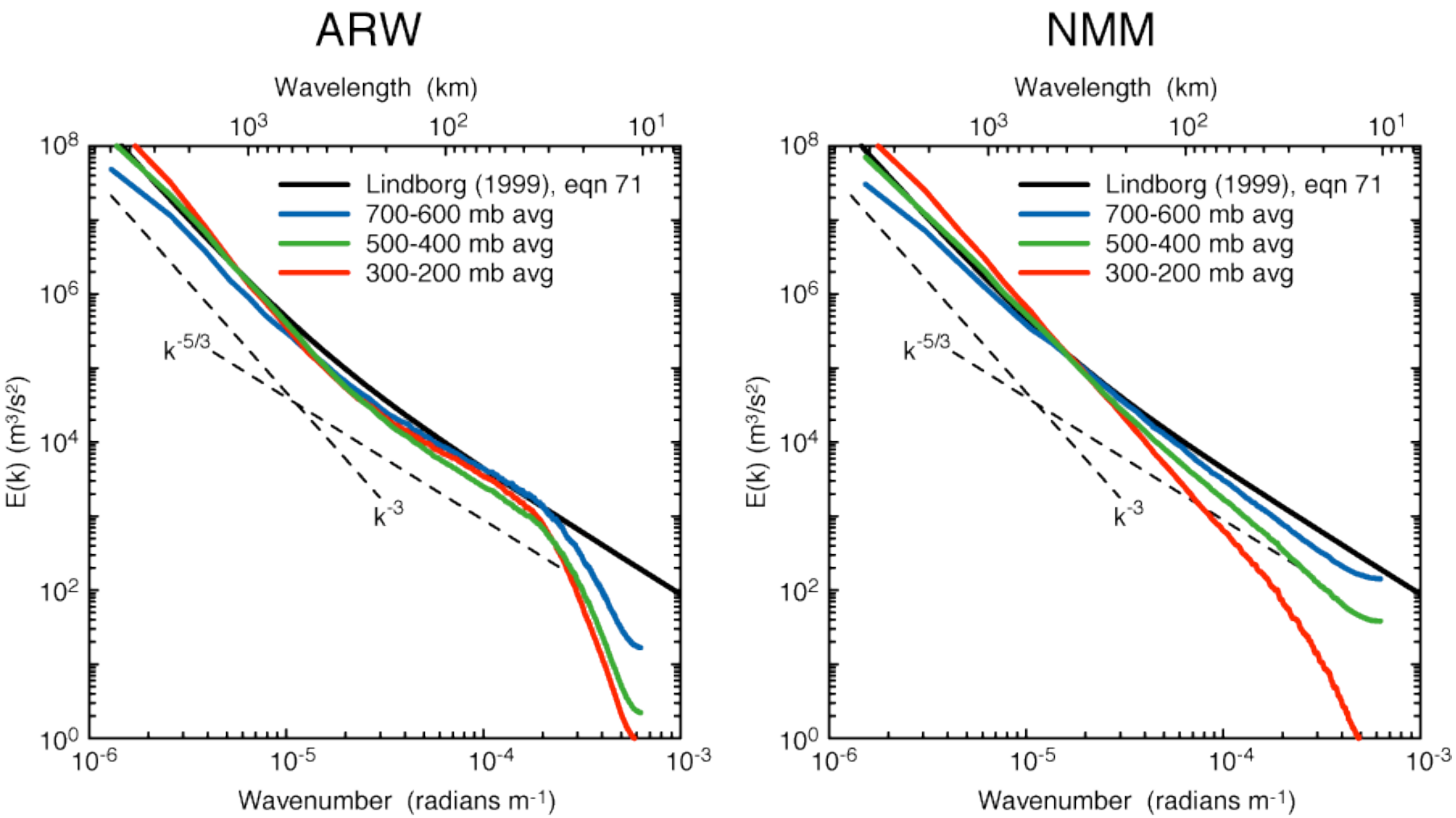
Schematic of some typical atmospheric spectra



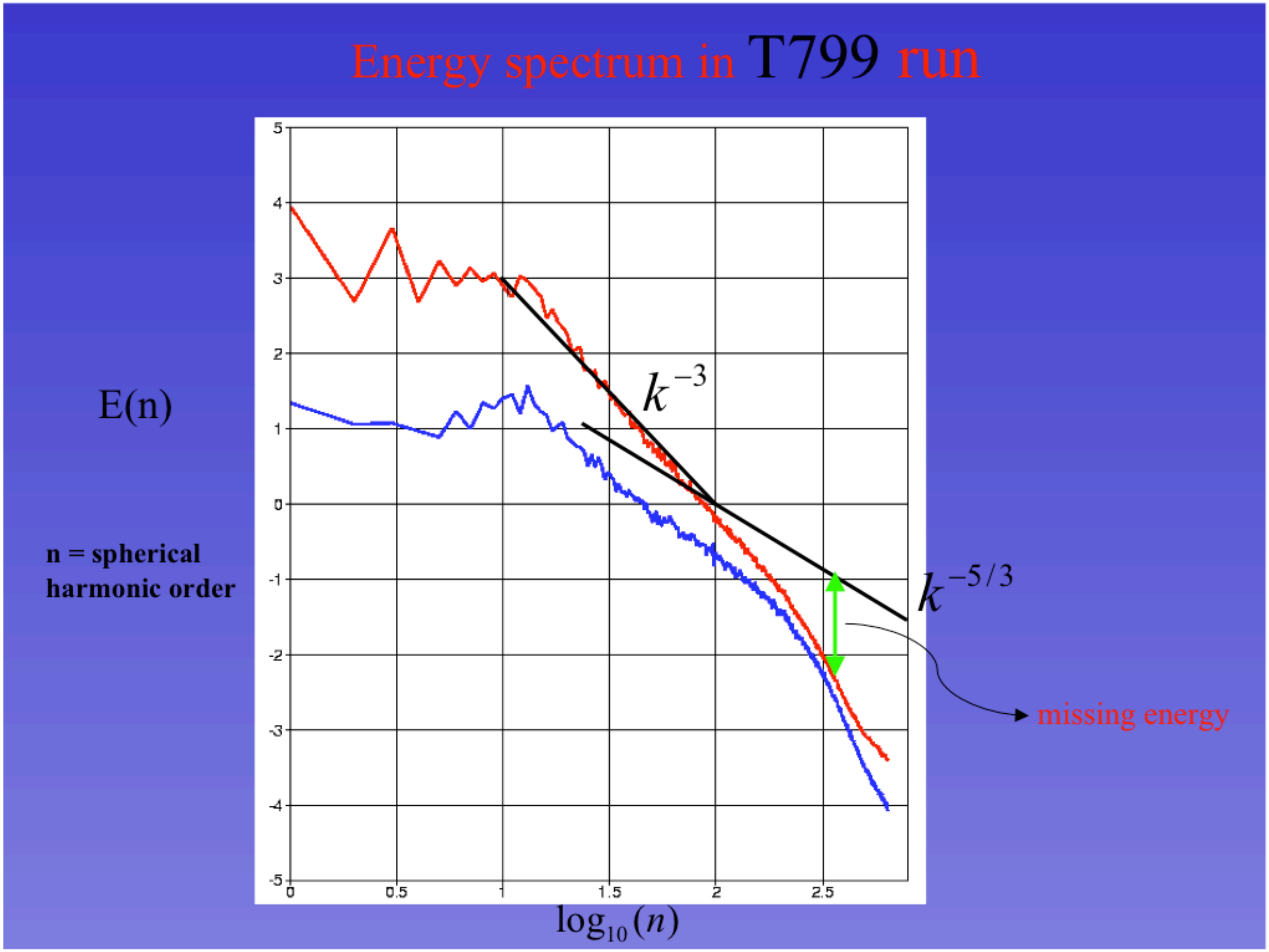


Spectra for WRF-ARW, -NMM
DWFE 7-25 January 2005 forecasts

(using 24, 27, 30, 33, 36, 39, 42 and 45 h forecast times)



ECMWF model



(courtesy of Tim Palmer, 2004)

Where are we?

- Some Eulerian models produce a k^{-3} - $k^{-5/3}$ spectral transition at mesoscale resolutions; effective resolution depends on filtering.
  - How should the filtering change with resolution?

- Other Eulerian models do not produce a clean $k^{-5/3}$ spectral transition - why?

- Semi-Lagrangian semi-implicit models?
  - At high resolution (dx ~ km's) the SLSI models show a transition.
  - At mesoscale resolutions (dx >= 10 km): No transition!

  What is causing this behavior in SLSI schemes?

Filtering in models

- Damping in time-integration schemes
- Filtering in interpolation schemes (SL)
- Dissipation implicit in transport schemes (temporal or spatial)
- Explicit filters

Spatial filters

$$\frac{\partial \phi}{\partial t} = \dots + \nu_2 \frac{\partial^2 \phi}{\partial x_i^2}; \quad \dots - \nu_4 \frac{\partial^4 \phi}{\partial x_i^4}$$

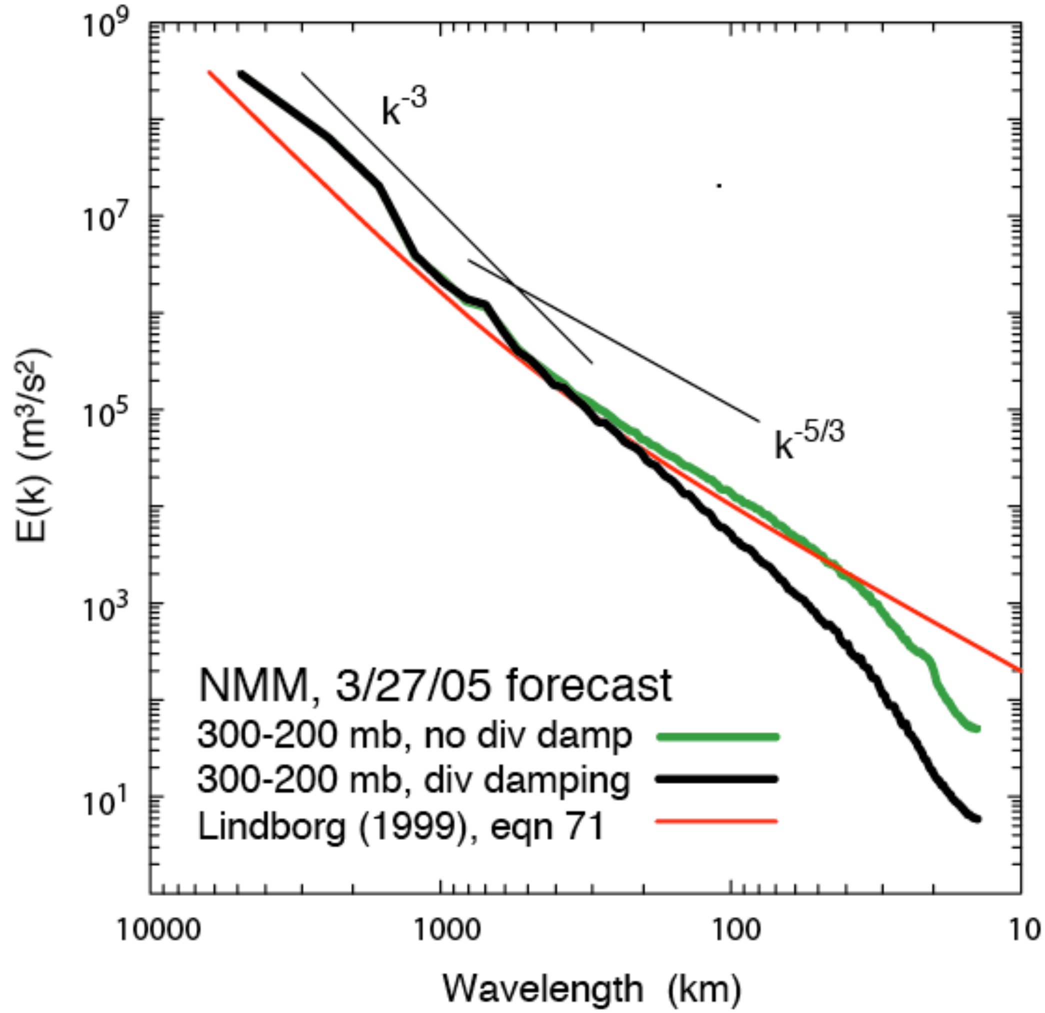
Horizontal divergence damping

$$\frac{\partial u_i}{\partial t} = \dots \; + \nu_d \frac{\partial}{\partial x_i} \nabla_h \cdot \mathbf{V}$$

Eulerian Cores
(NMM)

Horizontal divergence damping examples

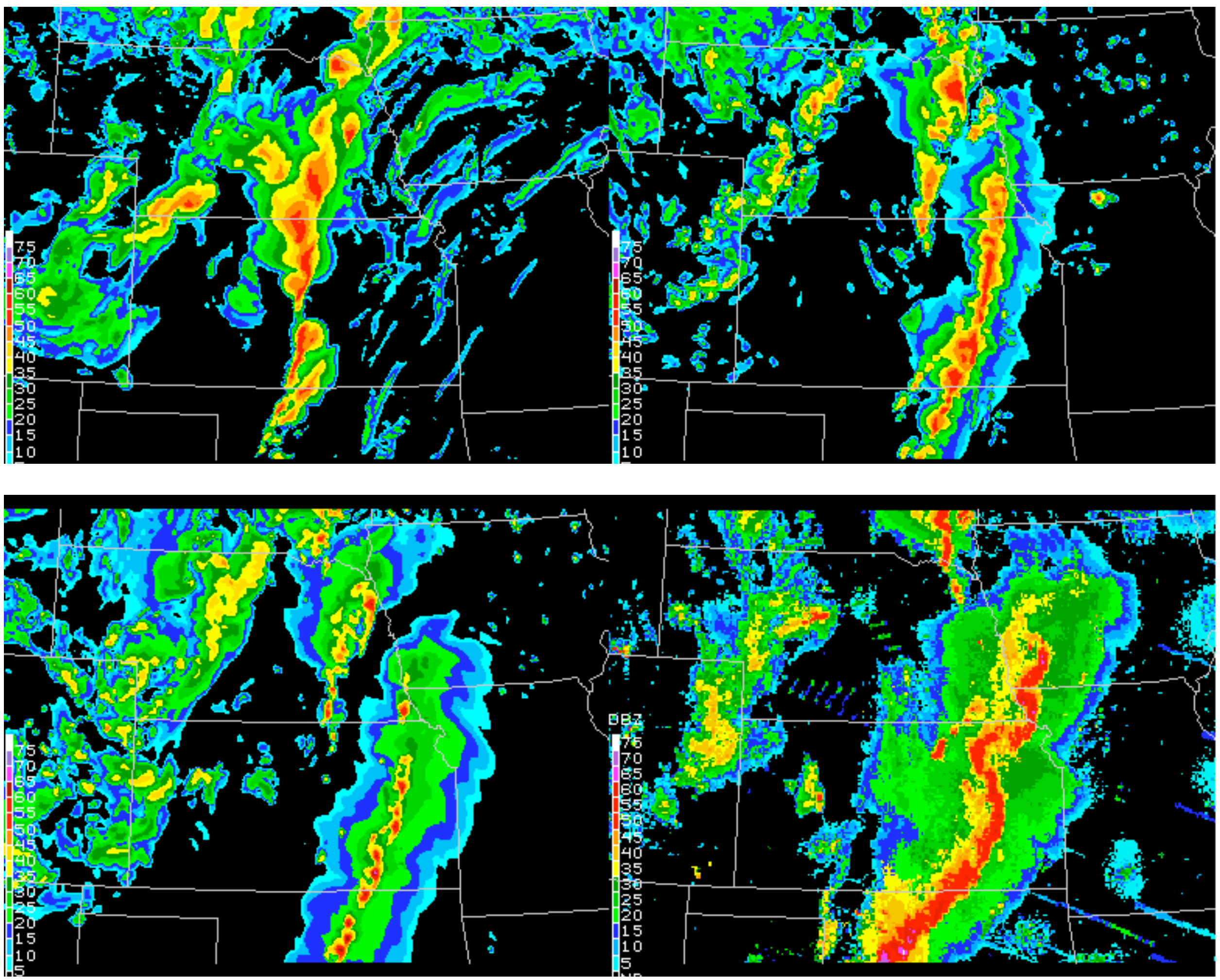
The current operational NWP model (regional North-American Model), 2005 Winter Forecast Experiment.



(Skamarock and Dempsey 2005)

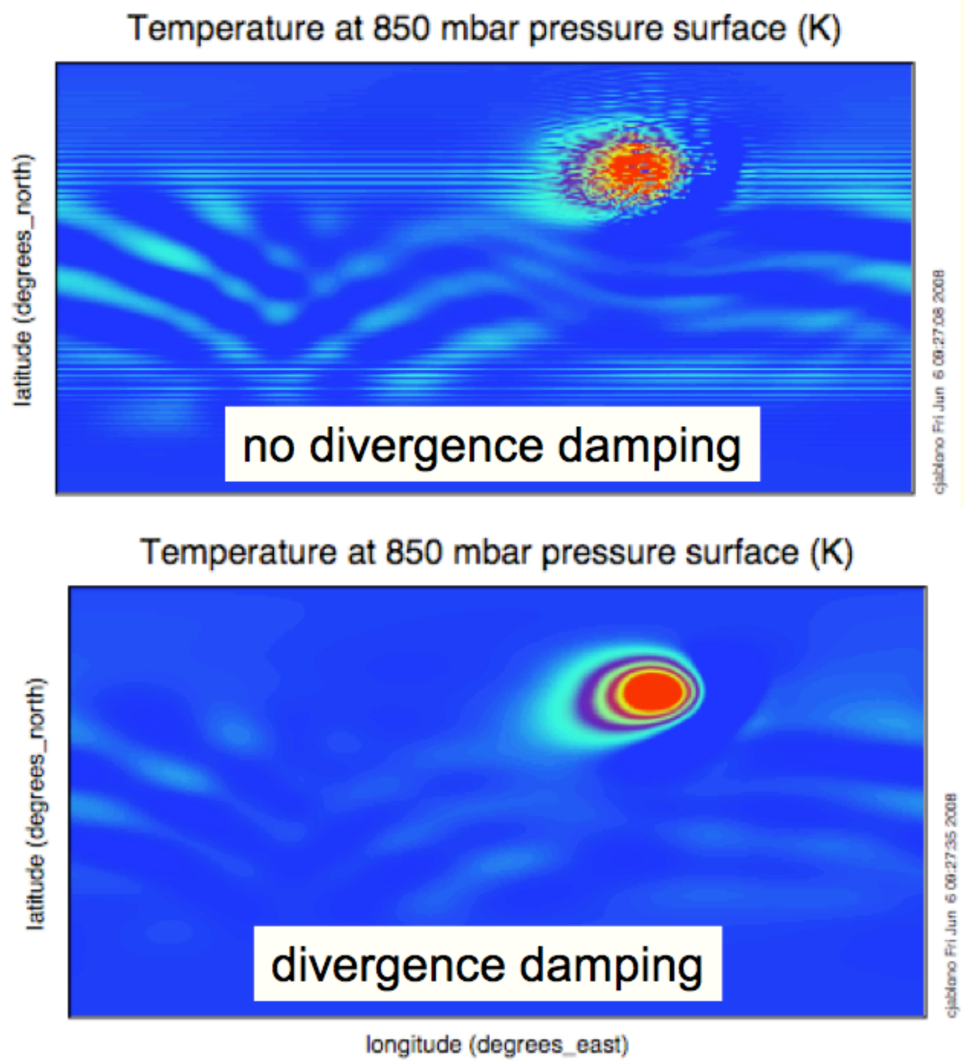
Which Model Uses Horizontal Divergence Damping?

Radar reflectivity (080606, 7 pm CST)



Eulerian Cores
(FV Core)

From C. Jablonowski

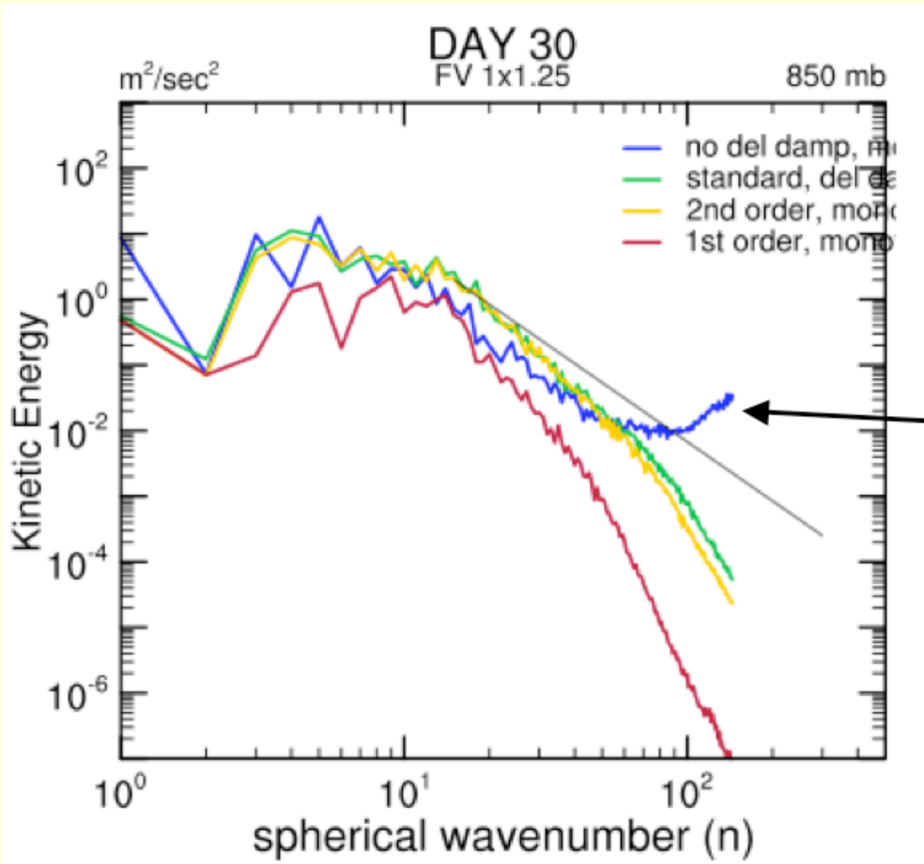


- Example: alternative 3D inertio-gravity wave test with background flow
- Model CAM FV 1°x 1° L20 at day 5.5, lat-lon cross section at 850 hPa
- Numerical stability of CAM FV depends on the resolution- and time step dependent choice of the divergence damping coefficient c

Eulerian Cores
(FV Core)

From C. Jablonowski, aquaplanet simulations

Effects of the divergence damping and order of accuracy on the Kinetic Energy spectrum (test 2-0-0)



Blue: PPM, **no** divergence damping

Green: PPM, standard divergence damping

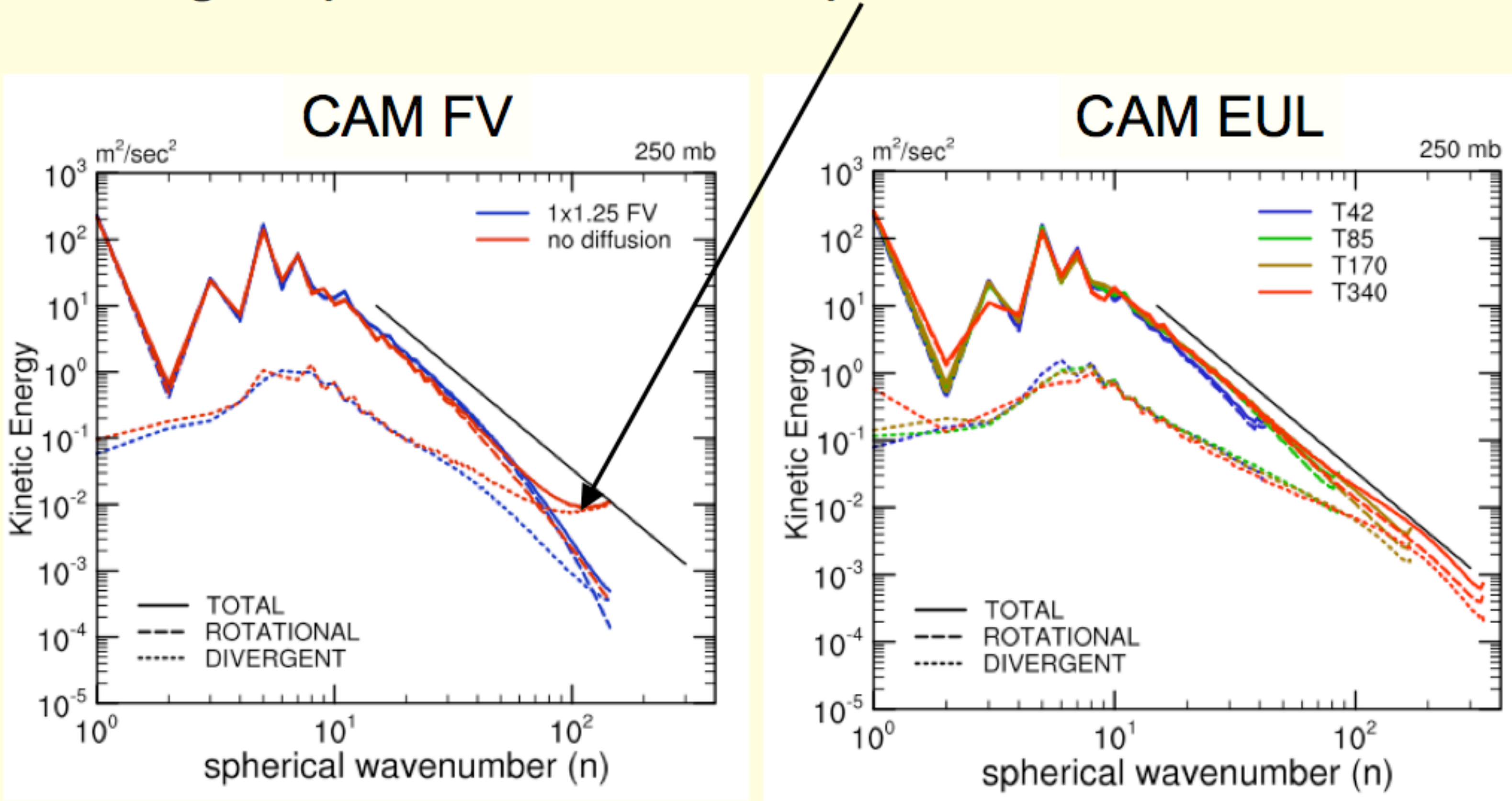
Accumulation of energy at small scales without divergence damping

Model: CAM FV, plot provided by D. Williamson (NCAR)

Eulerian Cores
(FV Core)

From C. Jablonowski, aquaplanet simulations

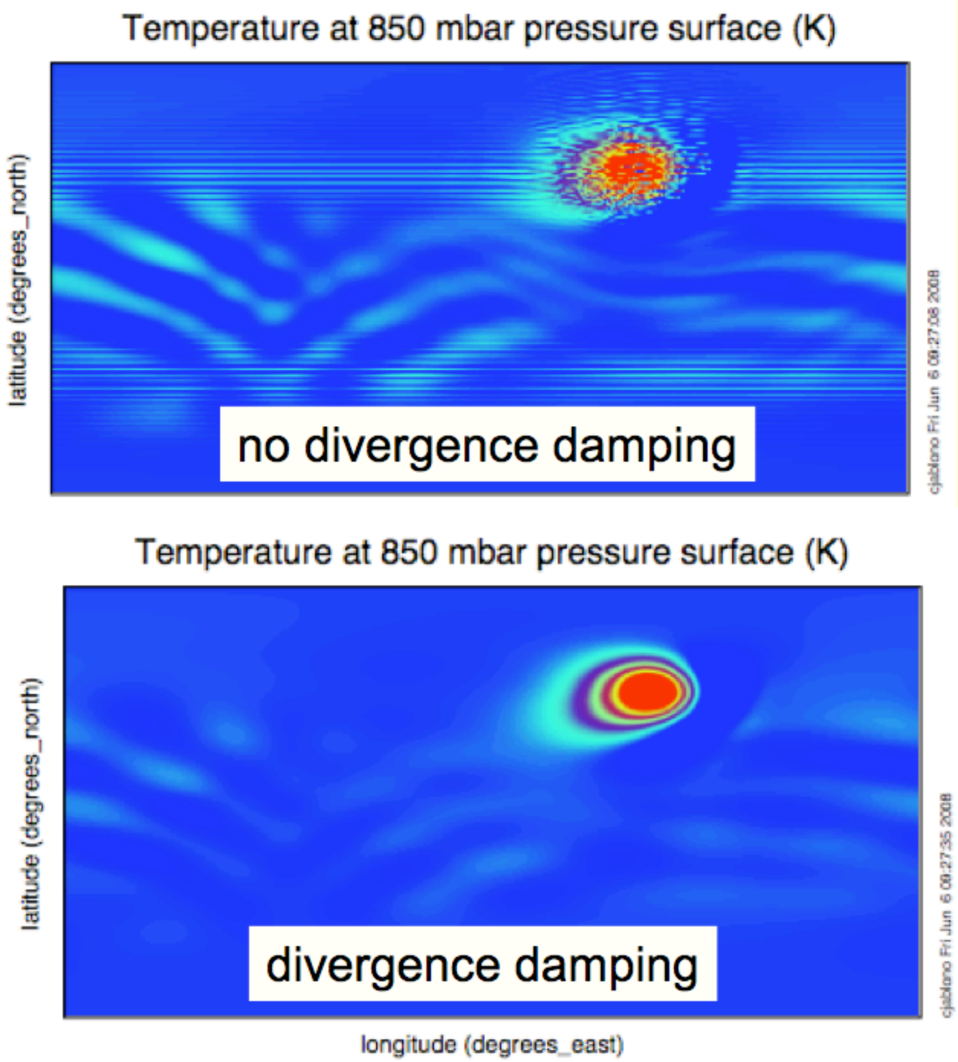
- Without diffusion (here divergence damping): divergent part of the flow responsible for the hook



plots provided by D. Williamson (NCAR)

Eulerian Cores
(FV Core)

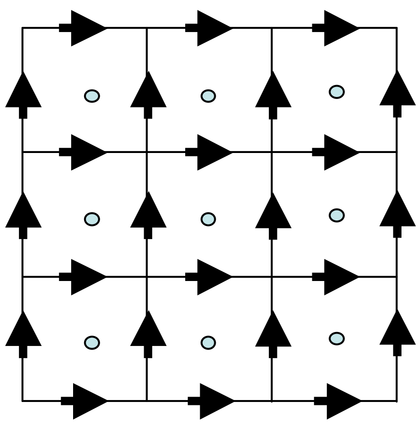
From C. Jablonowski



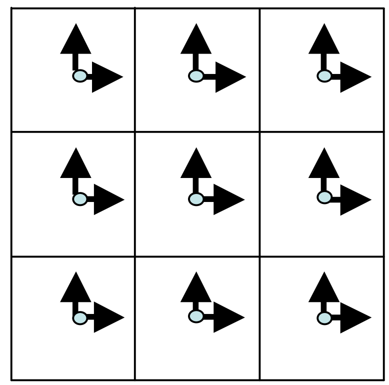
- Example: alternative 3D inertio-gravity wave test with background flow
- Model CAM FV 1°x 1° L20 at day 5.5, lat-lon cross section at 850 hPa
- Numerical stability of CAM FV depends on the resolution- and time step dependent choice of the divergence damping coefficient c

Eulerian Cores
(FV Core)

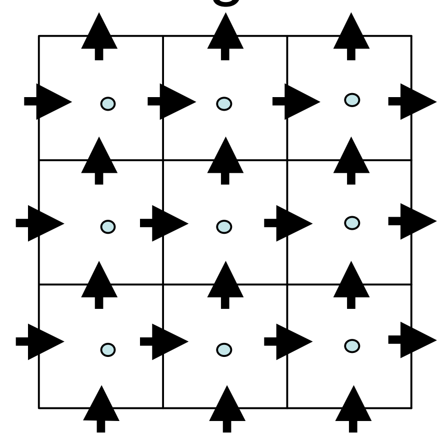
D grid (FV core)



A grid



C grid



Divergence operators:

$$\nabla \cdot V = \delta_x \overline{\overline{u}^x}^y + \delta_y \overline{\overline{v}^x}^y$$
$$\overline{\overline{\delta_x u}^x}^y + \overline{\overline{\delta_y v}^x}^y$$
$$\overline{\overline{\delta_x u + \delta_y v}^x}^y$$

$$\nabla \cdot V = \delta_x \overline{u}^x + \delta_y \overline{v}^y$$
$$\overline{\delta_x u}^x + \overline{\delta_y v}^y$$

$$\nabla \cdot V = \delta_x u + \delta_y v$$

Semi-Lagrangian models
Consider the 1D linear shallow-water equations…

Continuous equations

$$\text{linearize} \;\rightarrow\; \mathrm{U} = U + u(x,t) \qquad \frac{du}{dt} + g\frac{\partial h}{\partial x} = 0$$

$$\mathrm{H} = H + h(x,t) \qquad \frac{dh}{dt} + H\frac{\partial u}{\partial x} = 0$$

SLSI discretization

$$u^{t+\Delta t} = \left(u^t - \frac{1-\epsilon}{2}\Delta t g \delta_x h\right)\Bigg|_d^t - \left(\frac{1+\epsilon}{2} g\delta_x h\right)\Bigg|^{t+\Delta t}$$

$$h^{t+\Delta t} = \left(h^t - \frac{1-\epsilon}{2}\Delta t H \delta_x u\right)\Bigg|_d^t - \left(\frac{1+\epsilon}{2} H\delta_x u\right)\Bigg|^{t+\Delta t}$$

SLSI Amplification Factor (Gravel et al, MWR 1993)

$$\frac{E}{\rho} = \frac{1 - \gamma_3(1-\epsilon^2) \pm 2\gamma_3^{1/2}}{1 + \gamma_3(1+\epsilon)^2}$$

where

E is the amplification factor

ρ is the response function for the SL advection

$$\gamma_3 = gHK^2\left(\frac{\Delta t}{2}\right)^2$$

$$\boxed{|\rho| \leq 1 \;\text{ and }\; 0 \leq \epsilon \leq 1 \;\; \rightarrow \text{absolutely stable}}$$

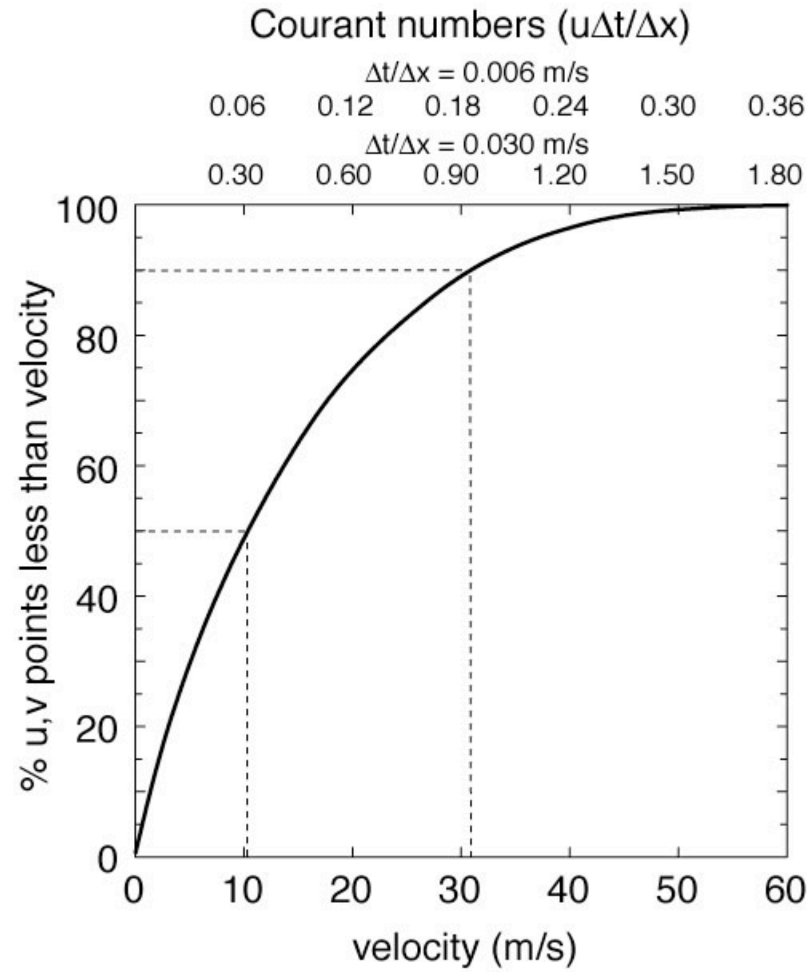
for stability, $0.1 \leq \epsilon \leq 0.2$ in NWP models

Time-steps and Courant Numbers

Damping arises from temporal off-centering and SL advection

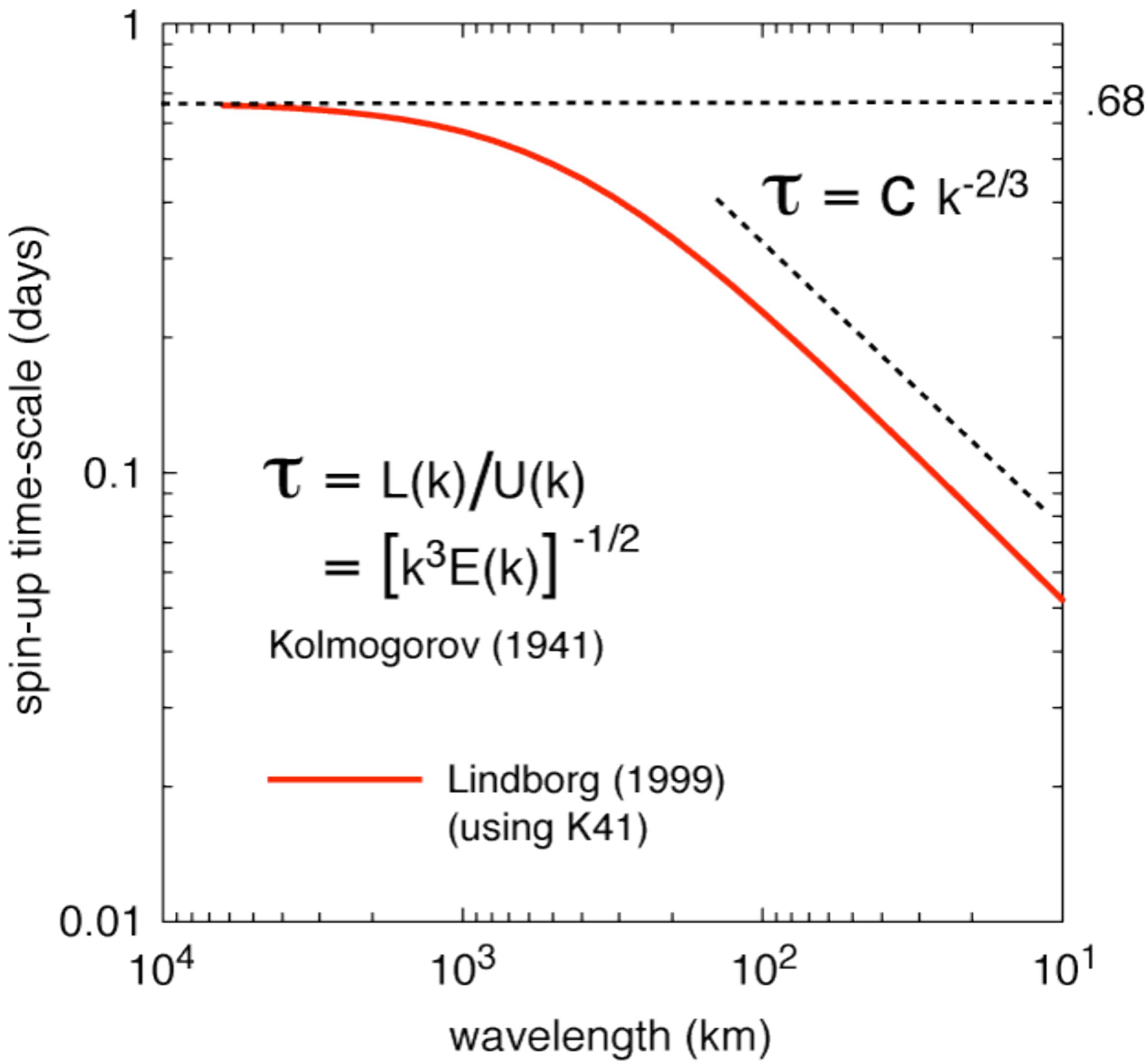
Typical Eulerian model
Adv. Courant numbers
0 < |Cr| < .2 (>90%)

Typical SLSI model
Adv. Courant numbers
0 < |Cr| < 1 (>90%)



Velocity distribution from 22 January DWFE CONUS WRF forecast.

Spectra Spin-up Time: Theory

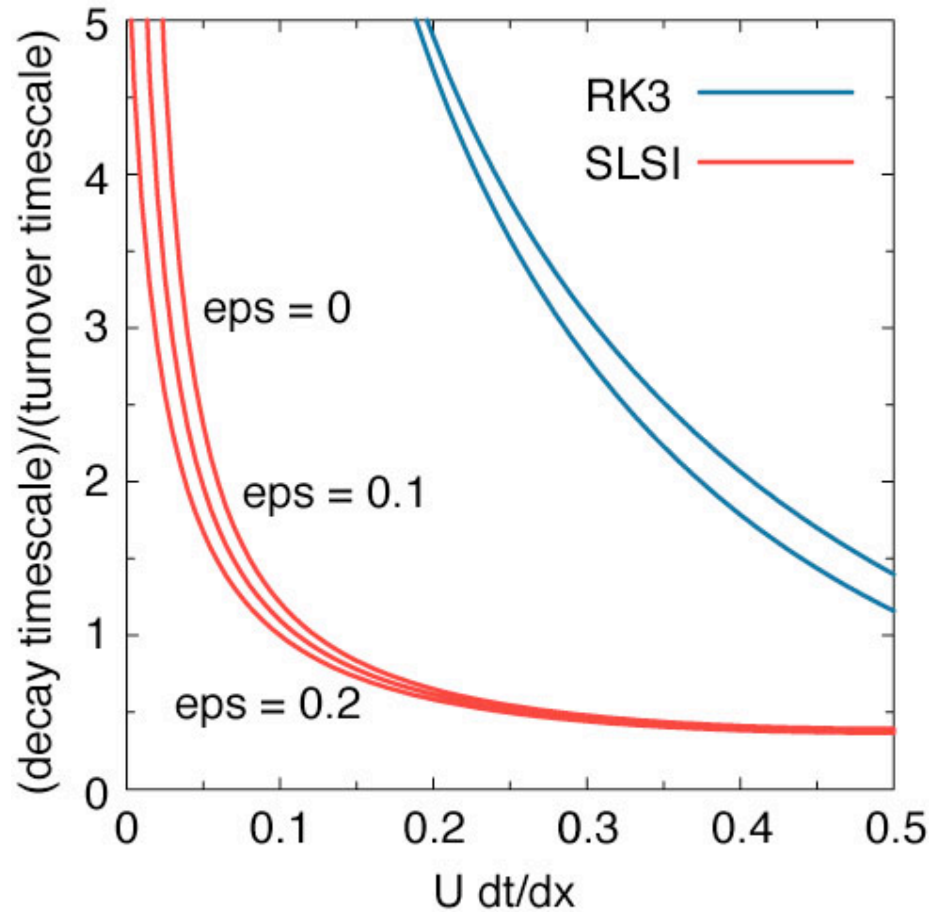


Damping in SLSI schemes

Consider a gravity wave…

10 km grid, 60 s RK3 timestep
60 s SLSI timestep
C dt/dx = 0.1
(C=16.67 m/s)
80 km wavelength
300 min eddy turnover time
Cubic SL interpolation

Result: Using an Eulerian timestep, damping in SLSI models arises almost entirely from the interpolation in the SL advection.

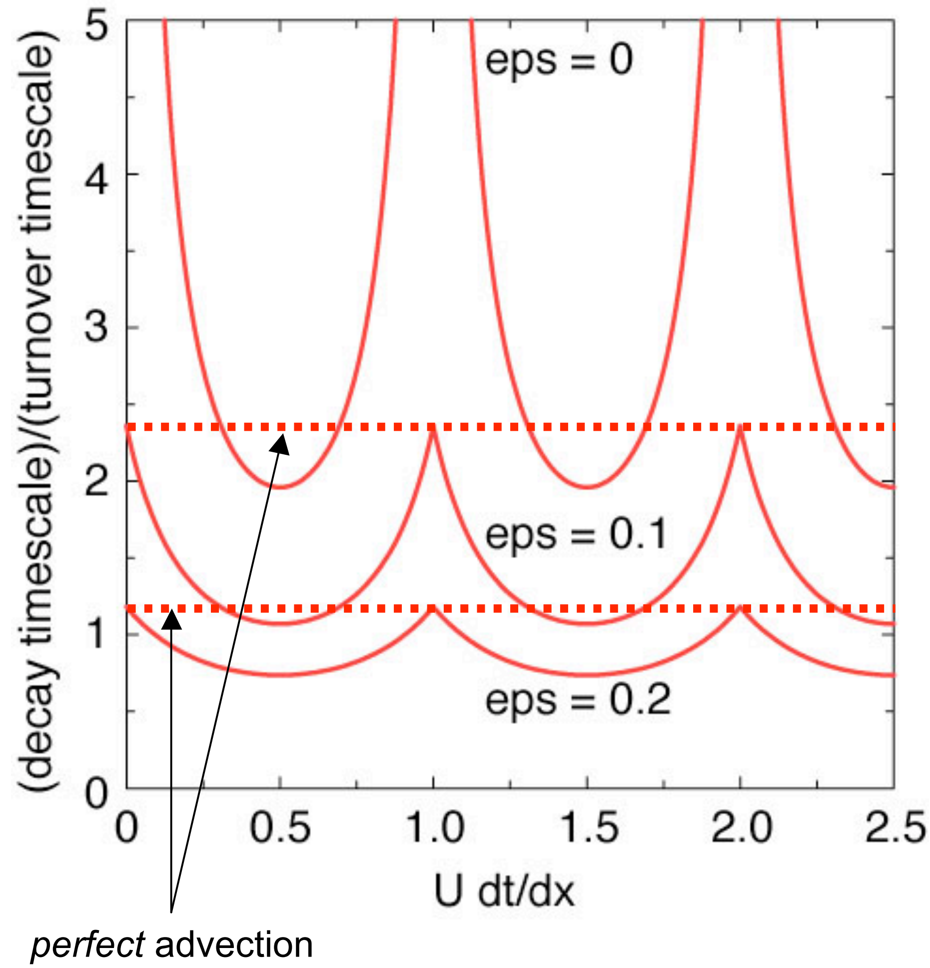


Damping in SLSI schemes

Consider a gravity wave…

10 km grid,
300 s SL timestep
C dt/dx = 0.5 (SLSI)
(C=16.6667 m/s)
80 km wavelength
300 min eddy turnover time
Cubic SL interpolation

Result: Using a typical SLSI timestep, damping in SLSI models arises primarily from the semi-implicit time-step off-centering



ECMWF model

Energy spectrum in T799 run

E(n)

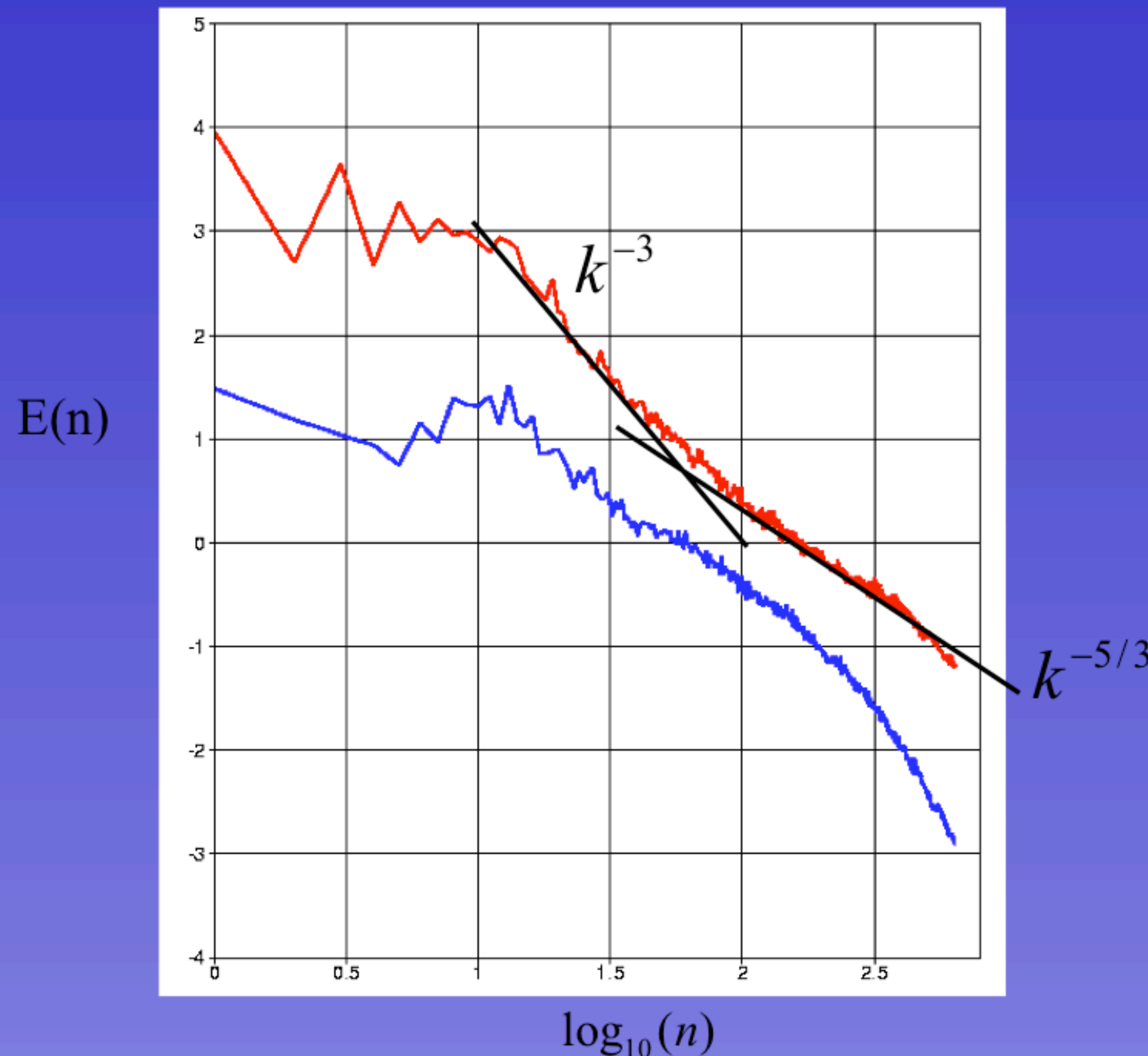
n = spherical harmonic order

$\log_{10}(n)$

(courtesy of Tim Palmer, 2004)

ECMWF model

Energy spectrum in ECMWF forecast model with backscatter



(courtesy of Tim Palmer, 2004)

Eulerian and SLSI schemes:

- Horizontal divergence damping inappropriate for meso/cloud scales.

SLSI schemes:

- Difficulties resolving spectral transition at mesoscale resolutions.
  - Eulerian timesteps - significant damping from interpolations (SL)
  - SL timesteps - significant damping from time-off-centering (SI)
- Alternatives?

Eulerian schemes:

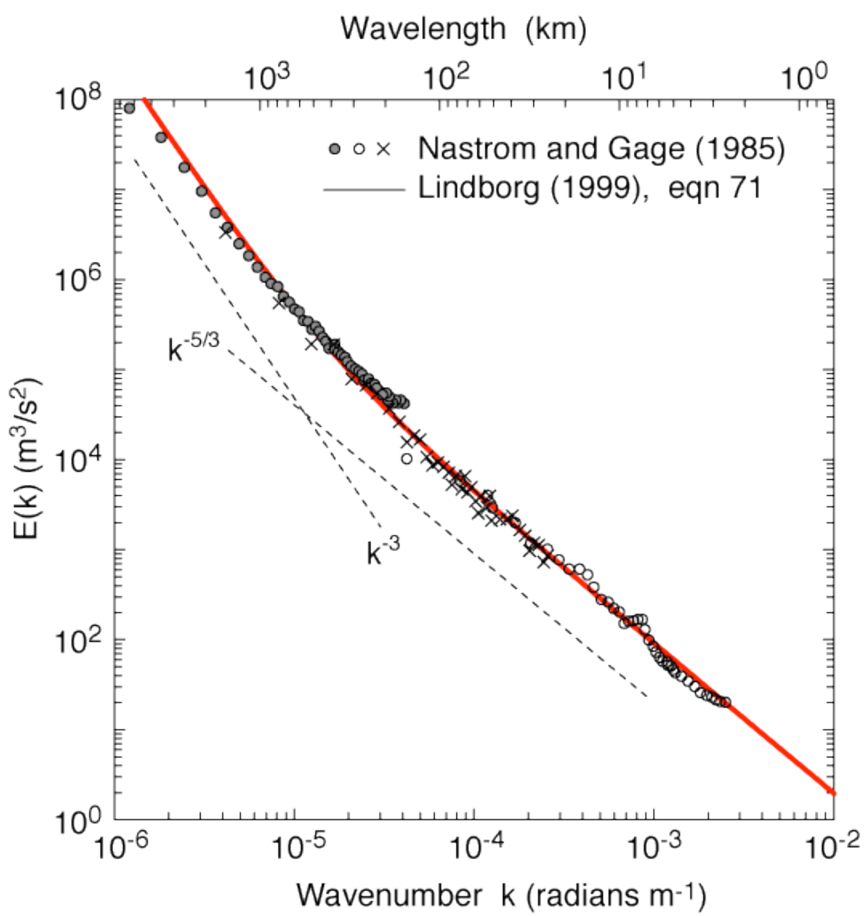
- More flexibility for “tuning” dissipation.
  - RK3, Leapfrog time-split schemes generally resolve mesoscale transition.
- Need tuning (additional dissipation) at cloud-permitting scales.

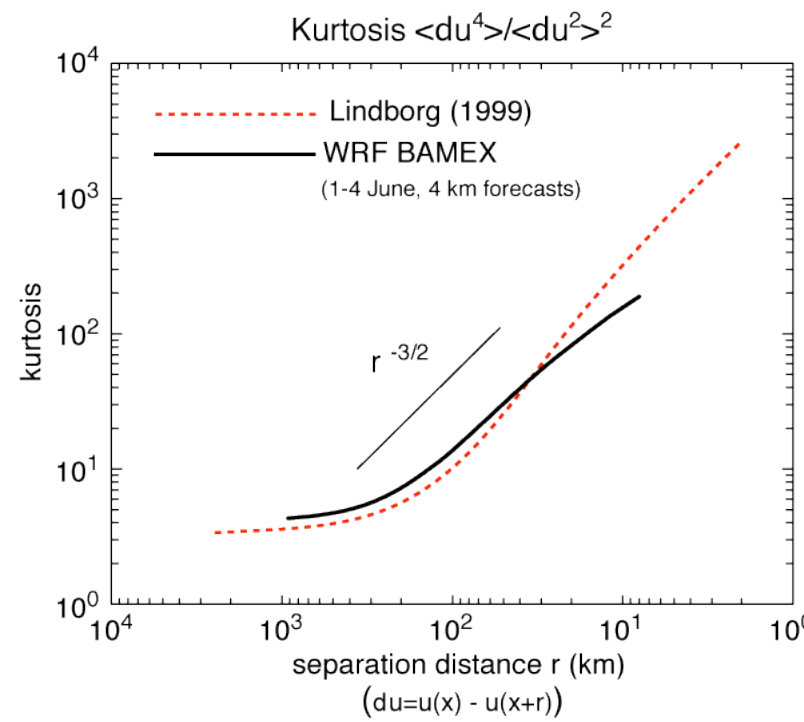
Mesoscale-Cloudscale Energetics:

- What is the character of the turbulence? (how do we parameterize it?)

Kinetic Energy Spectra and Model Filters

- Filters affect a model's ability to reproduce observed energetics.
- Large-scale and meso/cloud-scale energetics are fundamentally different.
- Global applications are moving to meso- and cloud- scale.





Filtering in Atmospheric Models

increasing length/time scales

climate

nwp (global)

scale	grid length (dx)	explicit filters	Implicit filters
		Weak/No theoretical basis	**Weak/No theoretical basis**
synoptic	dx > 50 - 100 km	n^{th} order spatial filters Smagorinsky (viscosity ~ deformation)	horz. divergence damping, temporal filtering, damping adv. schemes (SL, FCT, WENO, other upwind schemes)
mesoscale	dx < 50 - 100 km	n^{th} order spatial filters Smagorinsky (viscosity ~ deformation)	
hydrostatic			
nonhydrostatic			
cloud-scale	dx < 5 - 10 km dx > 200 m	n^{th} order spatial filters Smagorinsky (viscosity ~ deformation) LES-type subgrid mixing	temporal filtering, damping adv. schemes (SL, FCT, WENO, other upwind schemes)
LES	dx < 200 m dx > cm's	LES subgrid mixing model	
DNS	dx ~ cm's or less	Full Navier-Stokes, No approximations.	
		Strong theoretical basis	

Do some of these schemes adequately mimic LES models?

decreasing length/time scales