AOSS 321, Winter 2009
Earth System Dynamics

Lecture 5
1/22/2009

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Class News

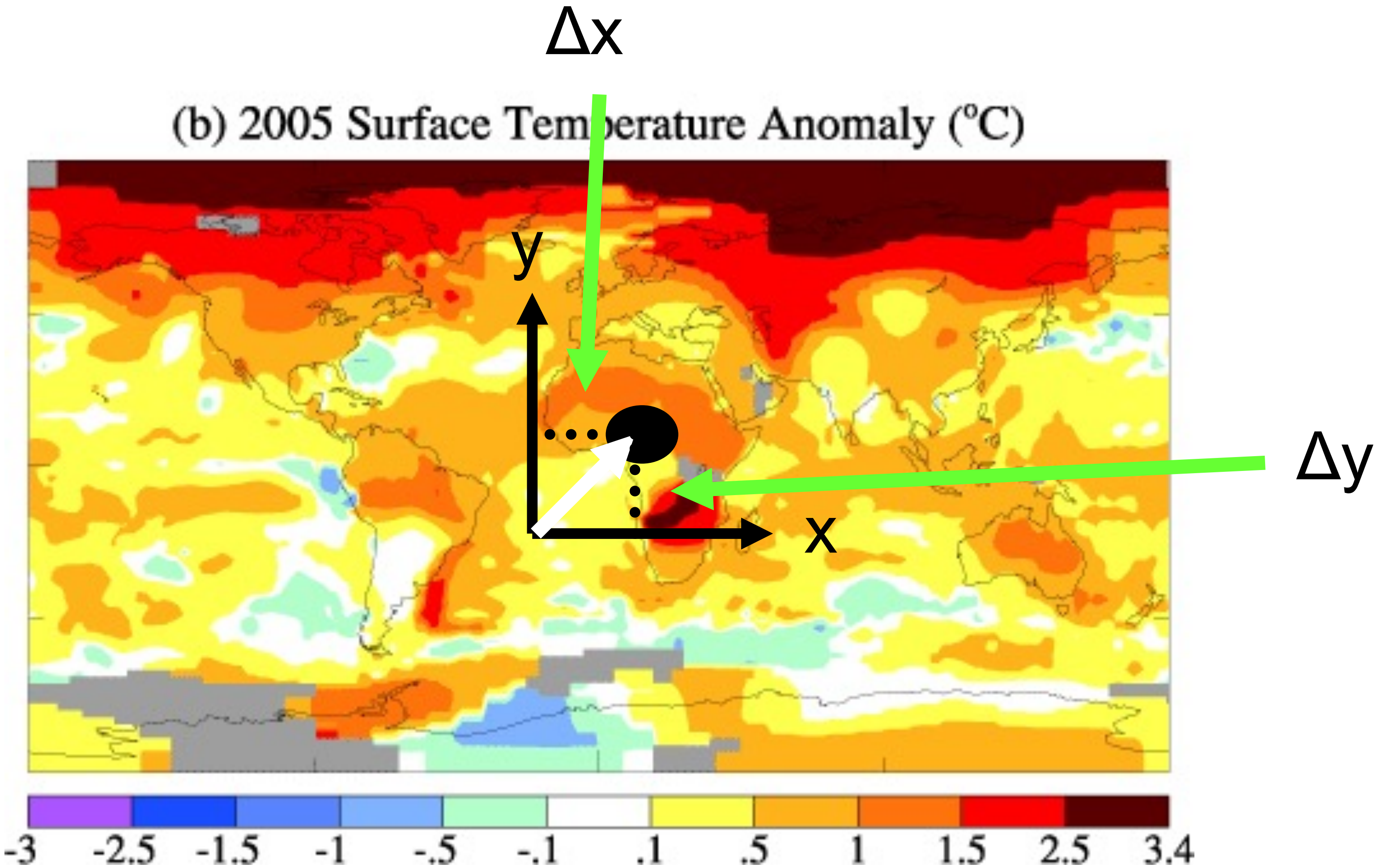
- Class web site: https://ctools.umich.edu/portal
- HW 1 due today
- Homework #2 posted today, due on Thursday (1/29) in class
- Our current grader is Kevin Reed (kareed@umich.edu)
- Office Hours
  - Easiest: contact us after the lectures
  - Prof. Jablonowski, 1541B SRB: Tuesday after class 12:30-1:30pm, Wednesday 4:30-5:30pm
  - Prof. Hetland, 2534 C.C. Little, TBA

Today's class

- Definition of the the Total (Material) Derivative
- Lagrangian and Eulerian viewpoints
- Advection
- Fundamental forces in the atmosphere:
  Surface forces:
  - Pressure gradient force
  - …

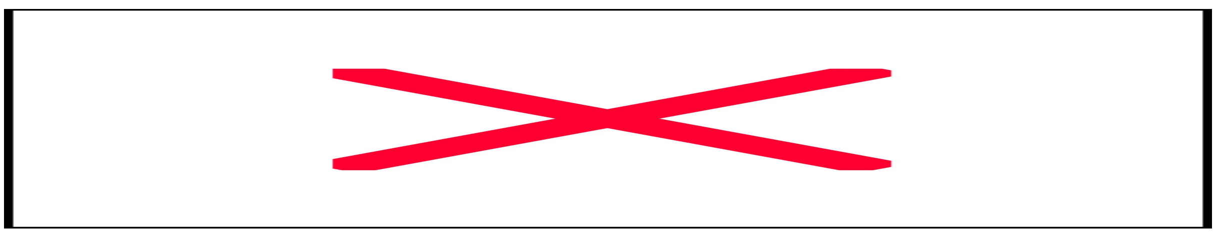
Total variations

Consider some parameter, like temperature, T



If we move a parcel in time Δt

Using Taylor series expansion

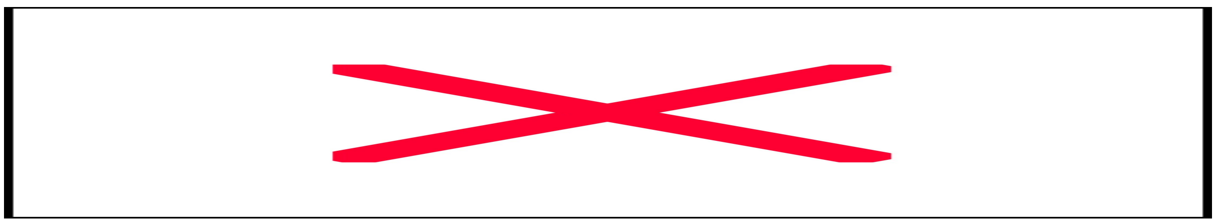


Higher Order Terms

Assume increments over Δt are small, and ignore Higher Order Terms

Total derivative

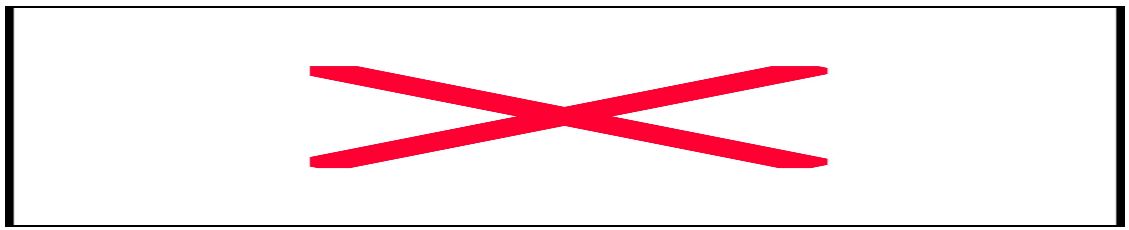
Total differential/derivative of the temperature T,
T depends on t, x, y, z



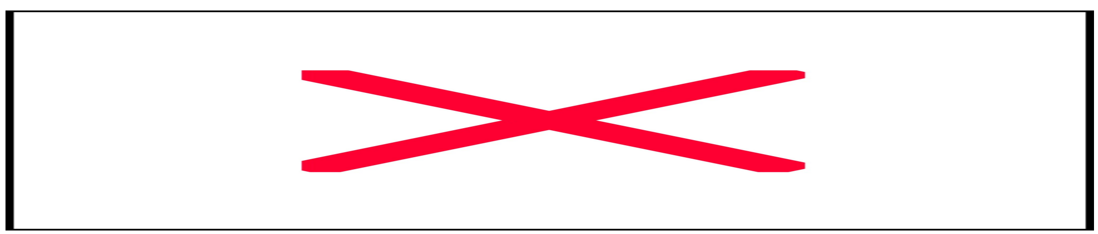
Assume increments over Δt are small

Total Derivative

Divide by Δt

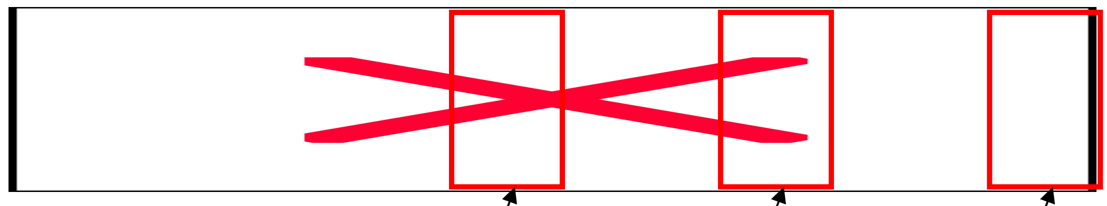


Take limit for small Δt



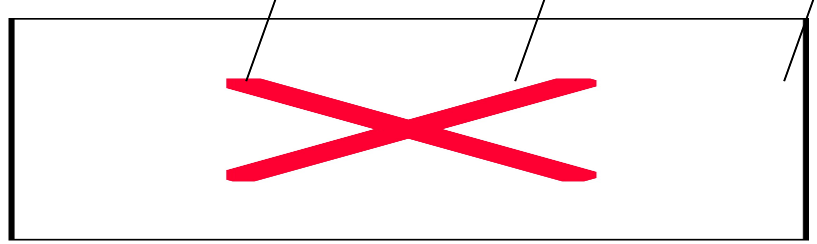
Total Derivative

Introduction of convention of d( )/dt ≡ D( )/Dt



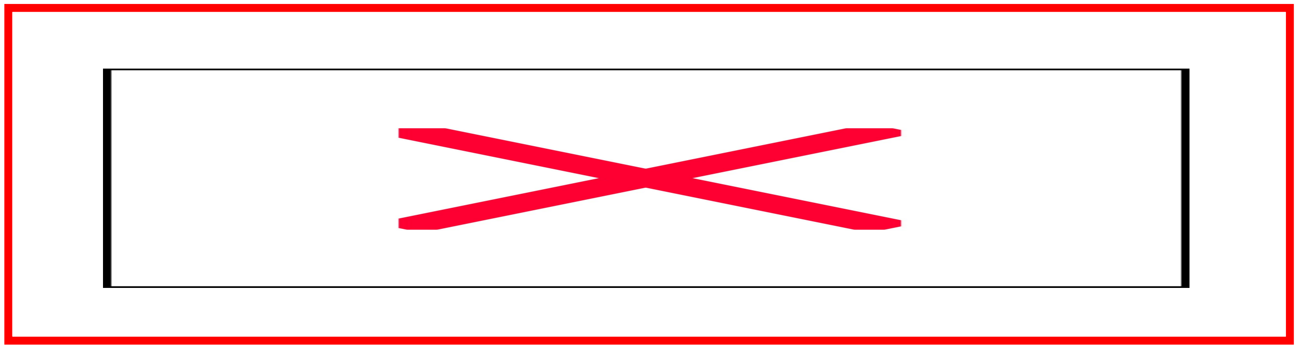
This is done for clarity.

By definition:

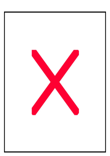


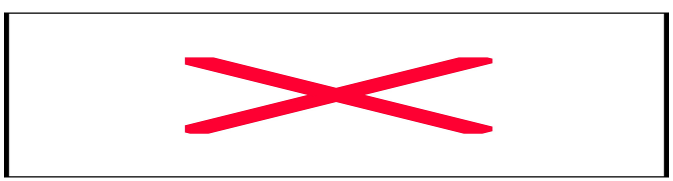
u,v,w: these are the velocities

Definition of the Total Derivative



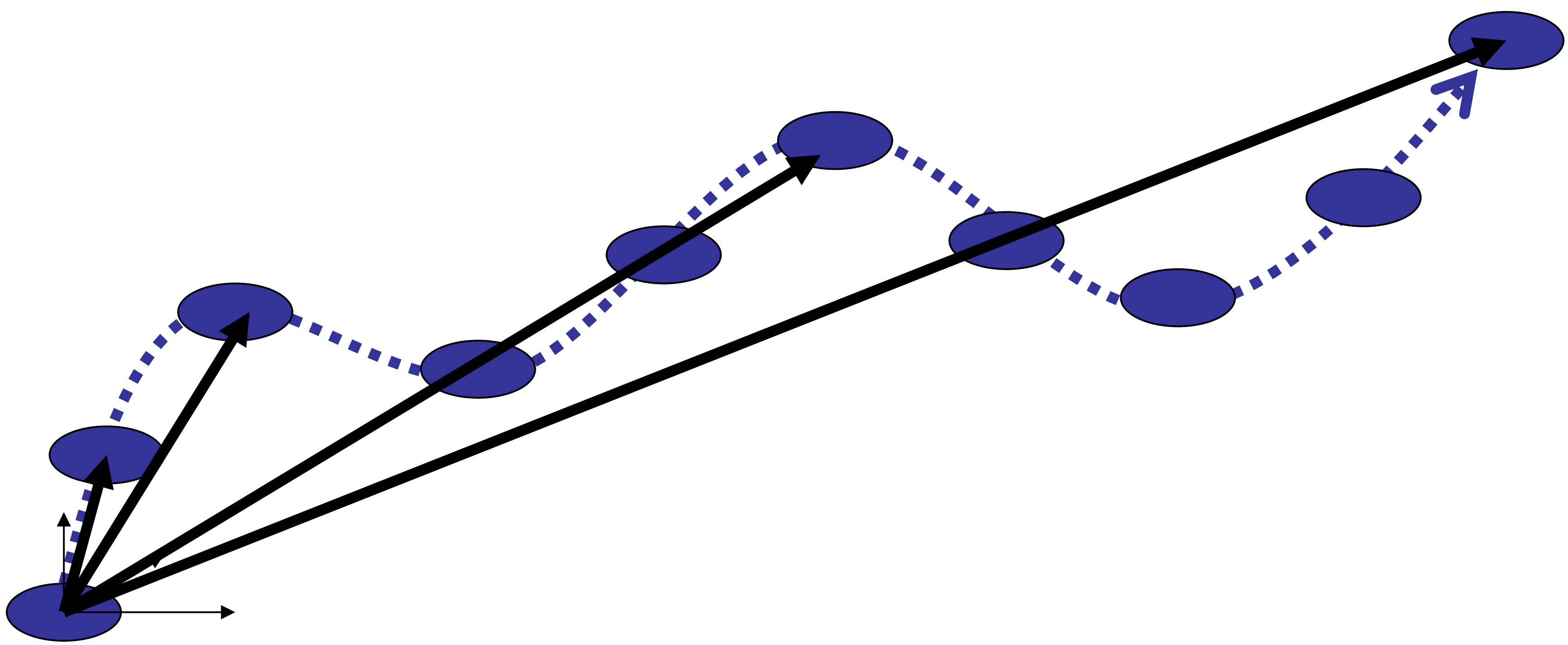
The **total derivative** is also called **material derivative.**

 describes a ‘**Lagrangian** viewpoint’

 describes an ‘**Eulerian** viewpoint’

Lagrangian view

Position vector at different times



Consider fluid parcel moving along some trajectory.

Lagrangian Point of View

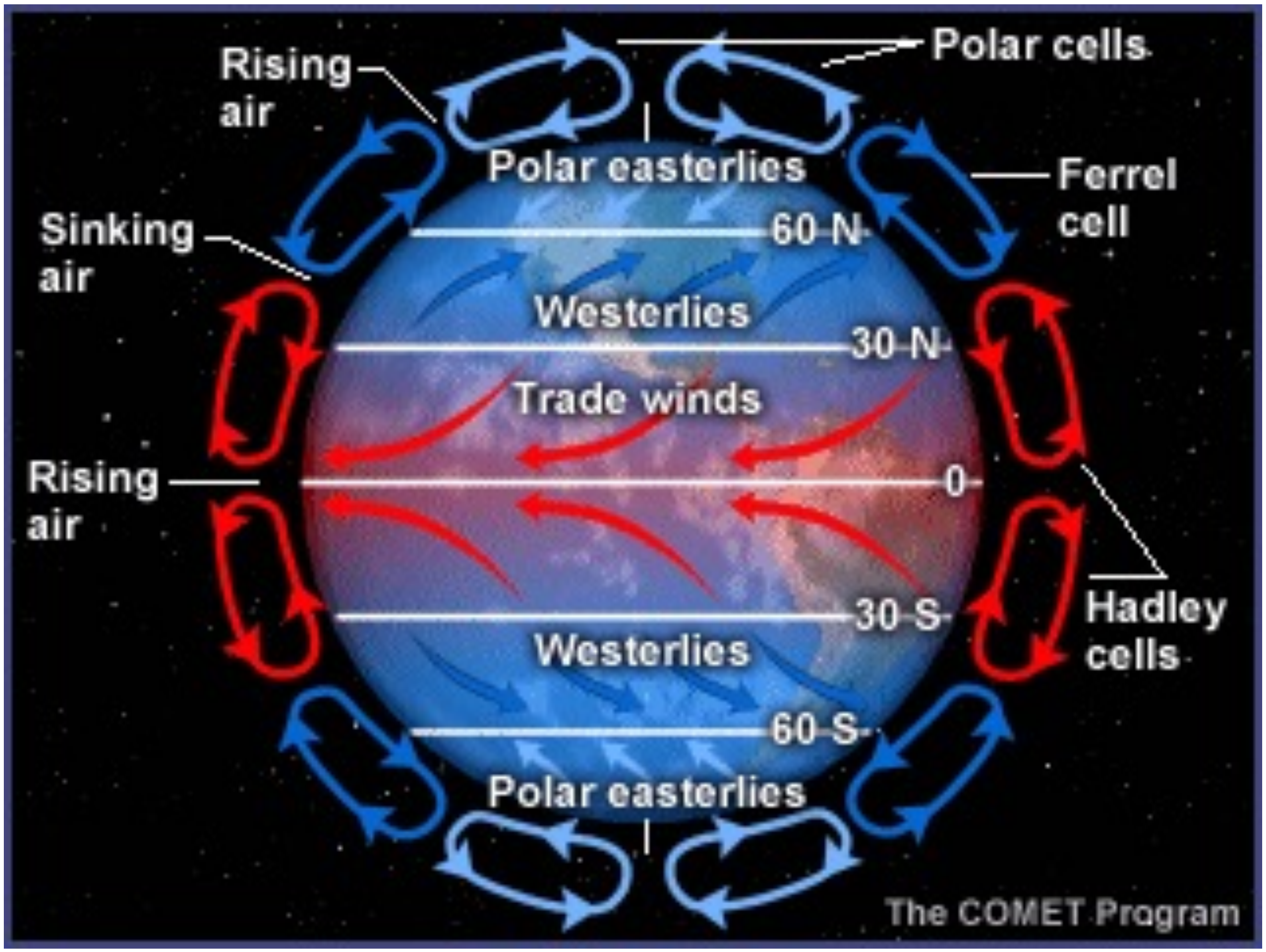
- This parcel-trajectory point of view, which follows a parcel, is known as the Lagrangian point of view.
  - Useful for developing theory
  - Requires considering a coordinate system for each parcel.
  - Very powerful for visualizing fluid motion

Lagrangian point of view: Eruption of Mount Pinatubo

- Trajectories trace the motion of individual fluid parcels over a finite time interval
- Volcanic eruption in 1991 injected particles into the tropical stratosphere (at 15.13 N, 120.35 E)
- The particles got transported by the atmospheric flow, we can follow their trajectories
- Mt. Pinatubo, NASA animation
- Colors in animation reflect the atmospheric height of the particles. Red is high, blue closer to the surface.
- This is a *Lagrangian* view of transport processes.

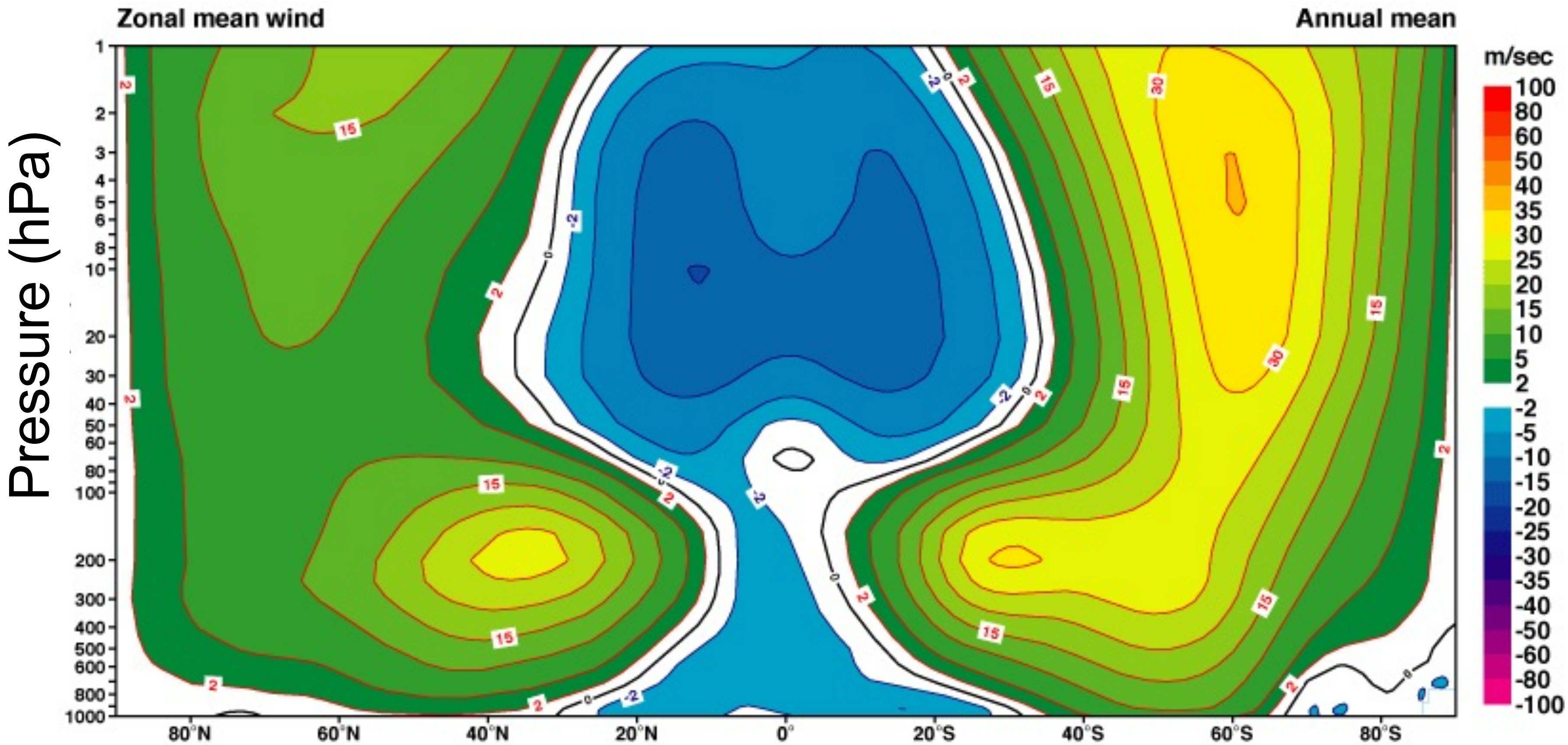
Global wind systems

- General Circulation of the Atmosphere



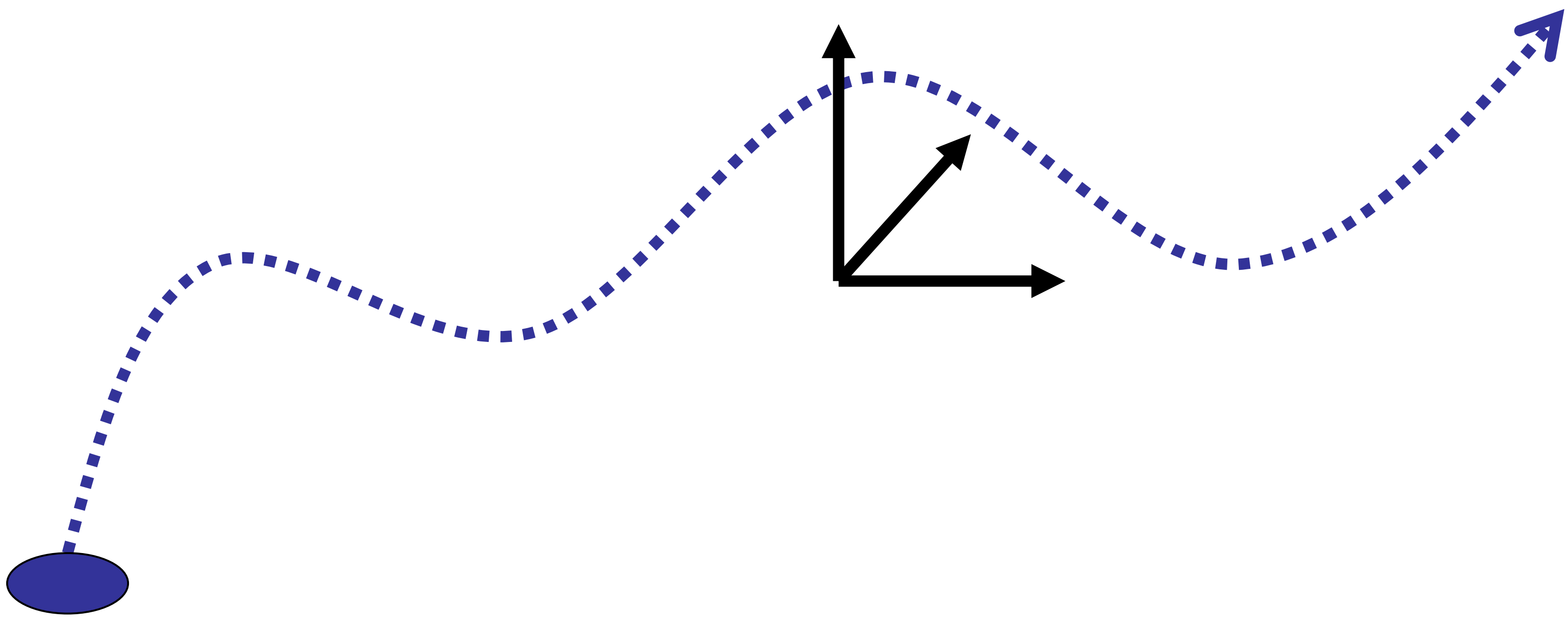
Zonally averaged circulation

- Zonal-mean annual-mean zonal wind



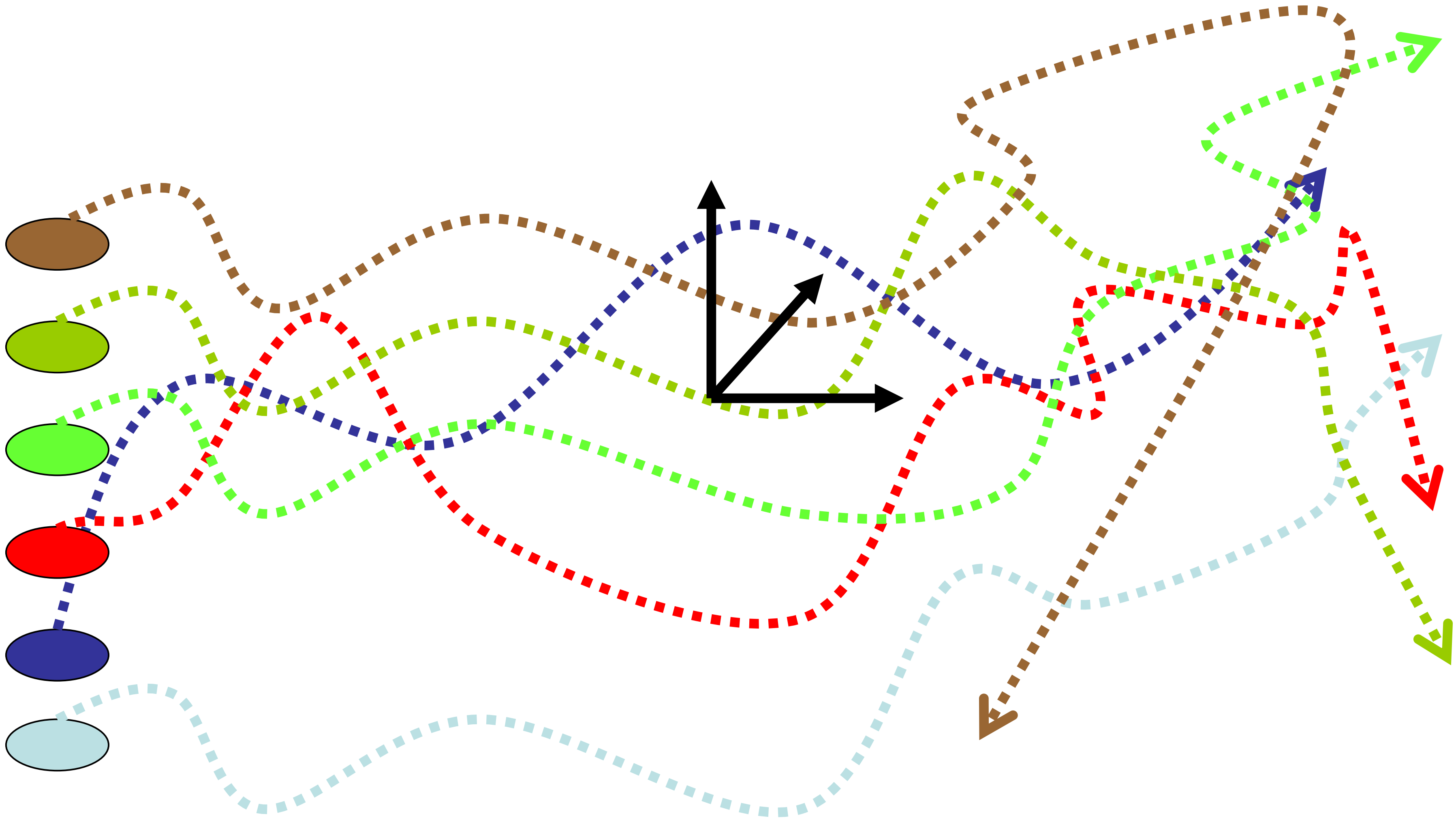
Eulerian view

Now we are going to really think about fluids.



Could sit in one place and watch parcels go by.

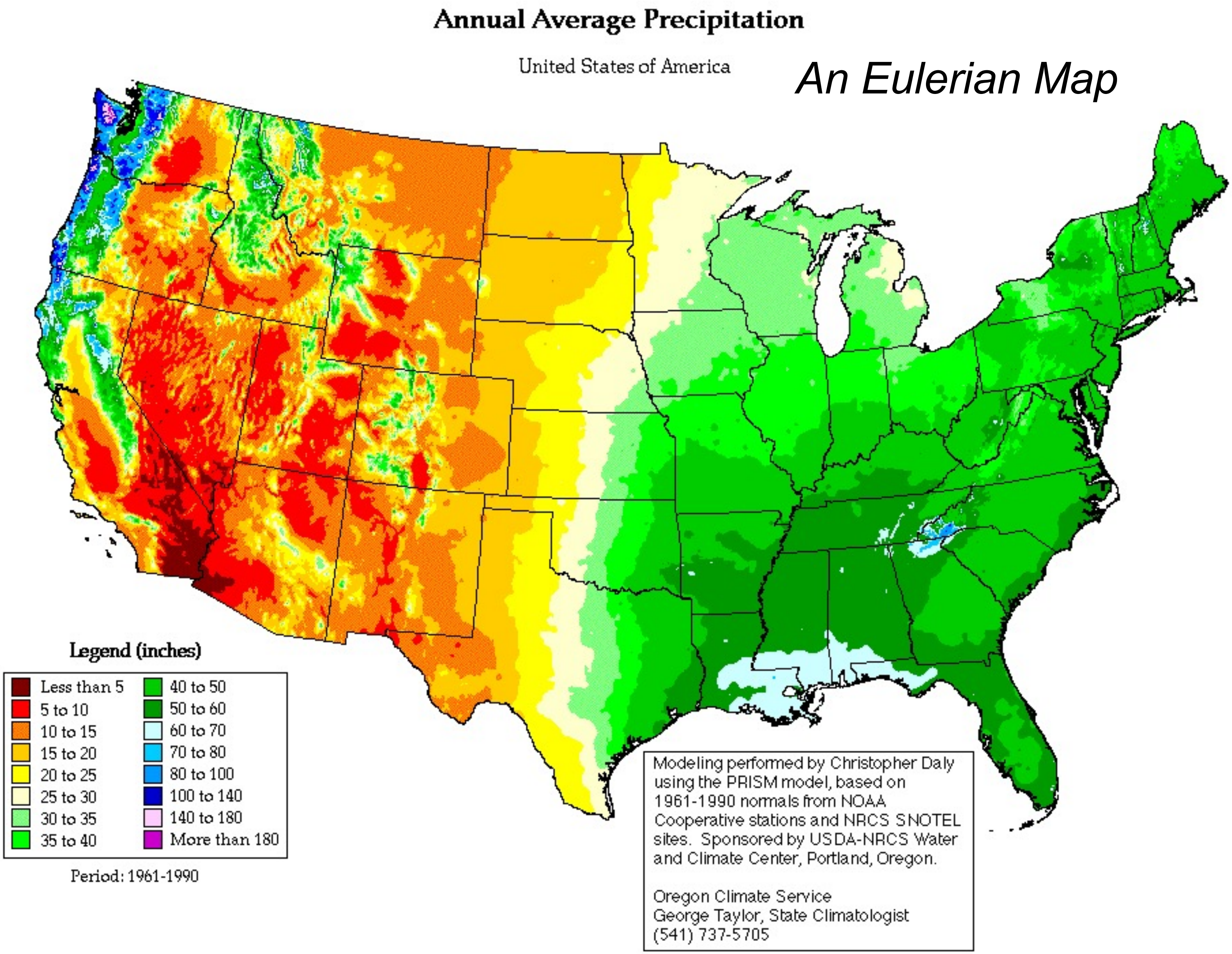
How would we quantify this?



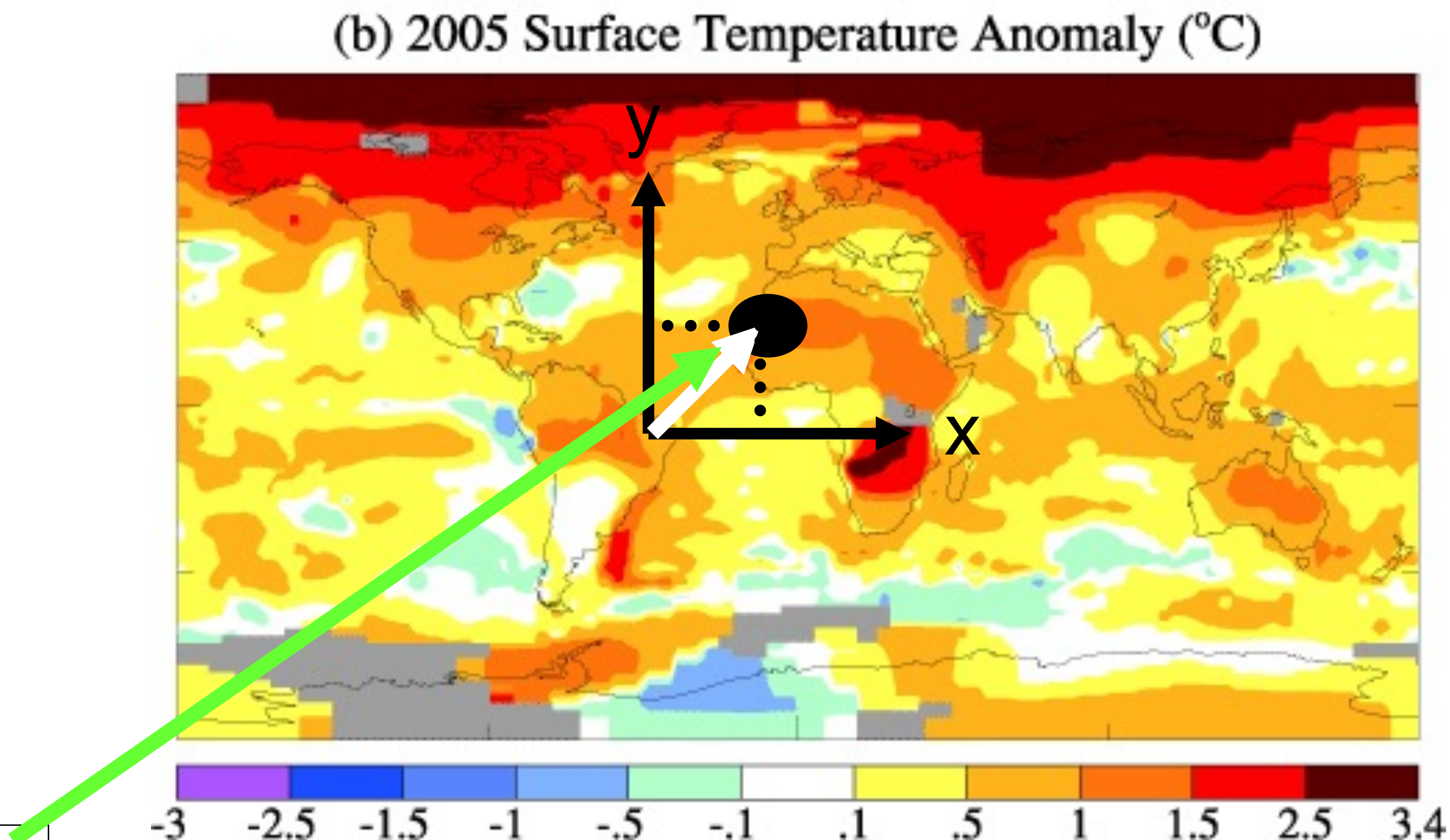
Eulerian Point of View

- This point of view, where is observer sits at a point and watches the fluid go by, is known as the Eulerian point of view.
  - Useful for developing theory
  - Looks at the fluid as a field.
  - Requires considering only one coordinate system for all parcels
  - Easy to represent interactions of parcels through surface forces
  - A value for each point in the field – no gaps or bundles of “information.”

An Eulerian Map

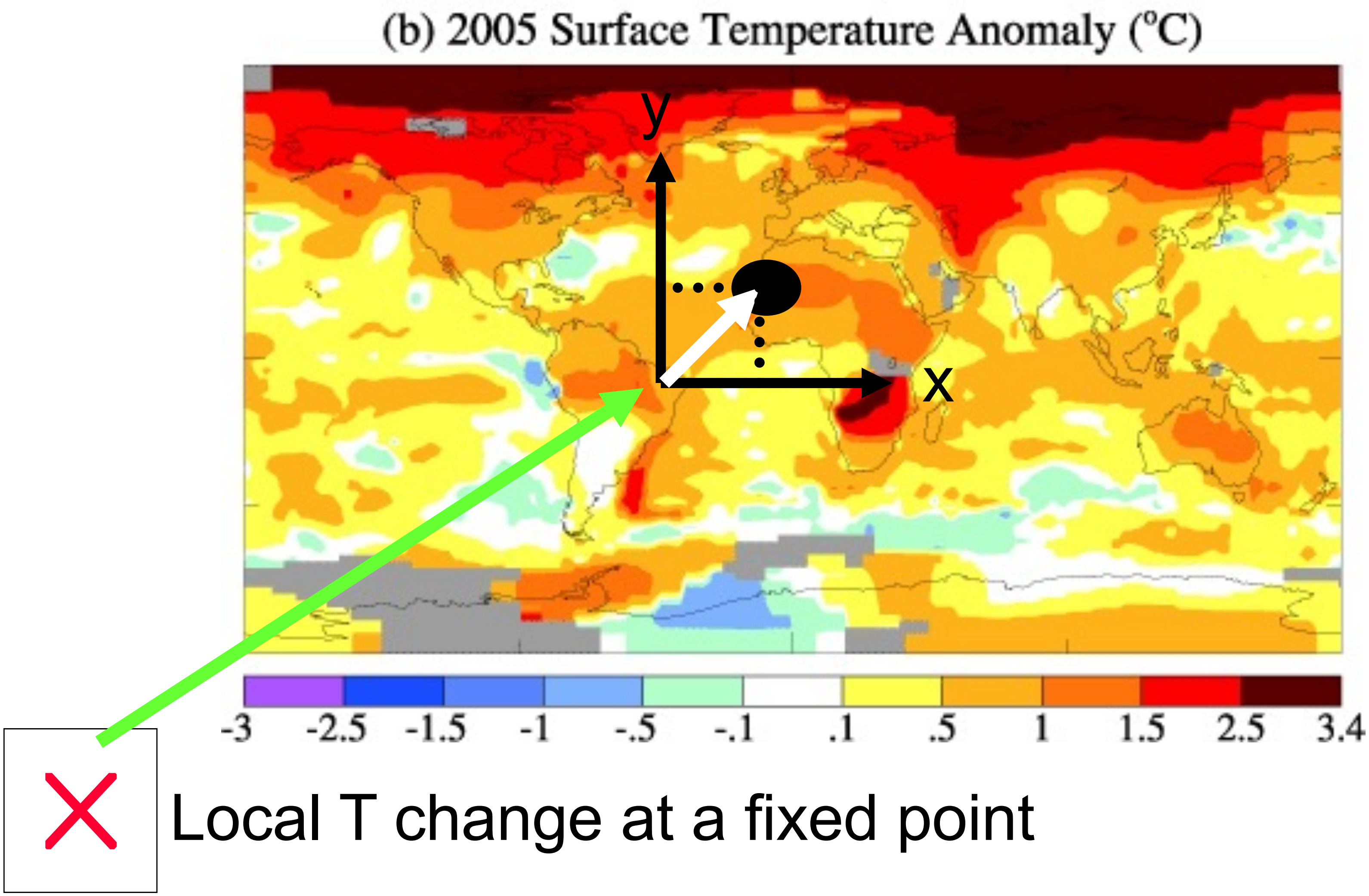


Consider some parameter, like temperature, T



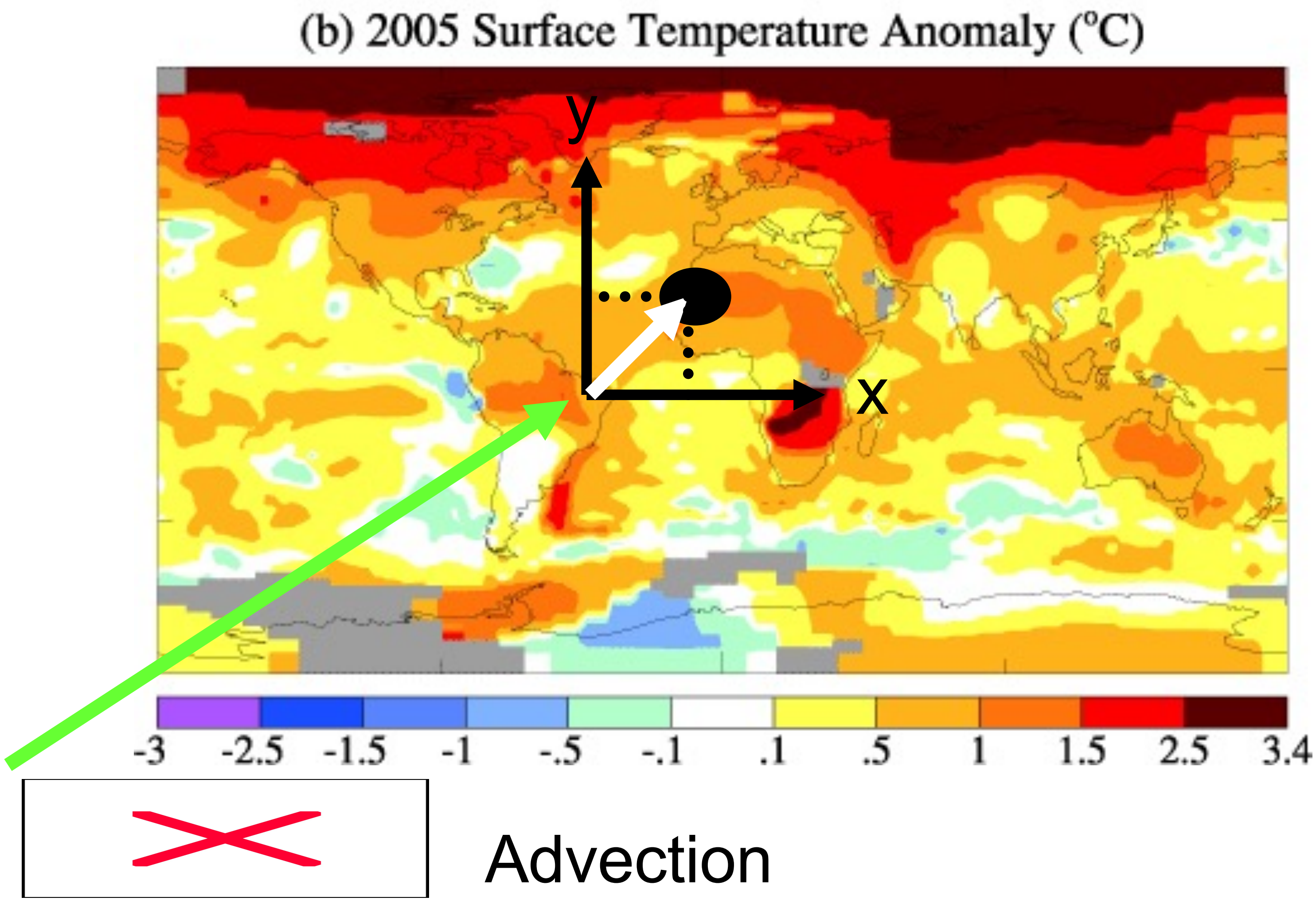
Material derivative, T change following the parcel

Consider some parameter, like temperature, T



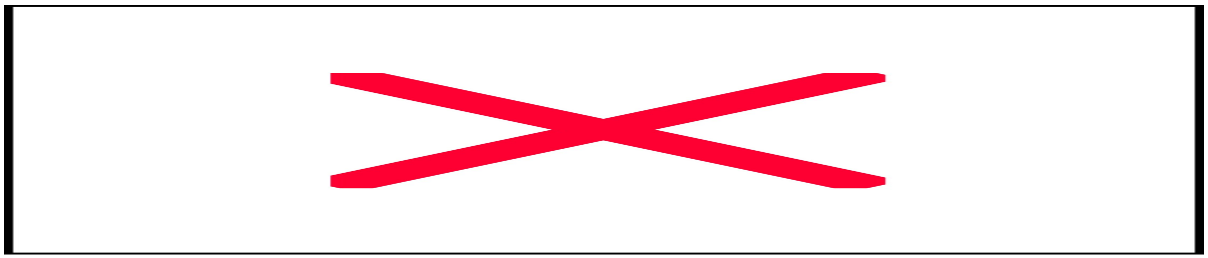
Local T change at a fixed point

Consider some parameter, like temperature, T

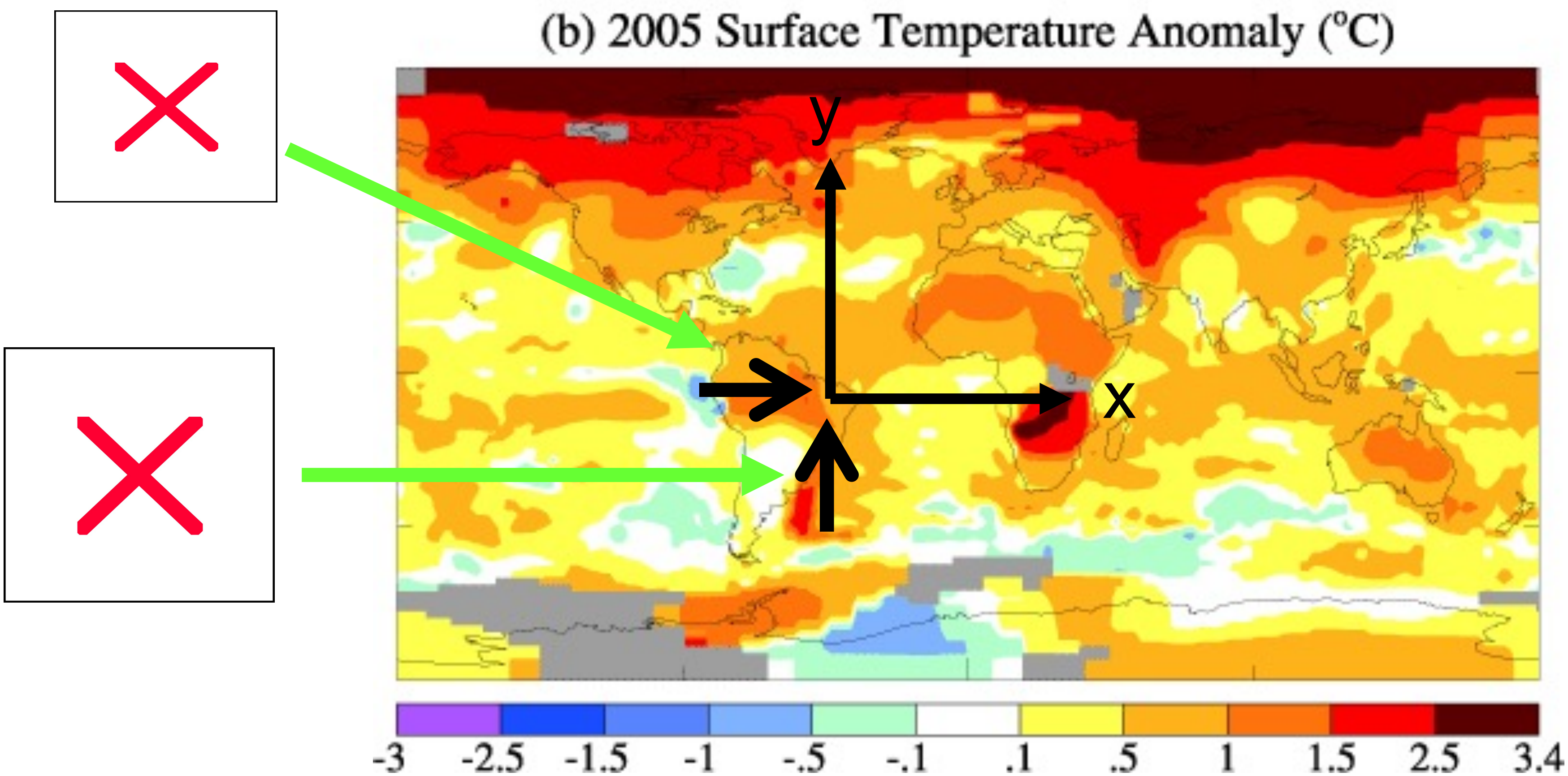


Advection

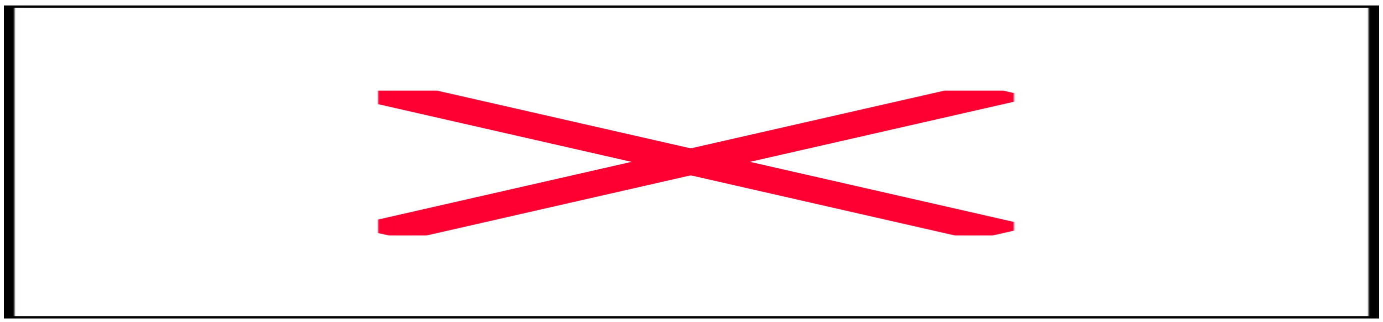
Temperature advection term



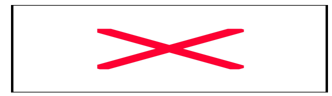
Consider some parameter, like temperature, T

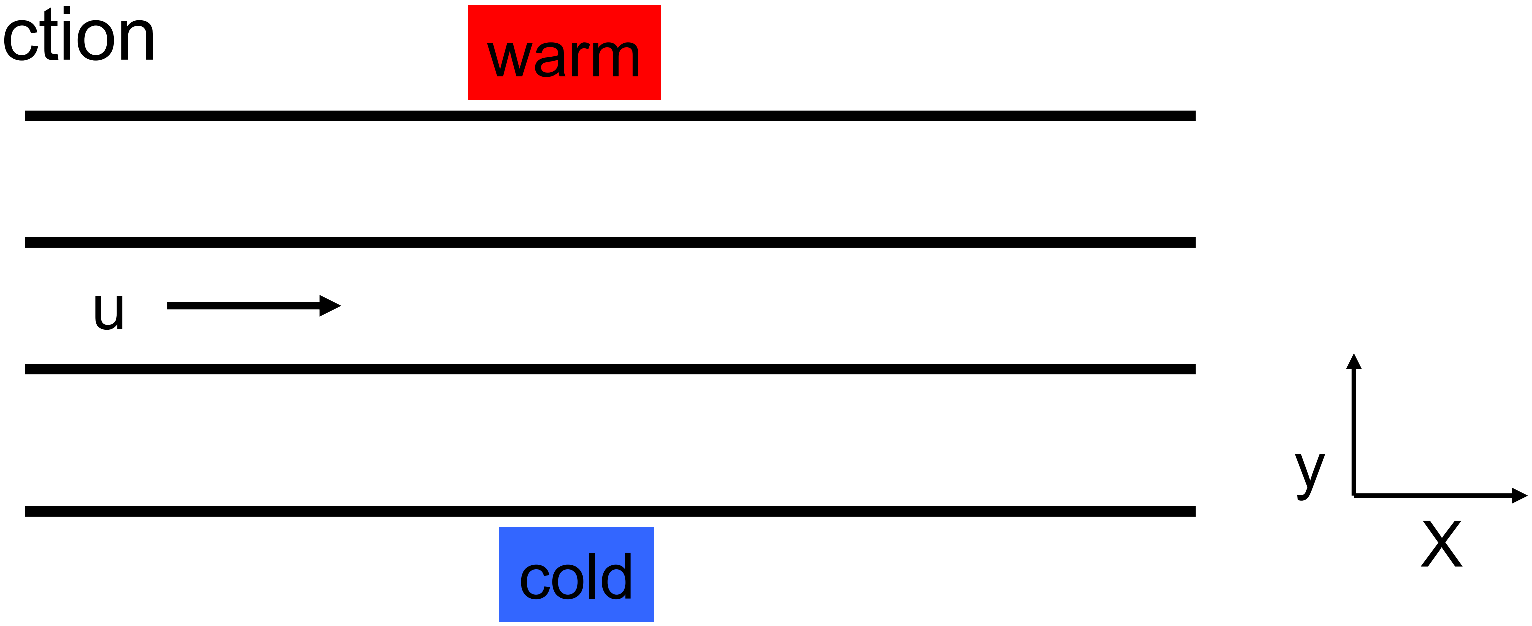


Temperature advection term



Advection of cold or warm air

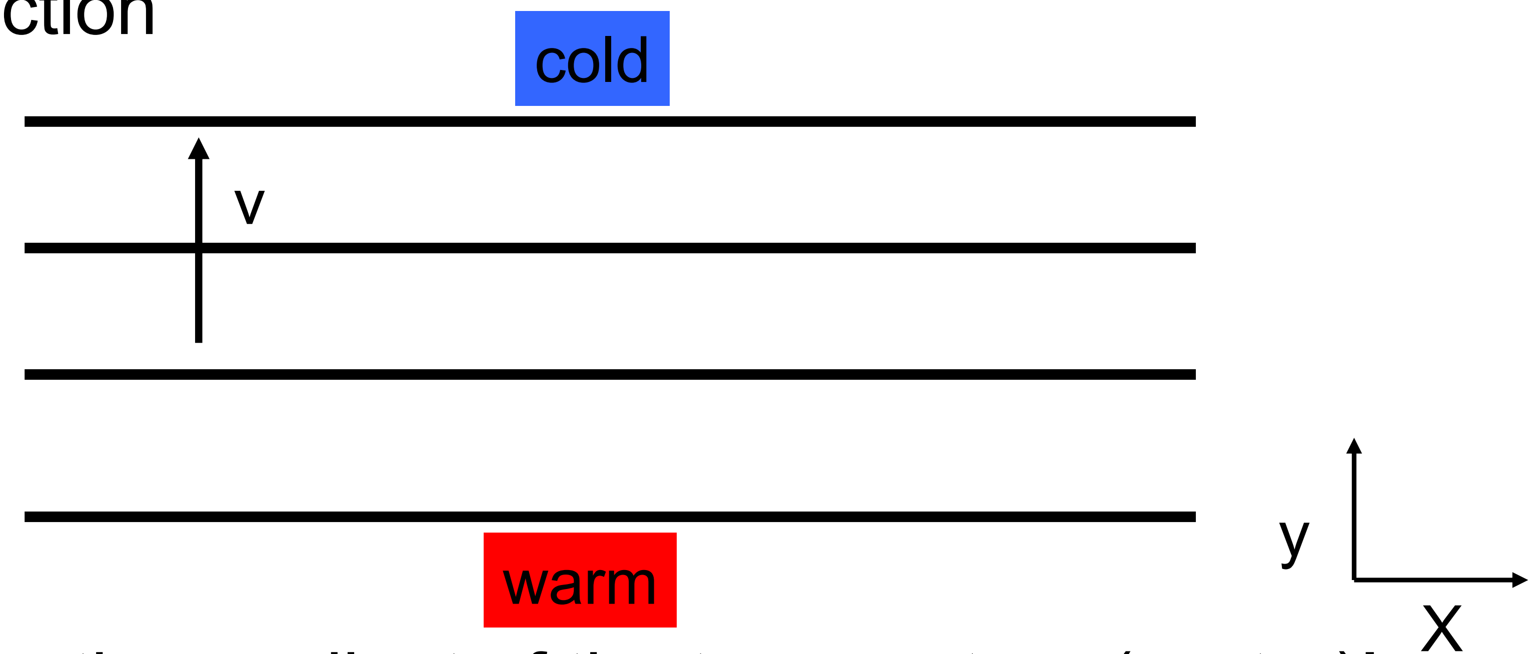
- Temperature advection: 
- Imagine the isotherms are oriented in the E-W direction



- Draw the horizontal temperature gradient vector!
- pure west wind u > 0, v=0, w=0: Is there temperature advection? If yes, is it cold or warm air advection?

Advection of cold or warm air

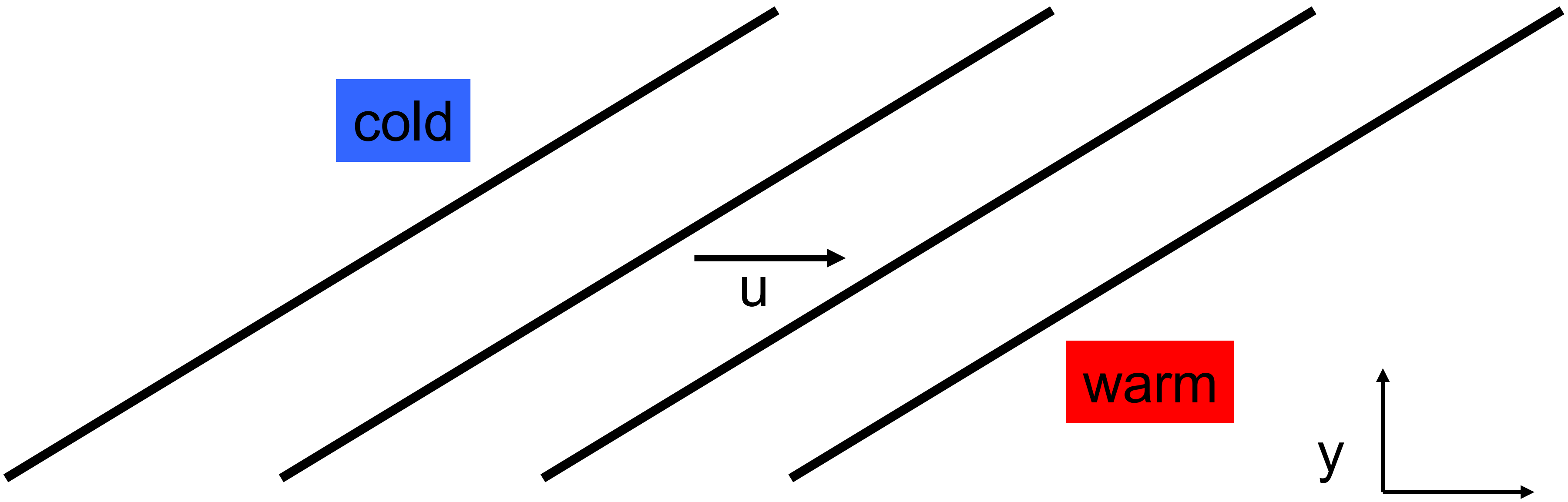
- Temperature advection:
- Imagine the isotherms are oriented in the E-W direction



- Draw the gradient of the temperature (vector)!
- pure south wind v > 0, u=0, w=0: Is there temperature advection? If yes, is it cold or warm air advection?

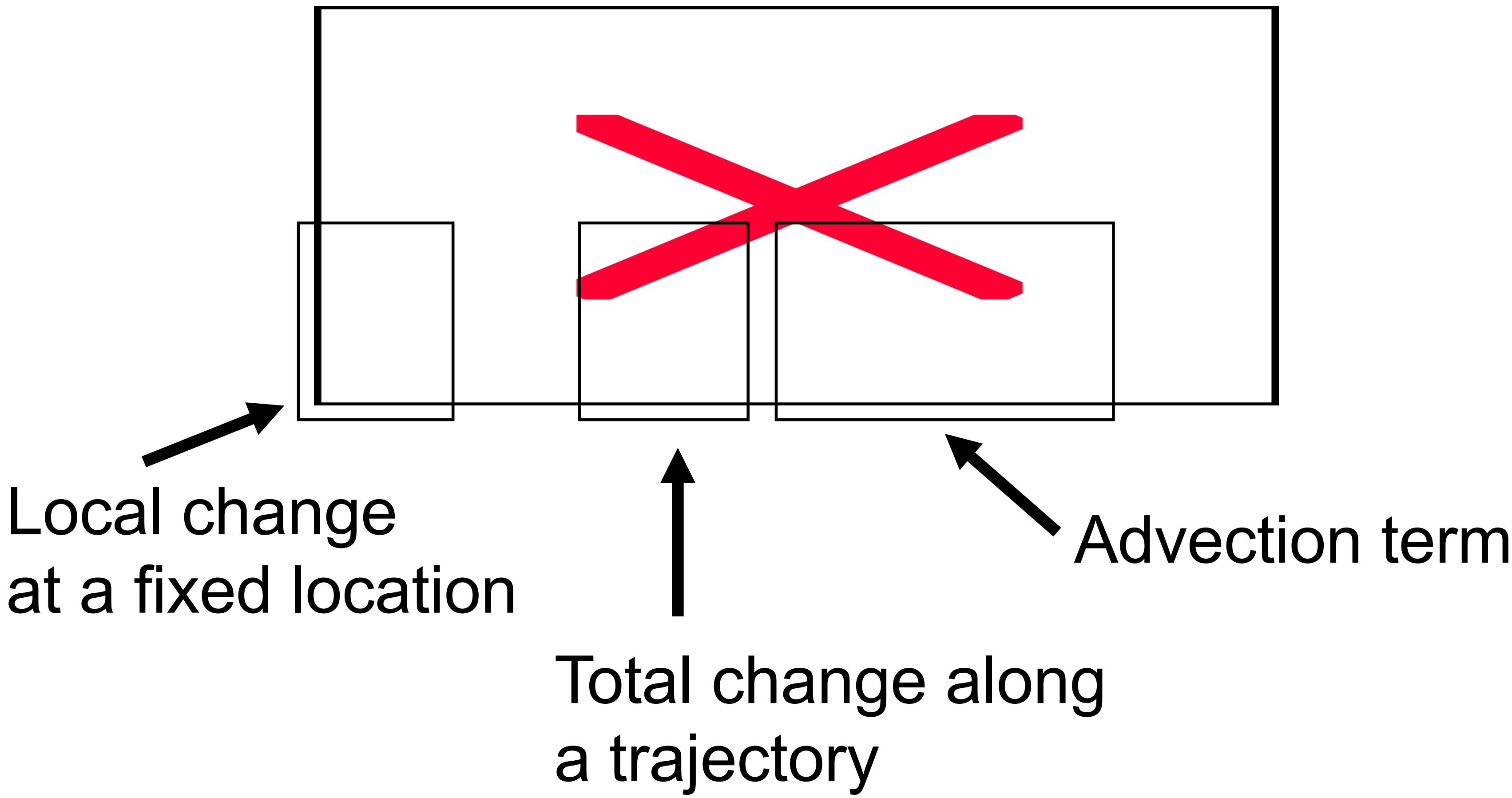
Advection of cold or warm air

- Temperature advection:
- Imagine the isotherms are oriented as

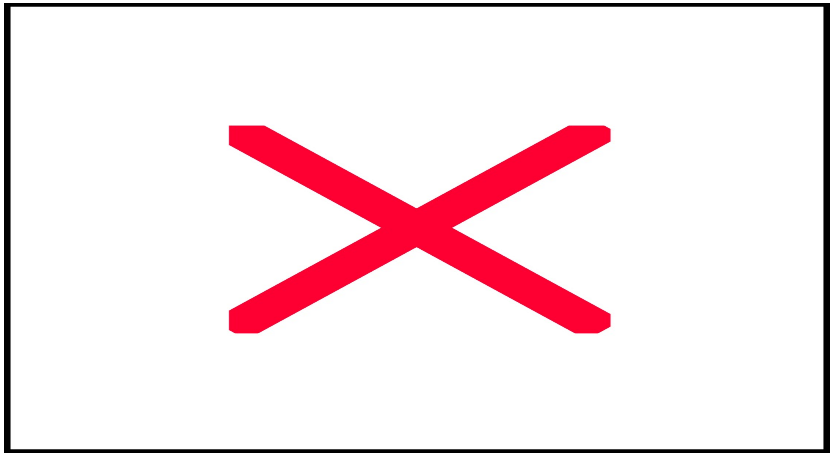


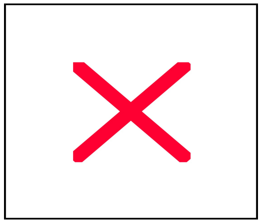
- Draw the horizontal temperature gradient!
- pure west wind u > 0, v=0, w=0: Is there temperature advection? If yes, is it cold or warm air advection?

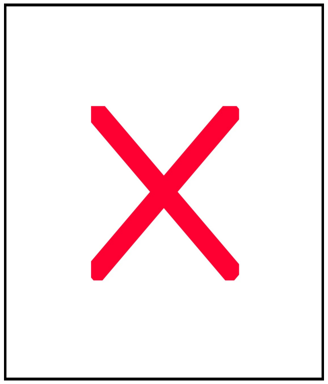
Summary: Local Changes & Material Derivative



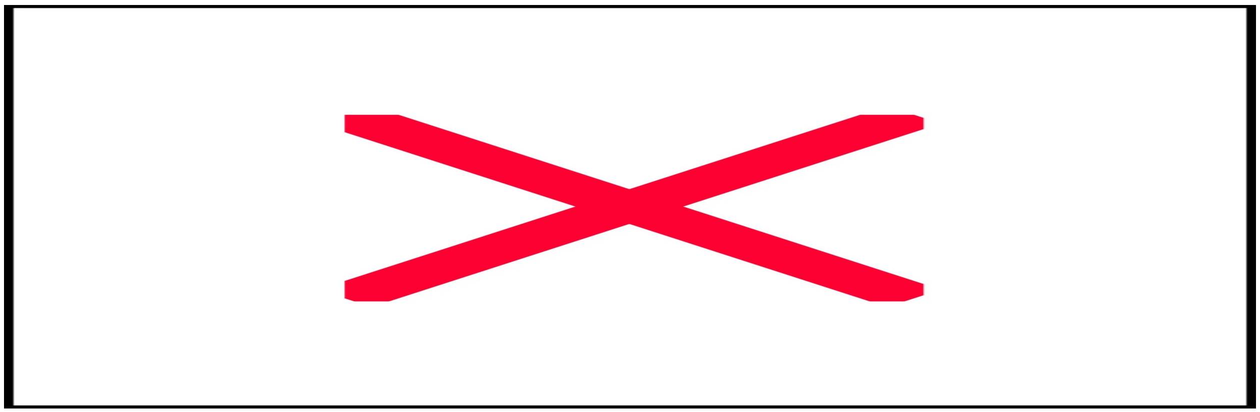
Summary: For 2D horizontal flows



with  horizontal wind vector and

 horizontal gradient operator

Conservation and Steady-State



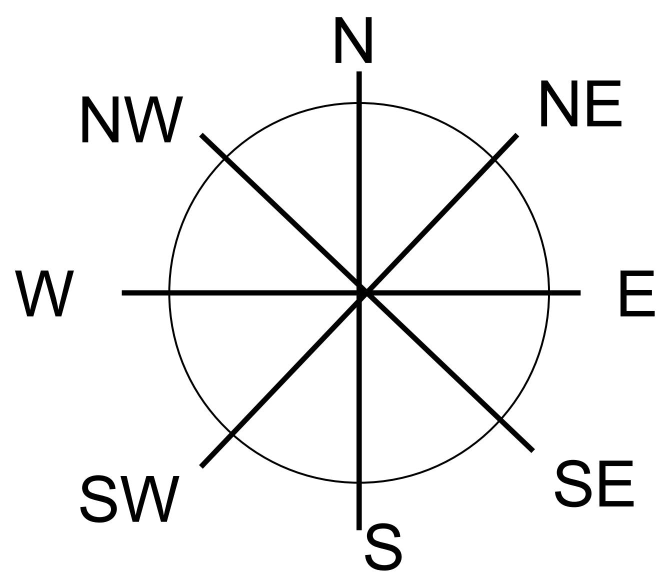
Remember: we talked about the conservation of money

Conservation principle is important for tracers in the atmosphere that do not have sources and sinks

Class exercise

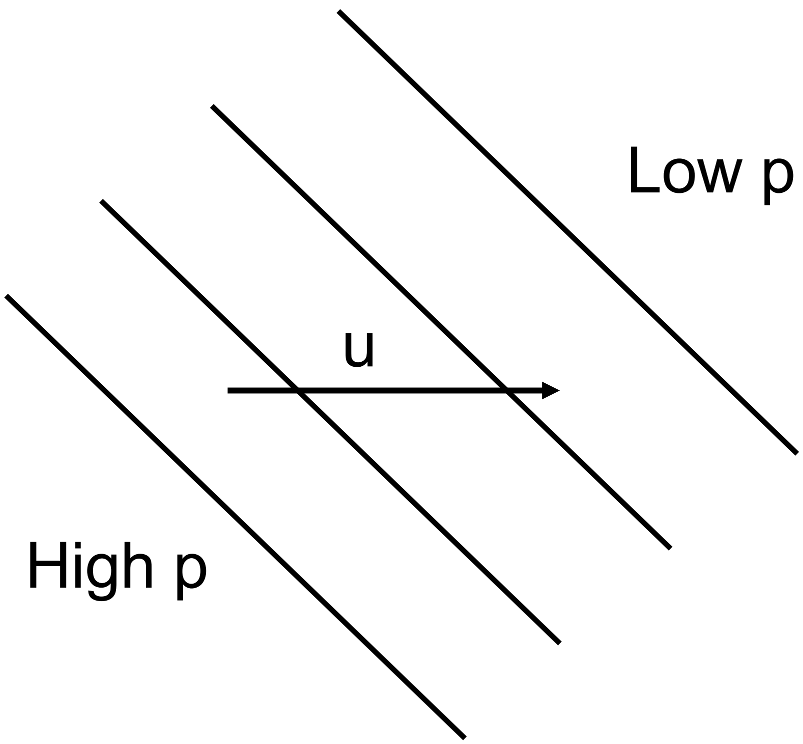
- The surface pressure **decreases** by 3 hPa per 180 km in the eastward direction.
- A ship steaming eastward at 10 km/h measures a **pressure fall** of 1 hPa per 3 hours.
- What is the pressure change on an island that the ship is passing?

Directions:



Food for thought

- Imagine a different situation.
- The surface pressure **decreases** by 3 hPa per 180 km in the **north-east** direction.
- Thus:

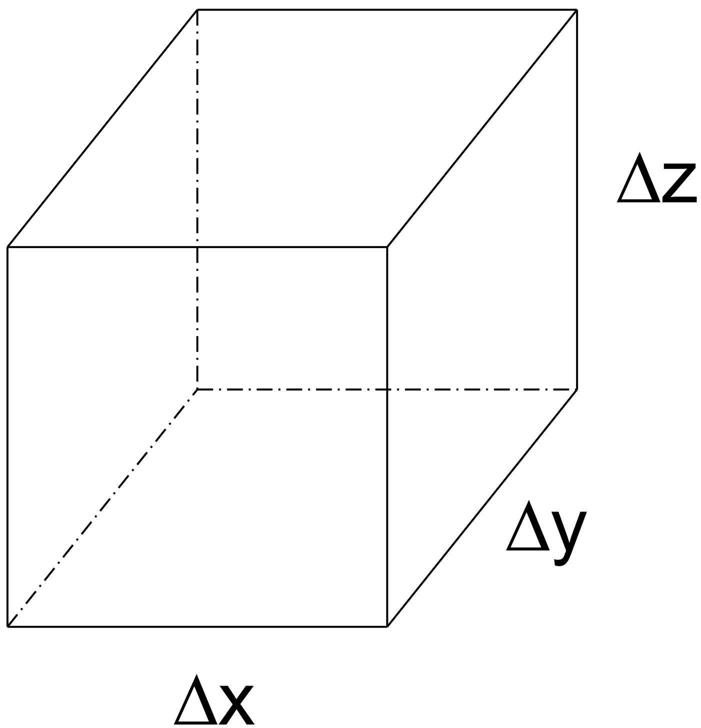


What are the fundamental forces in the Earth's system?

- Pressure gradient force
- Gravitational force
- Viscous force
- Apparent forces: Centrifugal and Coriolis
- Can you think of other classical forces and would they be important in the Earth's system?

- Total Force is the sum of all of these forces.

A particle of atmosphere



$\rho \equiv$ density = mass per unit volume (ΔV)

$\Delta V = \Delta x \Delta y \Delta z$

$m = \rho \Delta x \Delta y \Delta z$

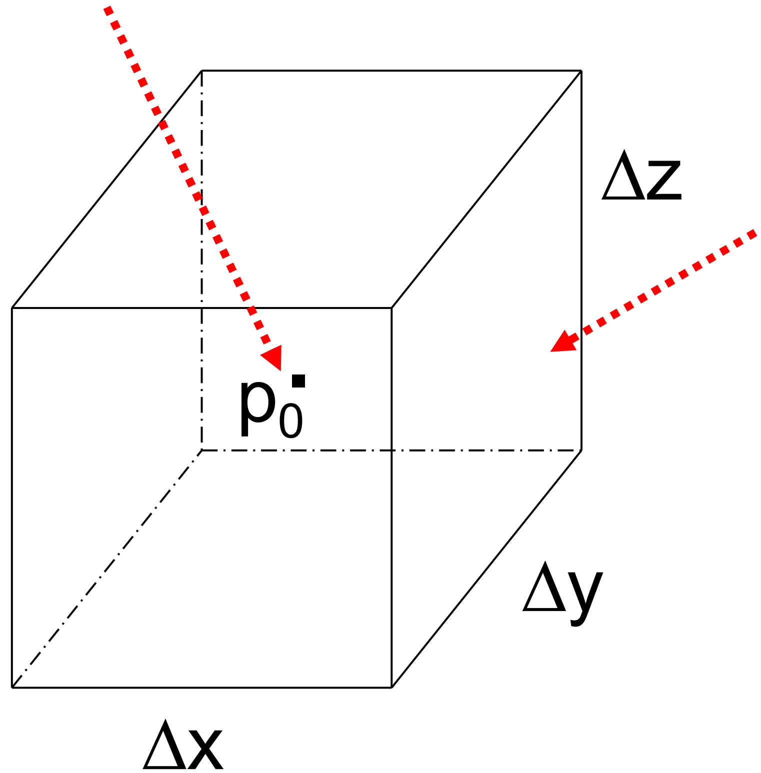
$p \equiv$ pressure = force per unit area acting on the particle of atmosphere

Check out Unit 6, frames 7-13:
http://www.atmos.washington.edu/2005Q1/101/CD/MAIN3.swf

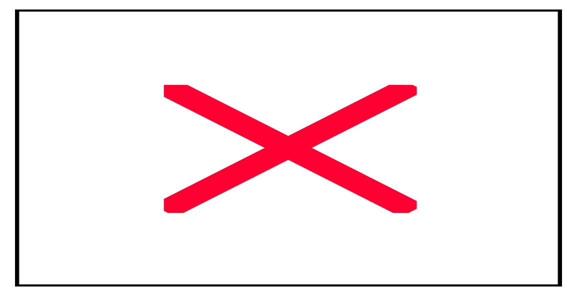
Pressure gradient force (1)

(x_0, y_0, z_0)

p_0 = pressure at (x_0, y_0, z_0)



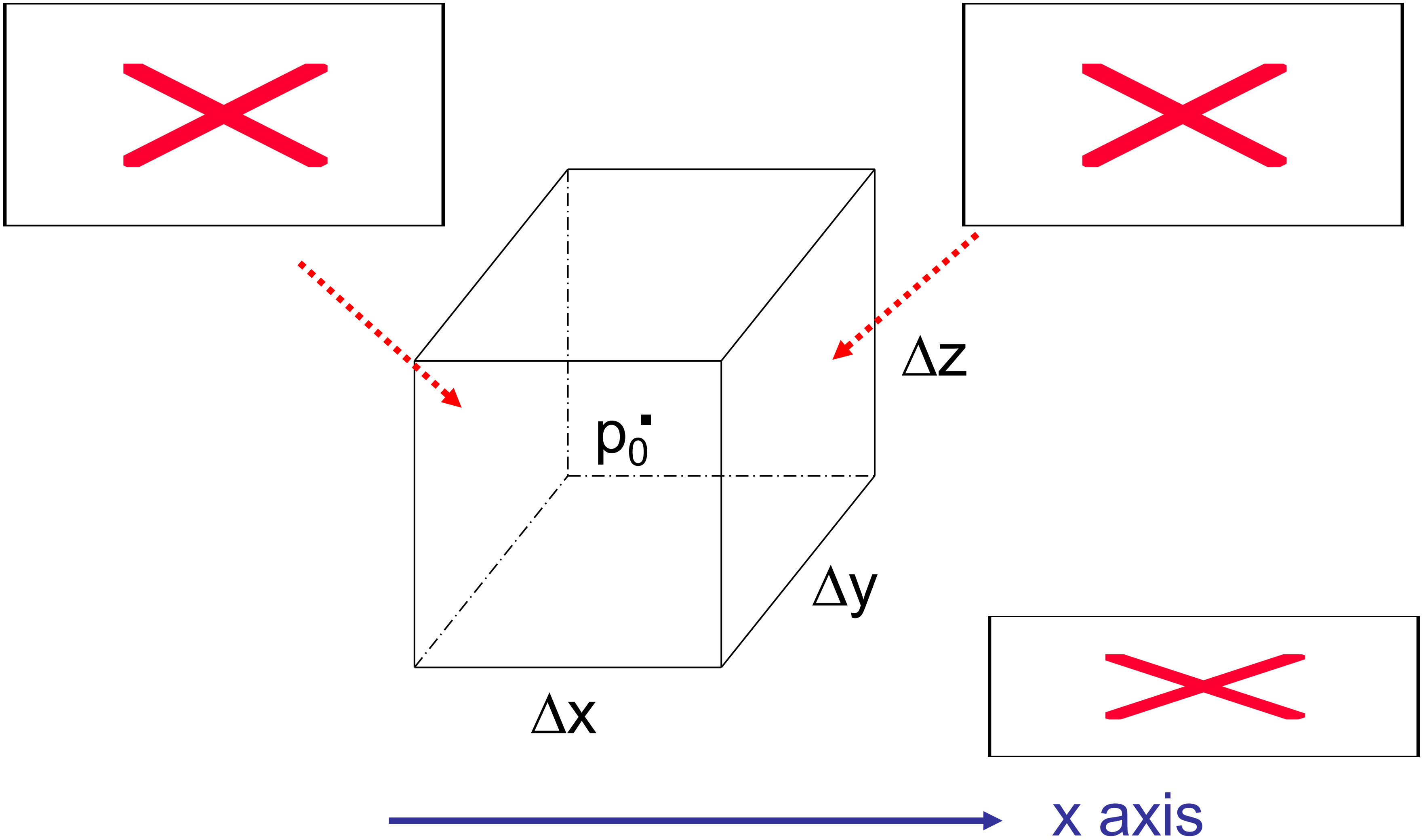
Pressure at the ‘wall’:



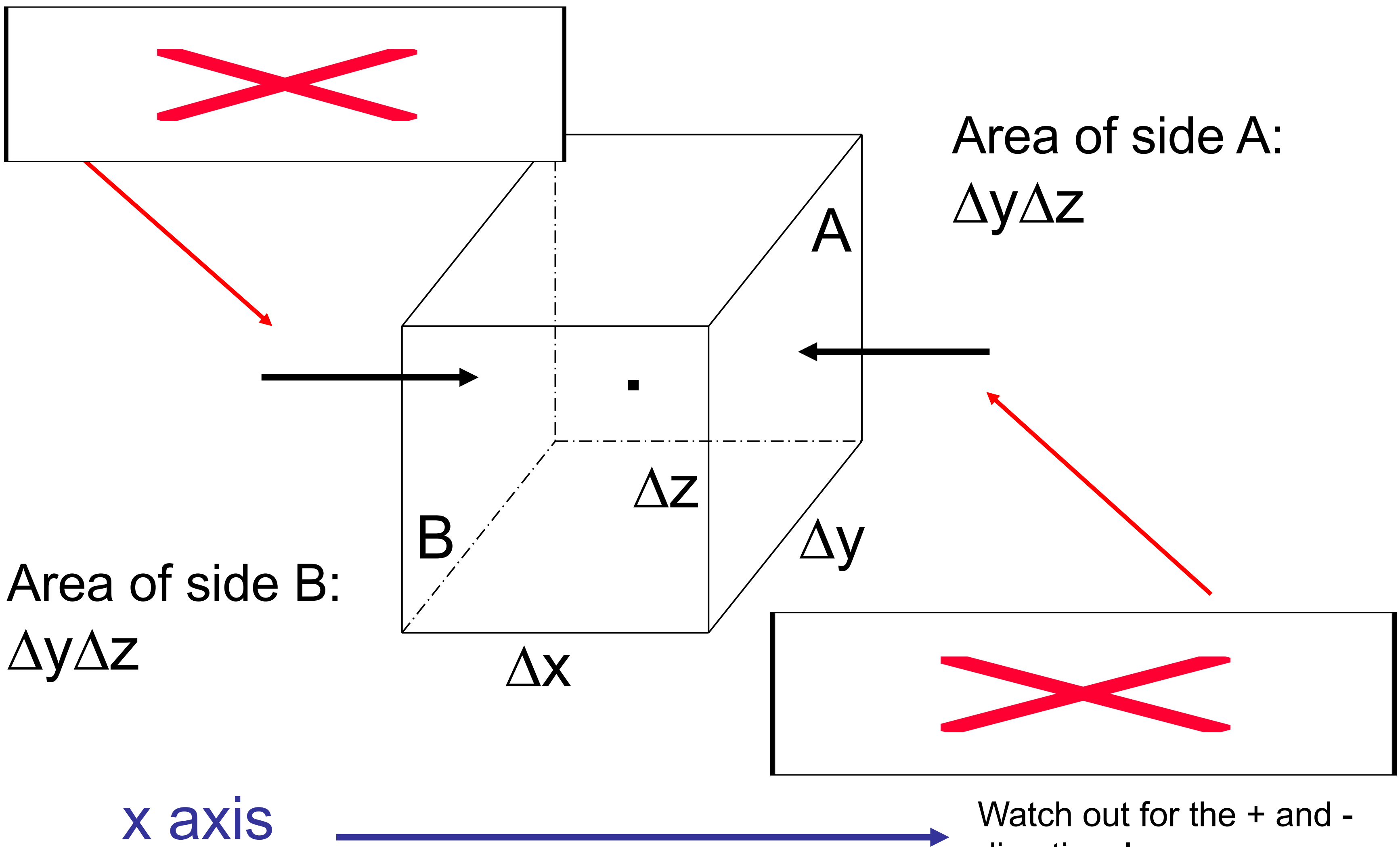
Remember the Taylor series expansion!

x axis

Pressure at the 'walls'

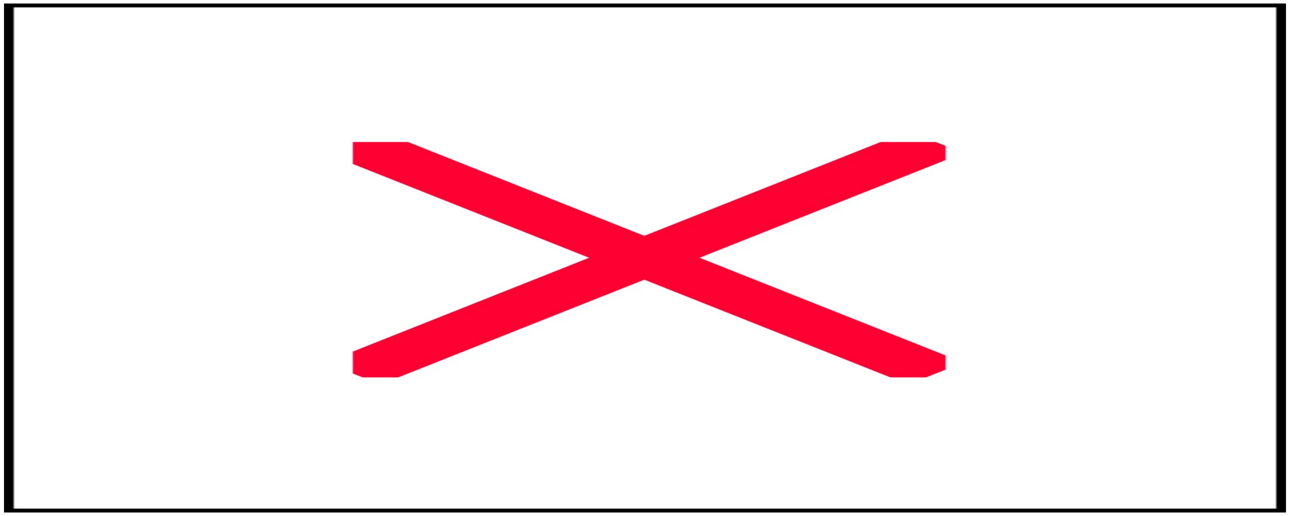


Pressure gradient force (3) (ignore higher order terms)

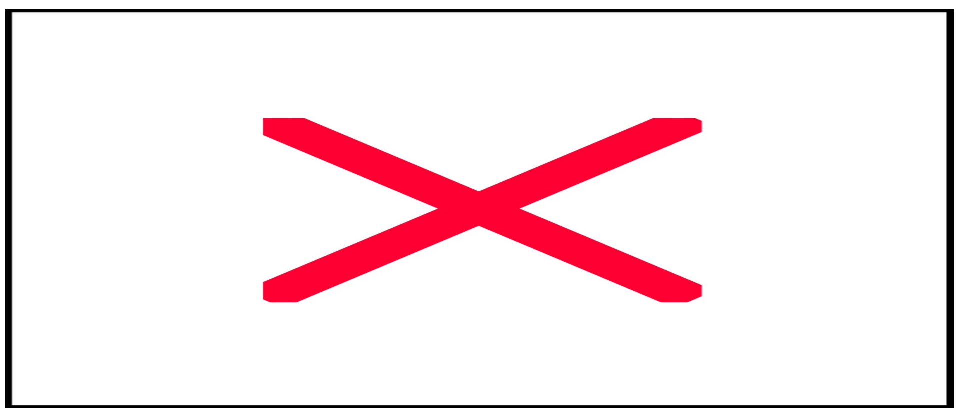


Watch out for the + and - directions!

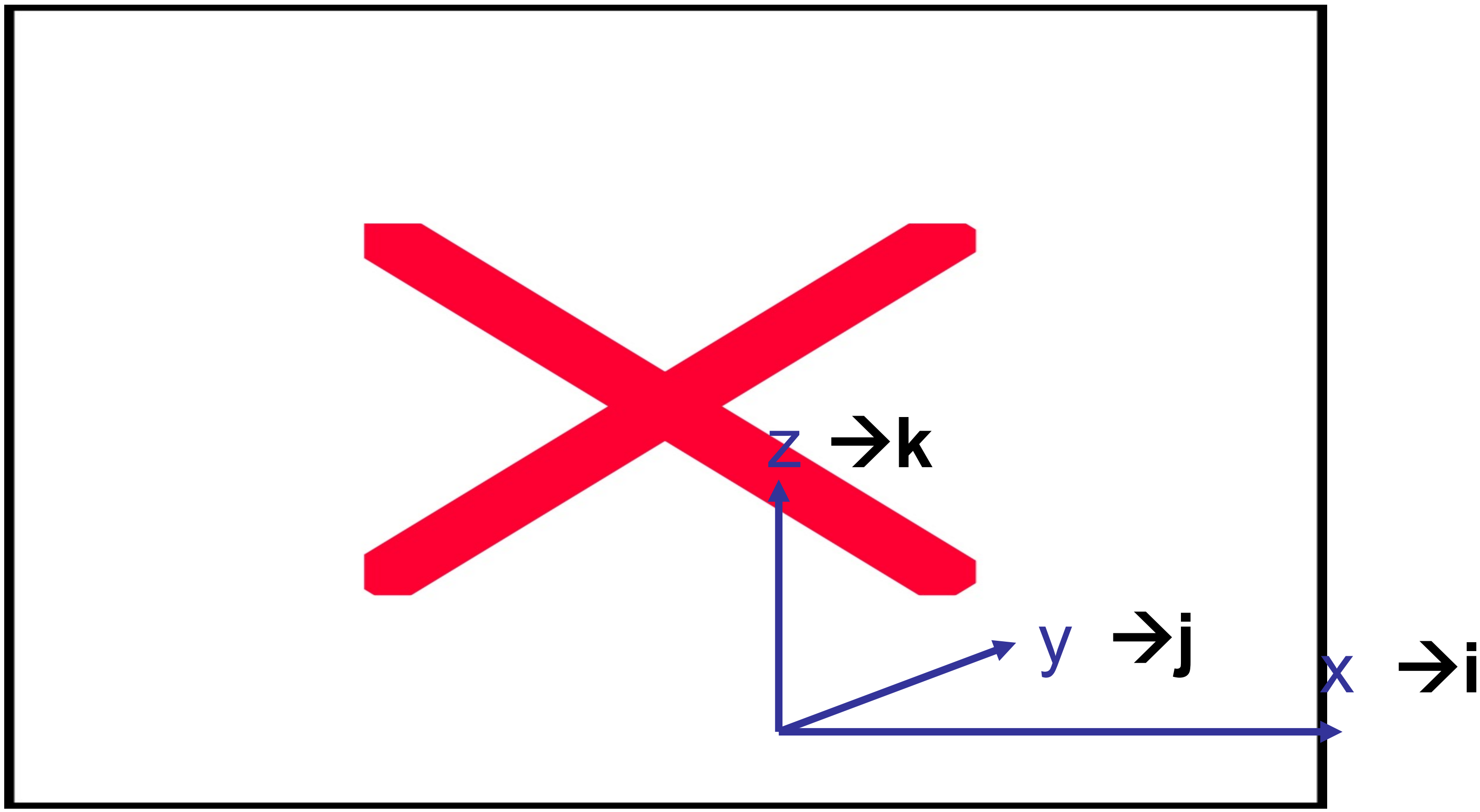
Pressure gradient force (4):Total x force



We want force per unit mass



Vector pressure gradient force



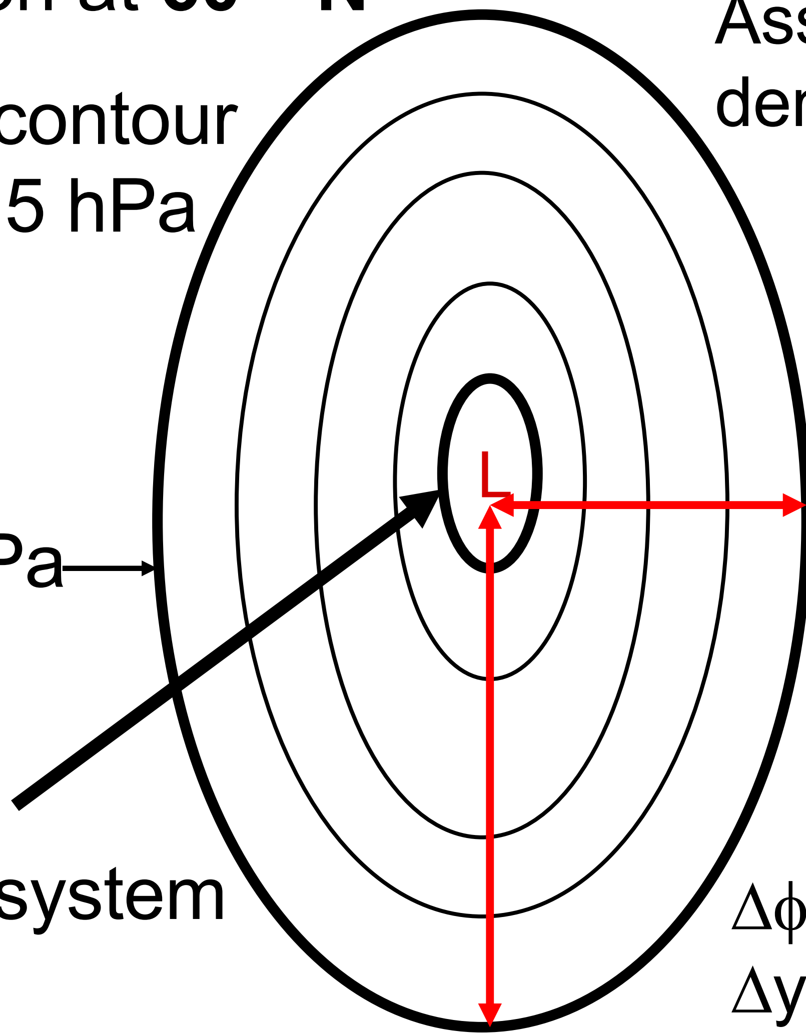
Class exercise

Compute the pressure gradient force at sea level in x and y direction at **60° N**

Isobars with contour interval Δp = 5 hPa

Assume constant density ρ = 1.2 kg/m^3 and radius a = 6371 km

1000 hPa



Low pressure system at 60° N

$\Delta\lambda = 20^o = \pi/9$

$\Delta x = a \cos\phi \, \Delta\lambda$

$\Delta\phi = 20^o = \pi/9$

$\Delta y = a \, \Delta\phi$

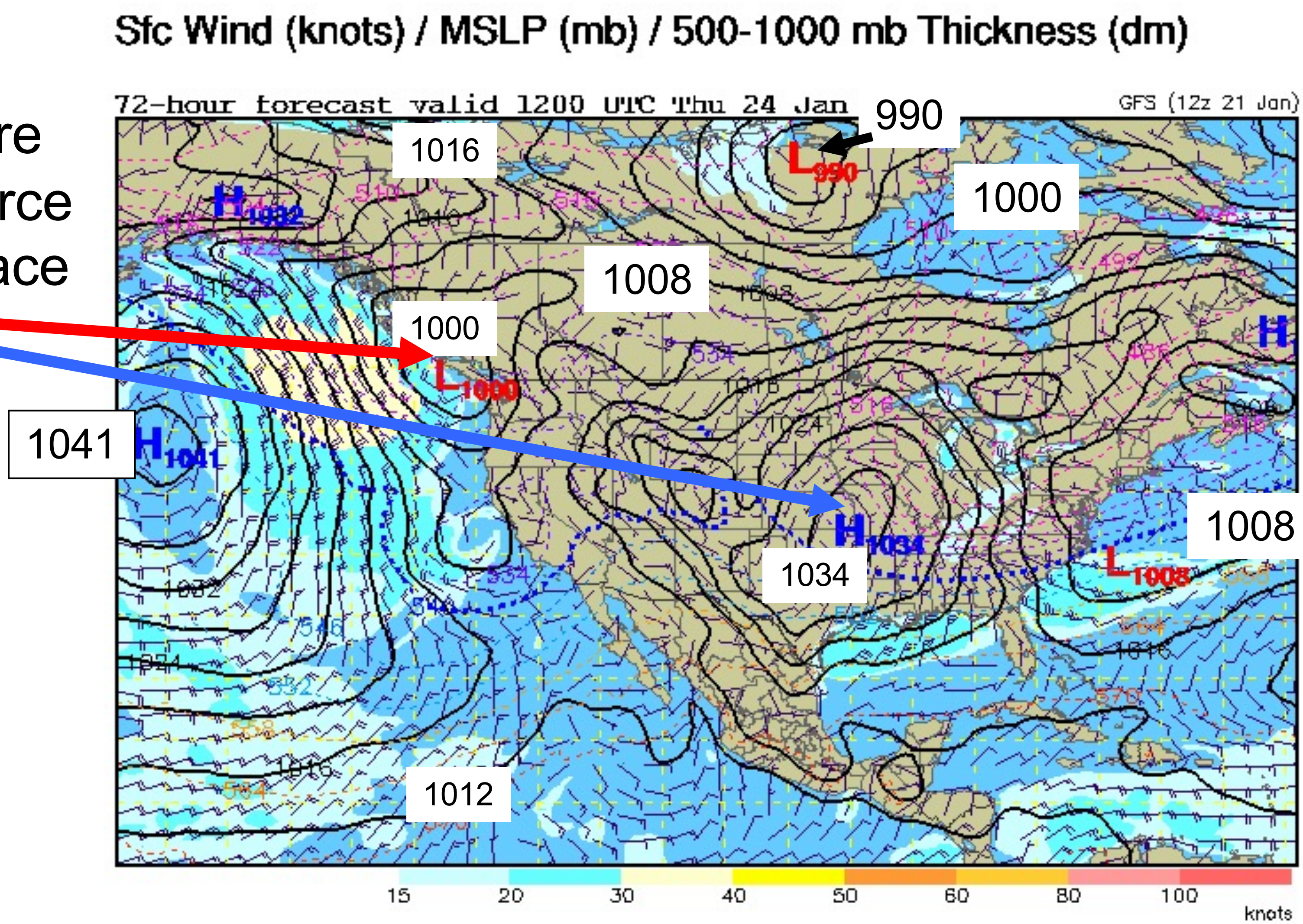
Class exercise

Compute the pressure gradient force at the surface

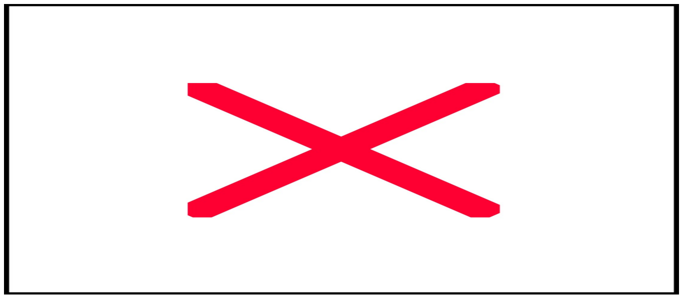
Contour interval: 4 hPa

Density?

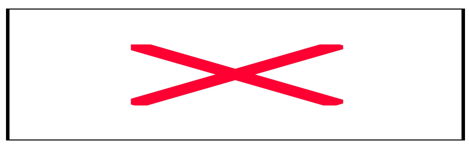
NCAR forecasts



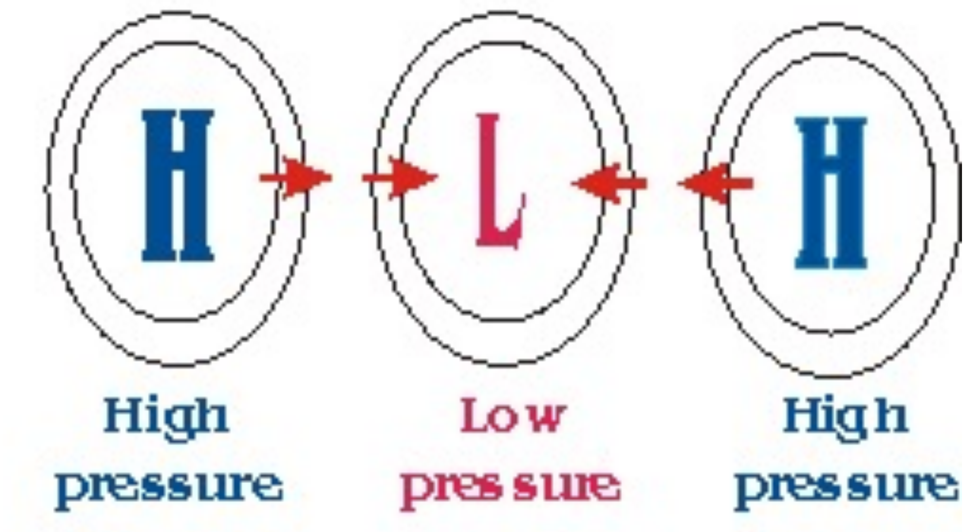
Our momentum equation so far

 other forces

Here, we use the text's convention that the velocity is



Highs and Lows



Pressure gradient force tries to eliminate the pressure differences