AOSS 321, Winter 2009
Earth System Dynamics

Lecture 9
2/5/2009

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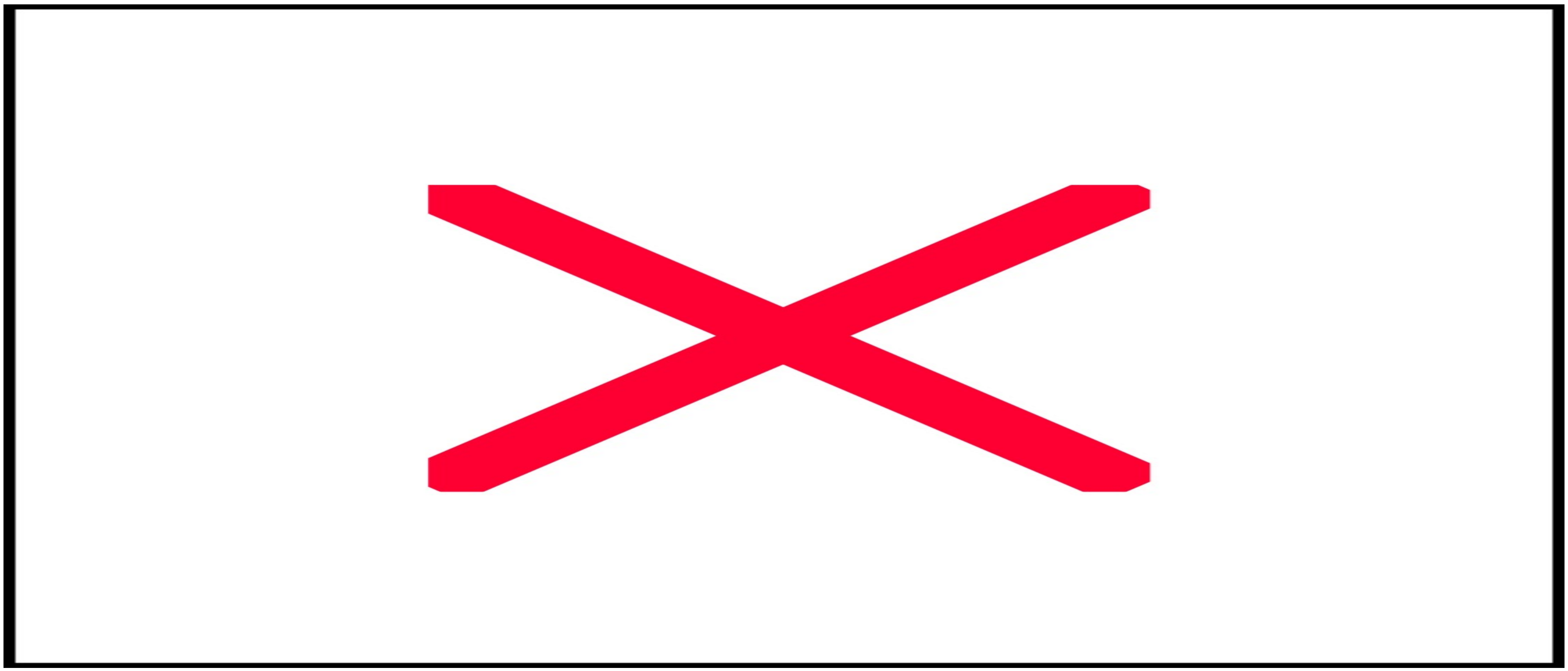
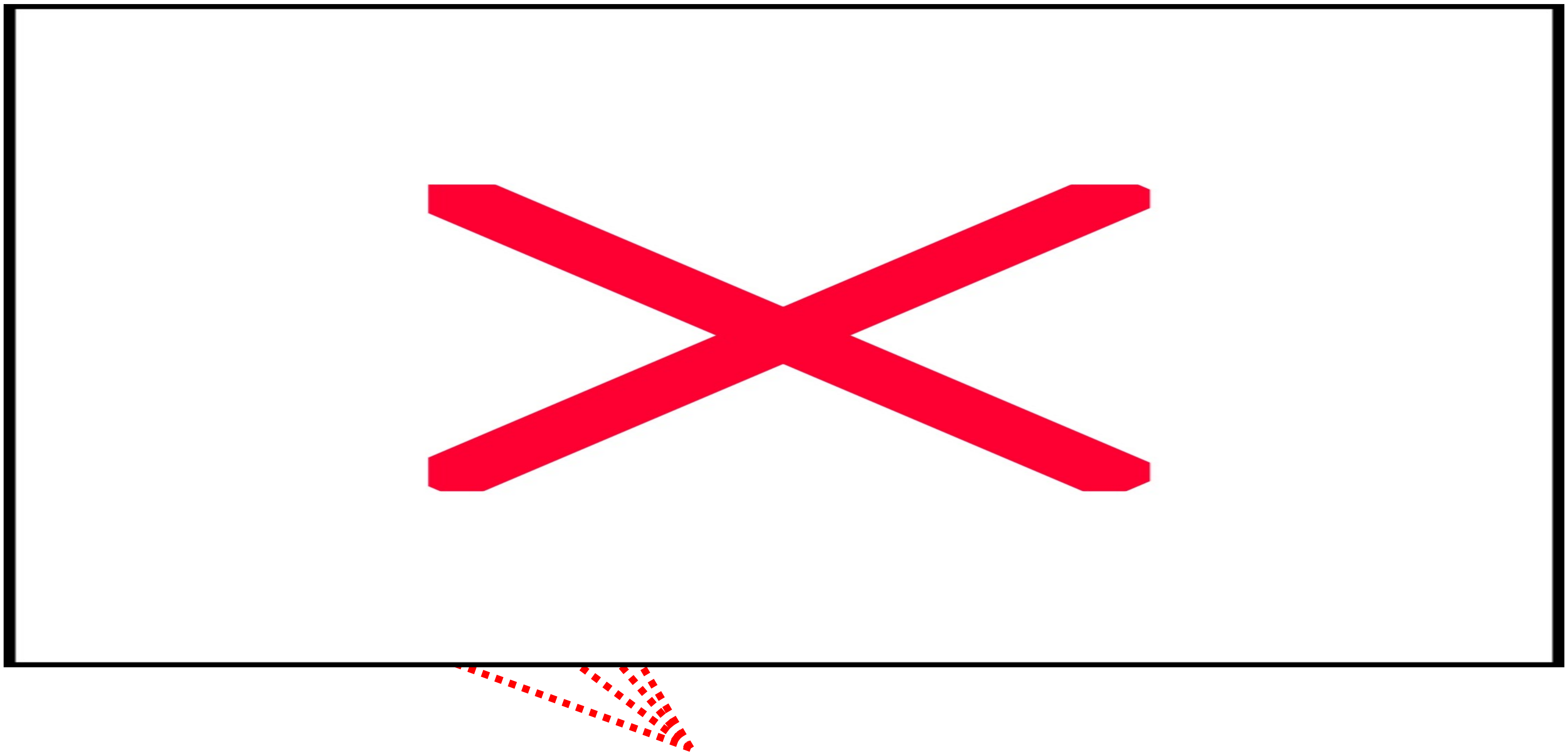
Today's class

- Discussion of the three momentum equations
- Scale analysis for the midlatitudes
- Geostrophic balance / geostrophic wind
- Ageostrophic wind
- Hydrostatic balance

Full momentum equation:
in component form

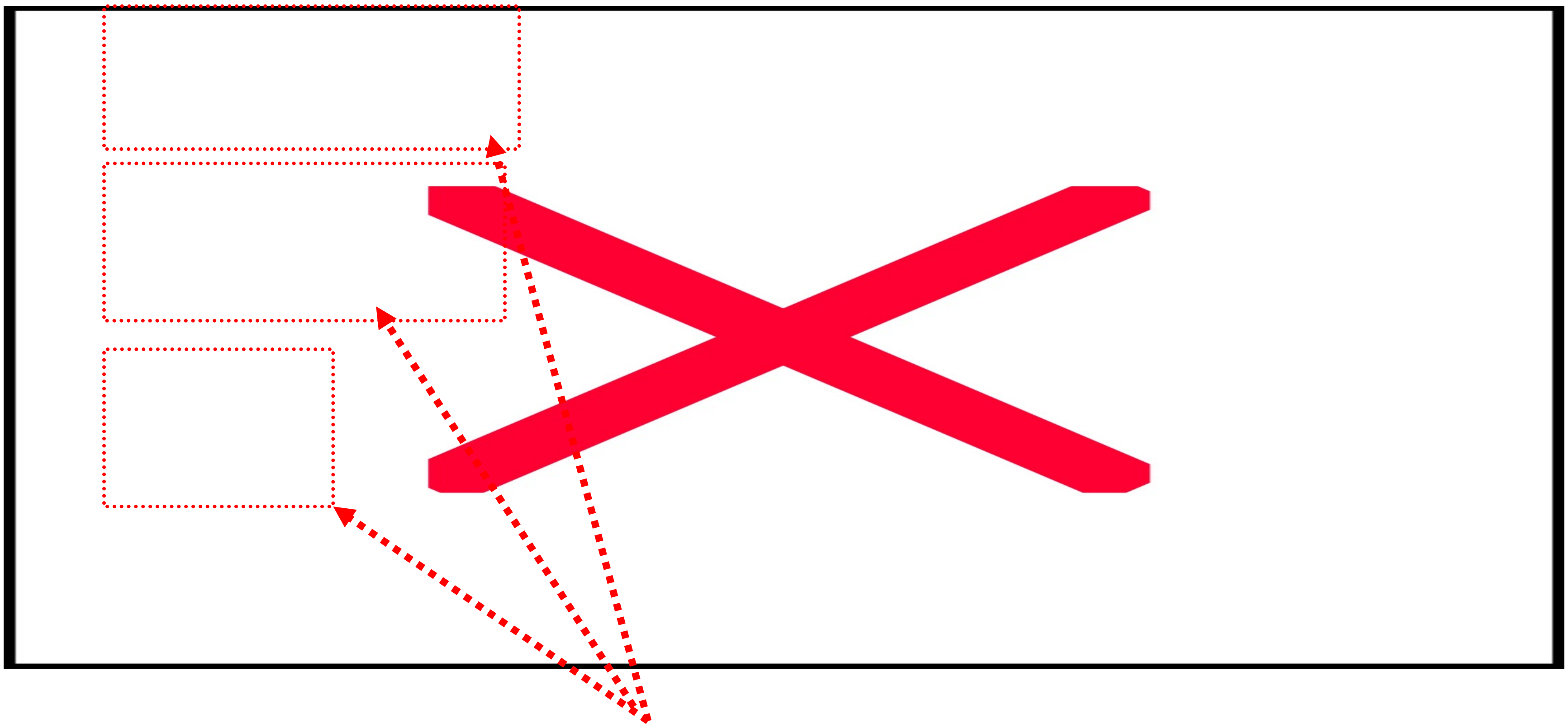


Let's think about this equation (1)



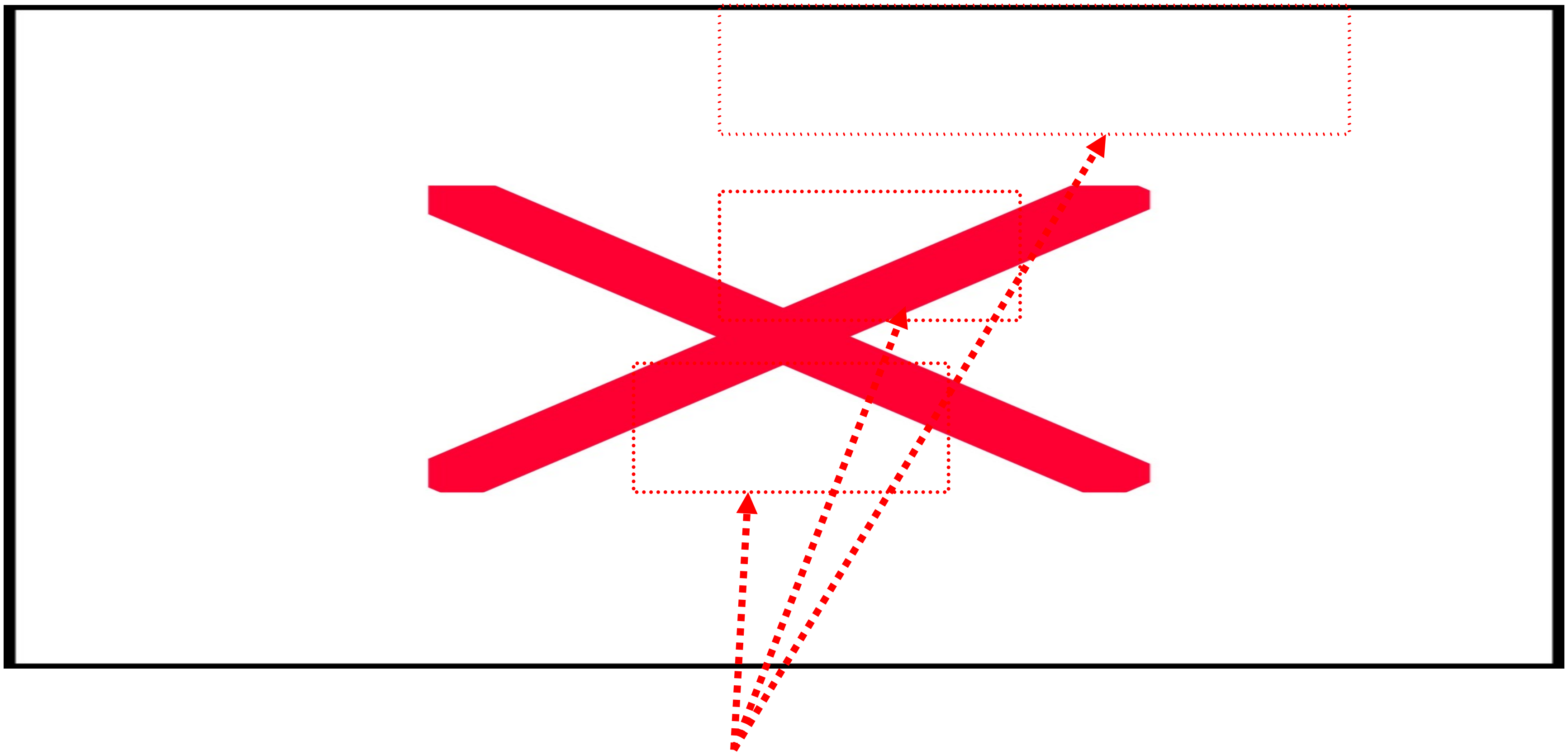
The equations are explicitly non-linear.

Let's think about this equation (2)



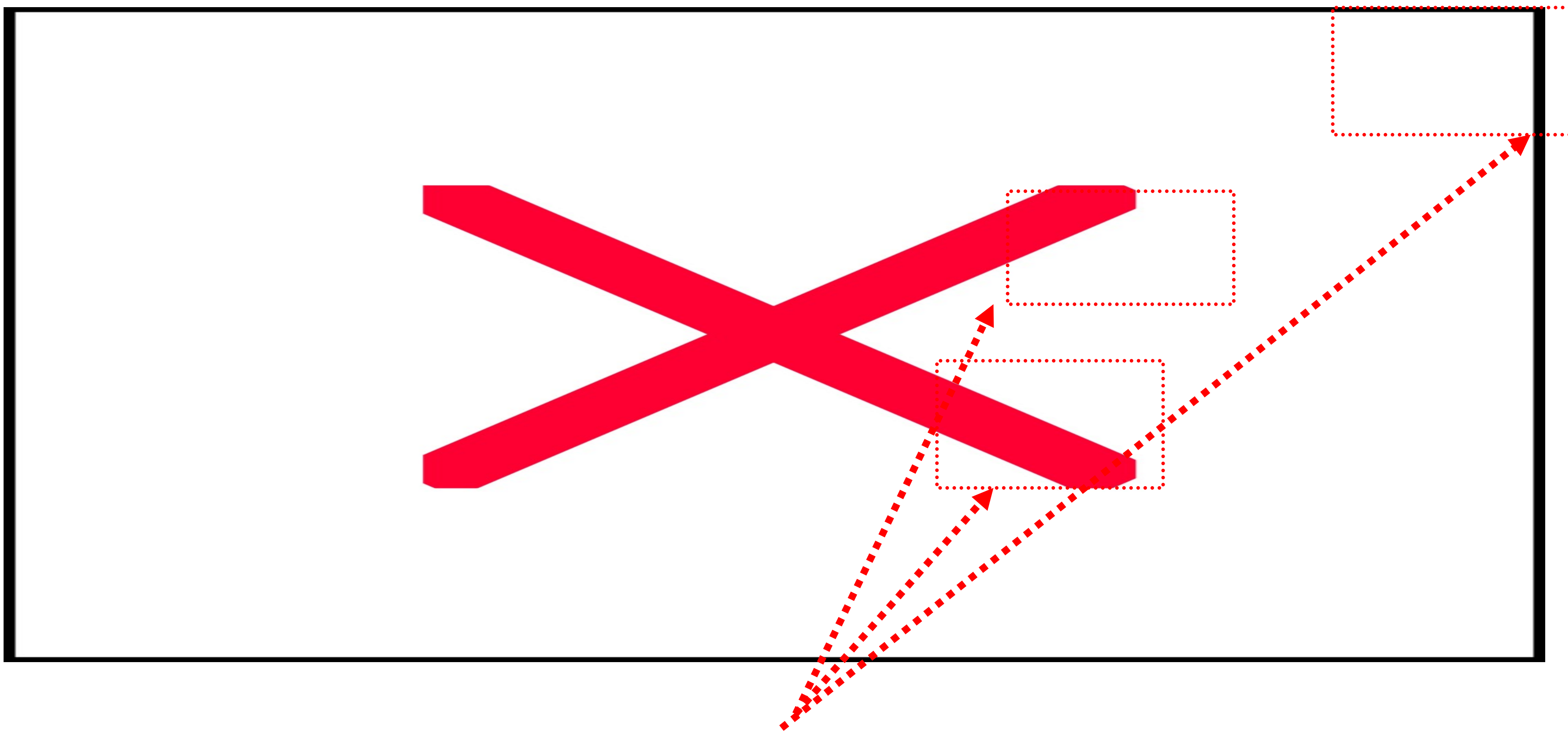
These are the curvature terms that arise because the tangential coordinate system curves with the surface of the Earth.

Let’s think about this equation (3)



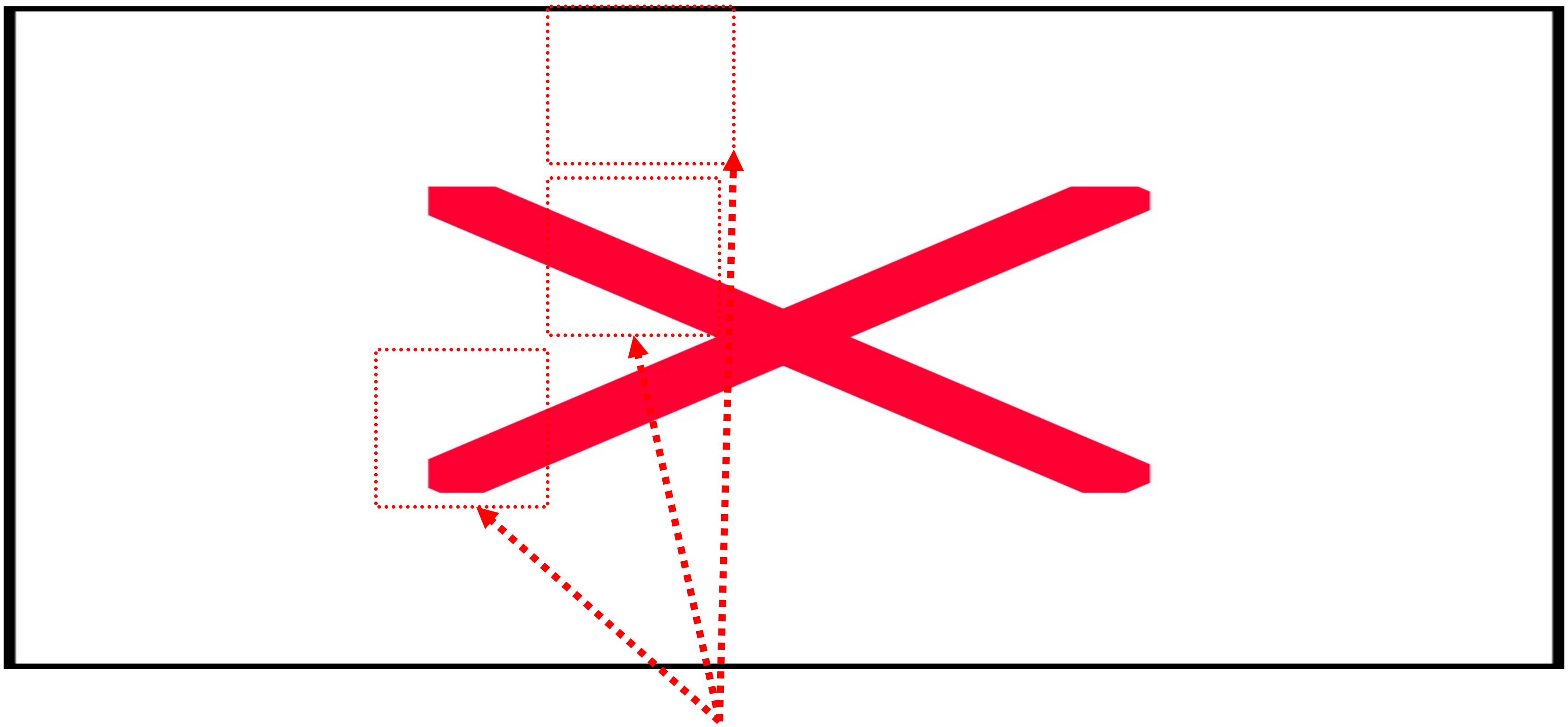
These are the Coriolis terms that arise because the tangential coordinate system rotates with the Earth.

Let's think about this equation (4)



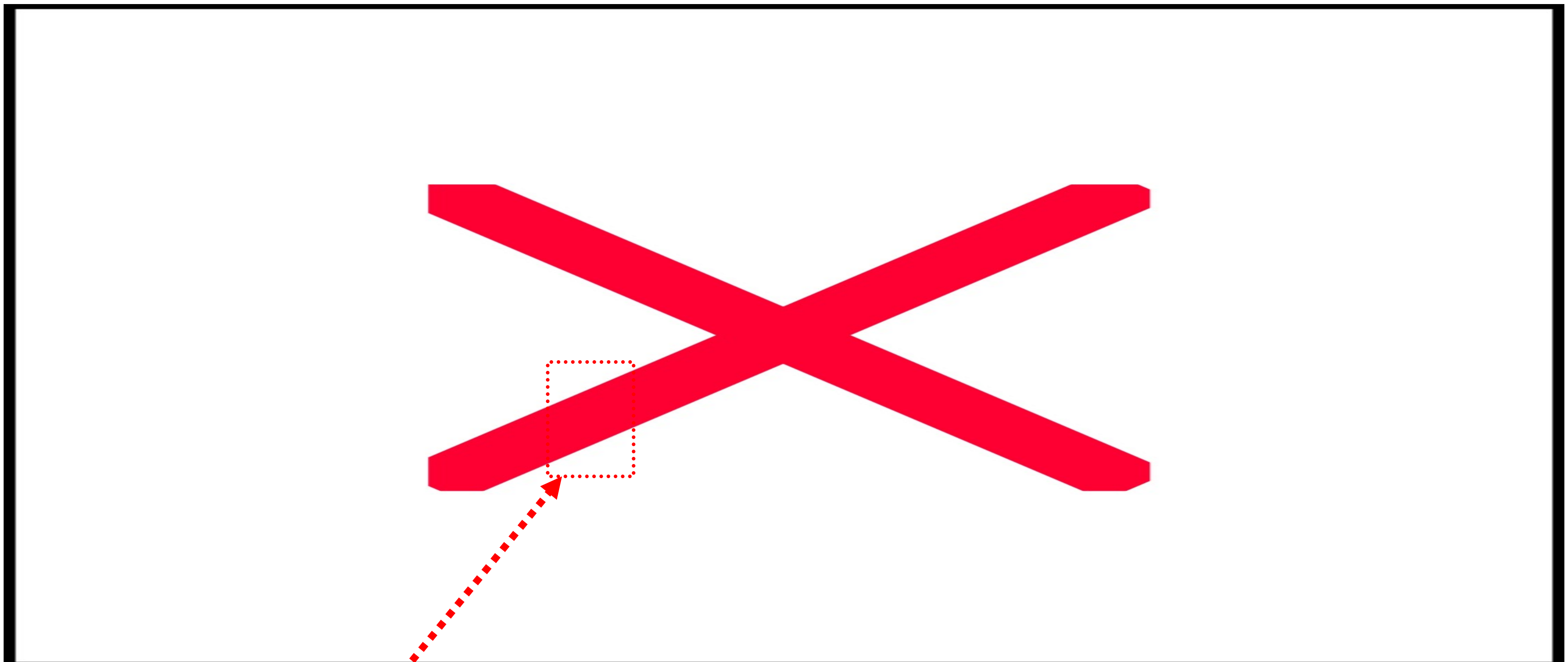
These are the friction terms, the viscous forces, that arise because the atmosphere is in motion. They oppose the motion.

Let's think about this equation (5)



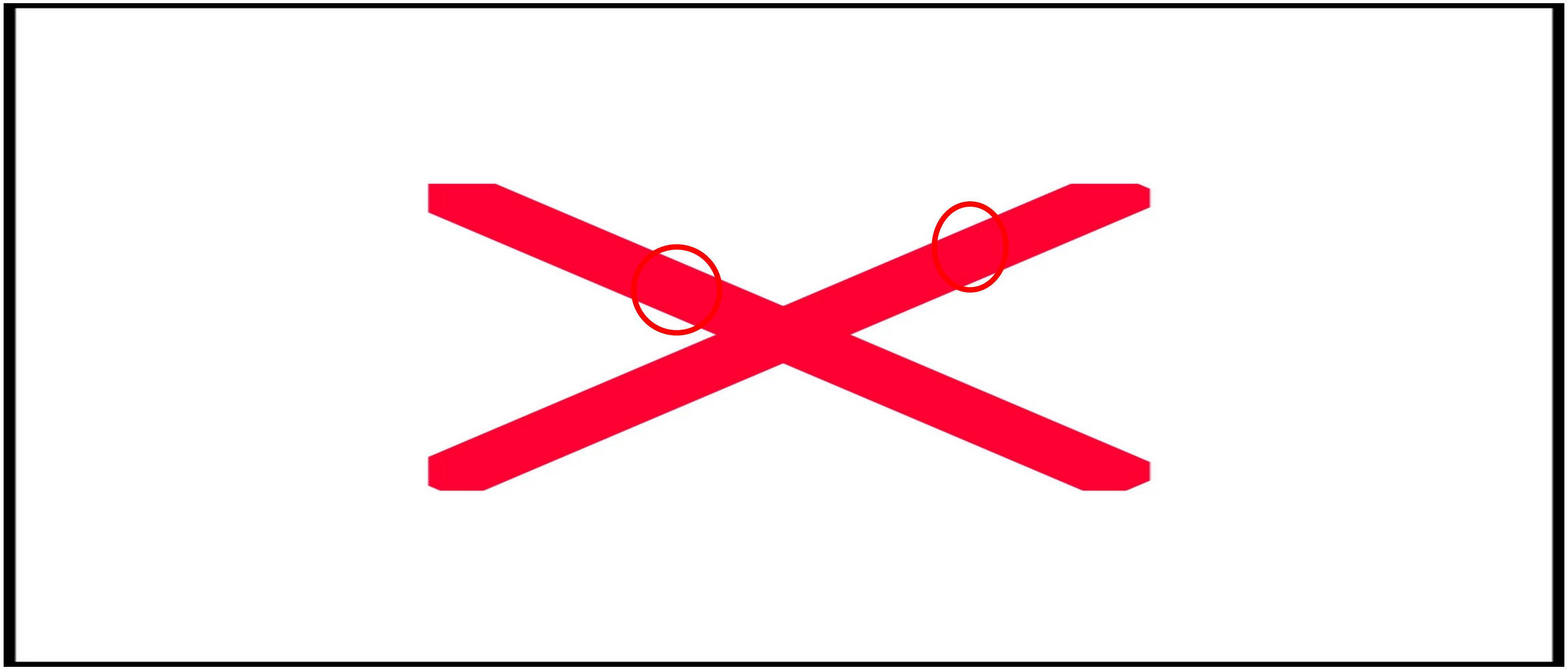
These are the pressure gradient terms. They initiate the motion.

Let's think about this equation (6)



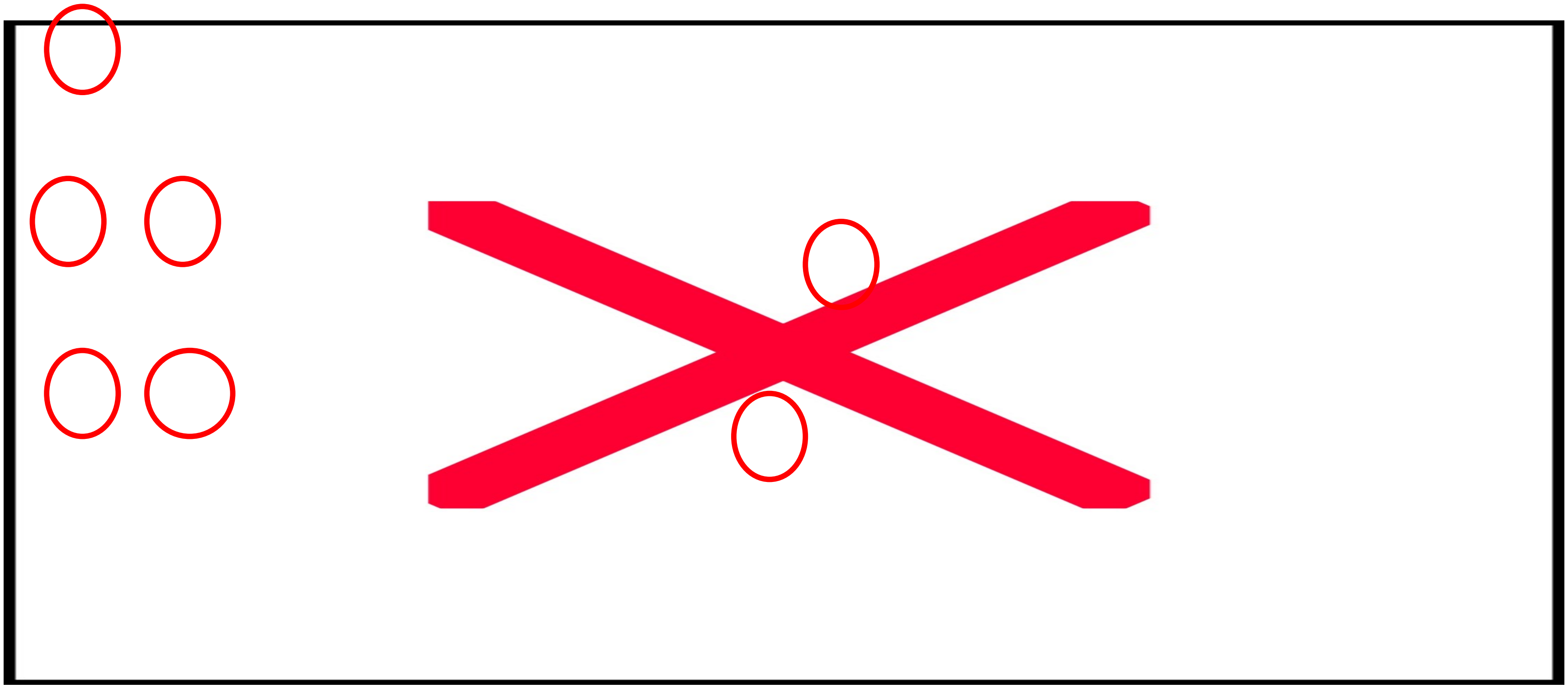
This is gravity (combined with the apparent centrifugal force). Gravity stratifies the atmosphere, with pressure decreasing with height. The motion of the atmosphere is largely horizontal in the x-y plane.

Let's think about this equation (7)



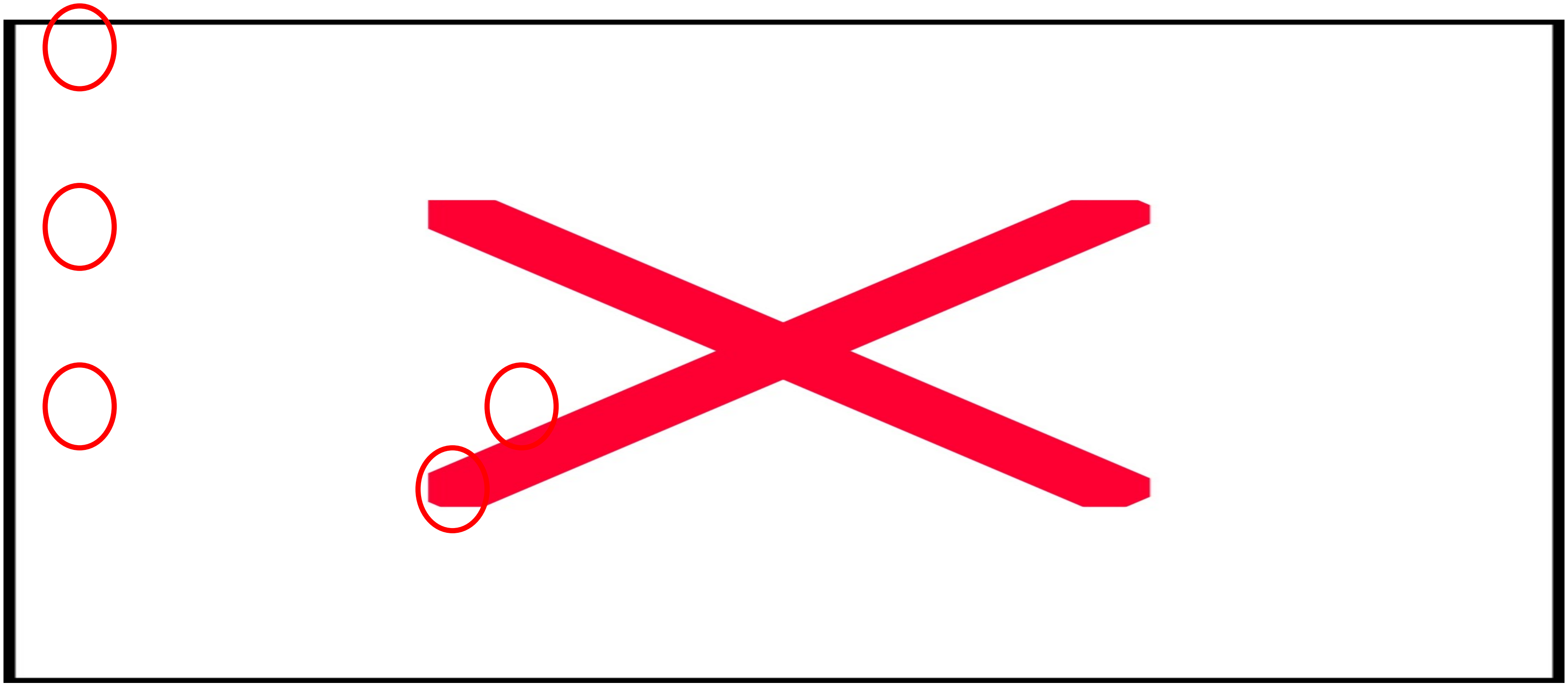
We are mixing (x, y, z) with (λ, Φ, z).

Let's think about this equation (8)



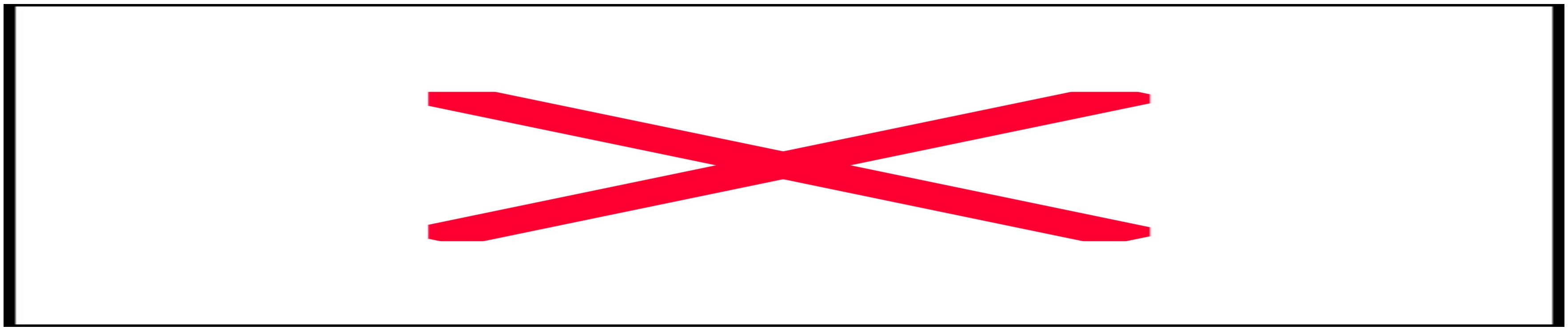
The equations are coupled.

Let's think about this equation (9)



We have u, v, w, ρ, p which depend on (x, y, z, t).
Can we solve this system of equations?

Consider x and y components of the momentum equation

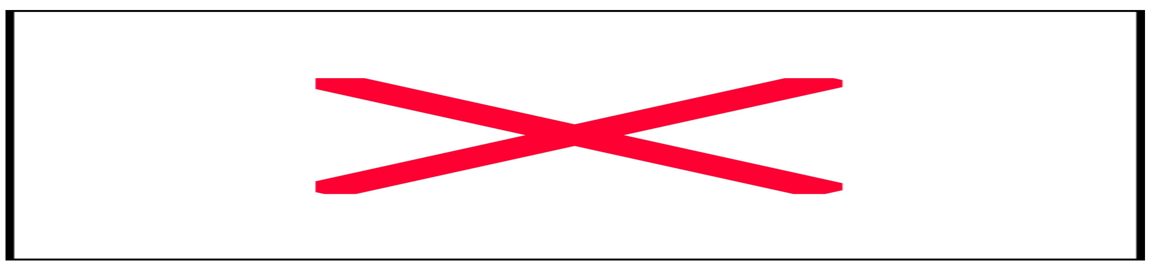


What are the units? Do they check out?

Time scales

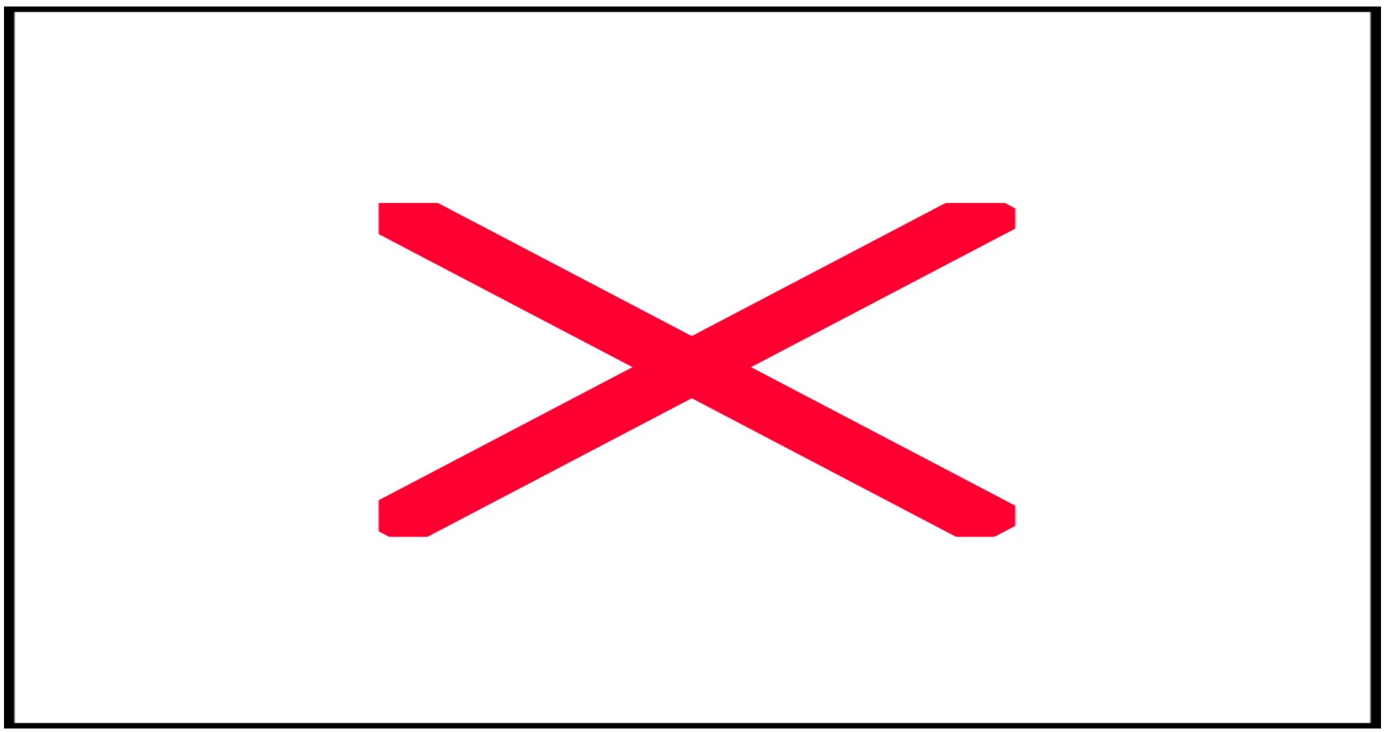
D ( )/Dt looks like 1/T (with T: time scale)

We recognize that time can be characterized by
a distance (length scale L) divided by a velocity (with velocity scale U): T = L/U
This gives us the unit 's' for the time scale.

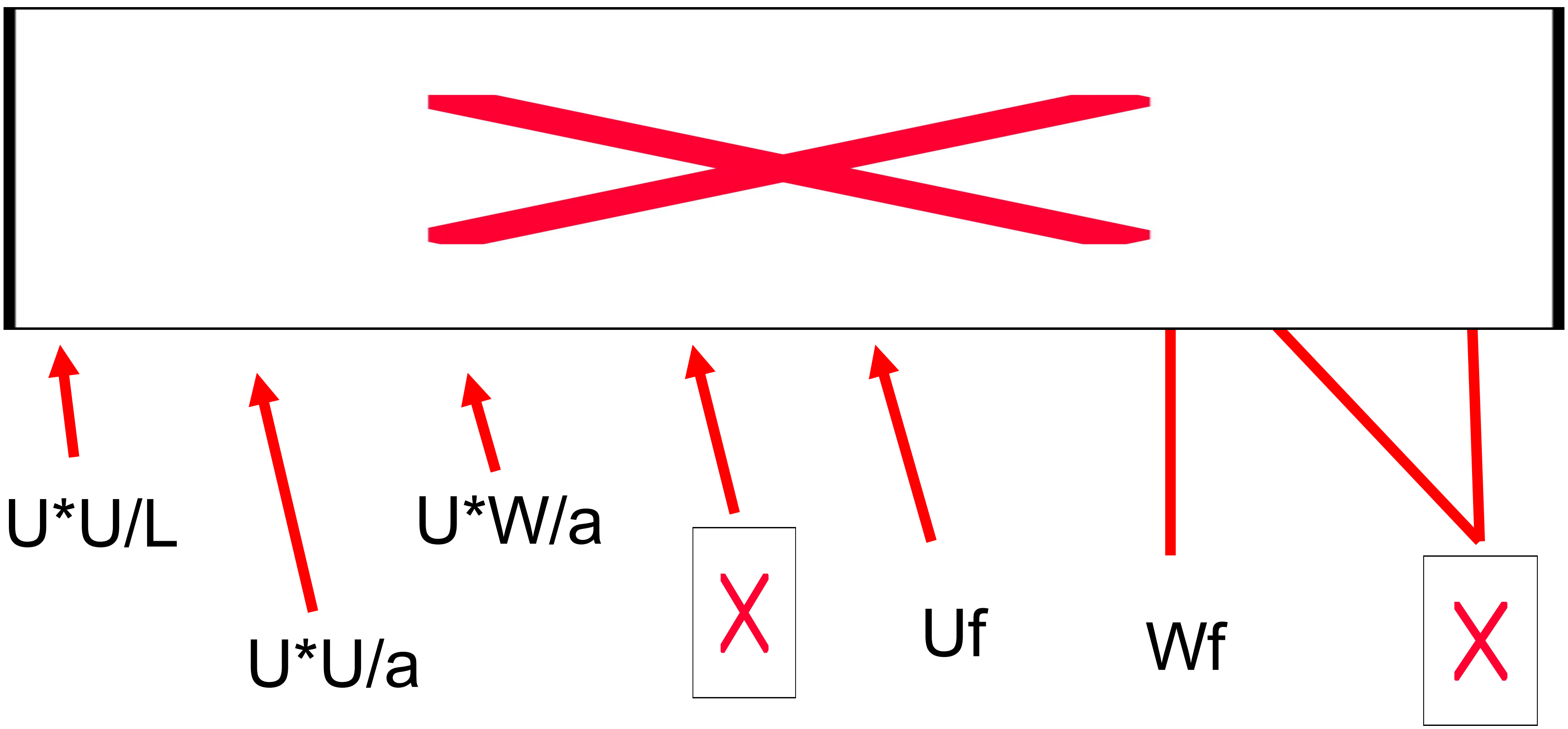


Scale Analysis:

Let us define



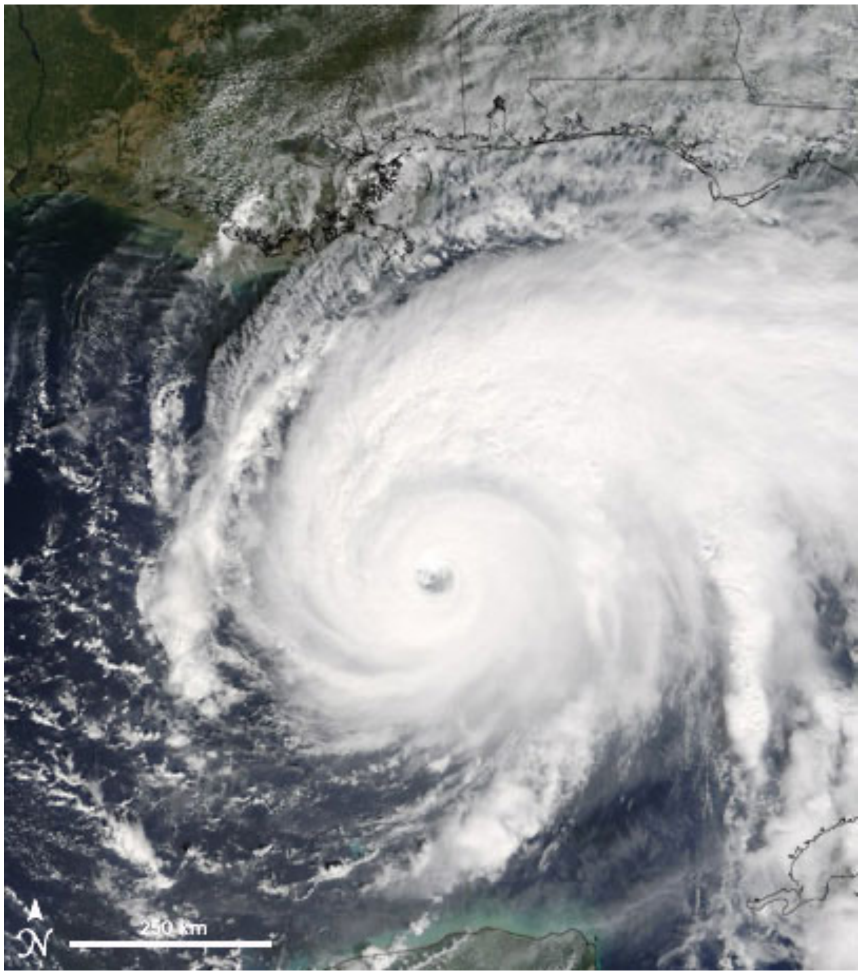
What are the scales of the terms?



Scales of atmospheric phenomena: Tornadoes

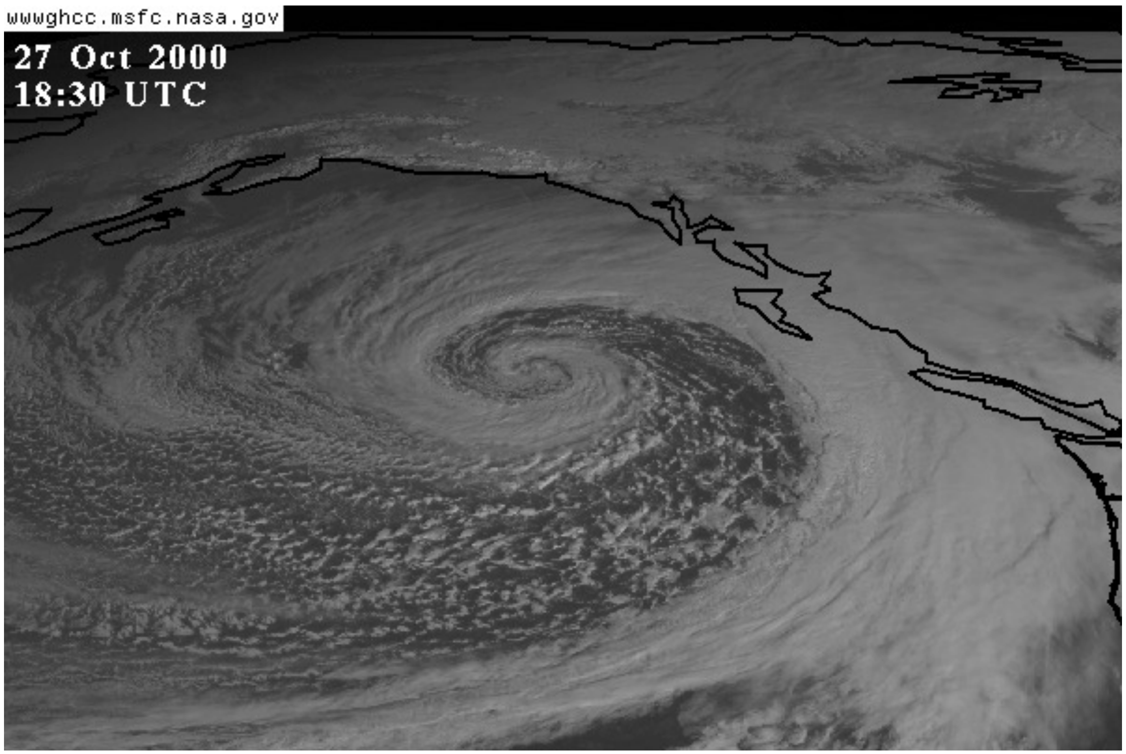
- Funnel cloud that emerges from a thunderstorm
- Scale of the motion?

Scales of atmospheric phenomena: Hurricanes



- Satellite image
- Tropical storm that originates over warm ocean water
- Scale of the motion?

Scales of atmospheric phenomena: Extratropical storm systems



- Satellite image
- Storm system in the Gulf of Alaska
- Scale of the motion ?

Scales for large mid-latitude systems

horizontal wind

vertical wind

length

height

radius

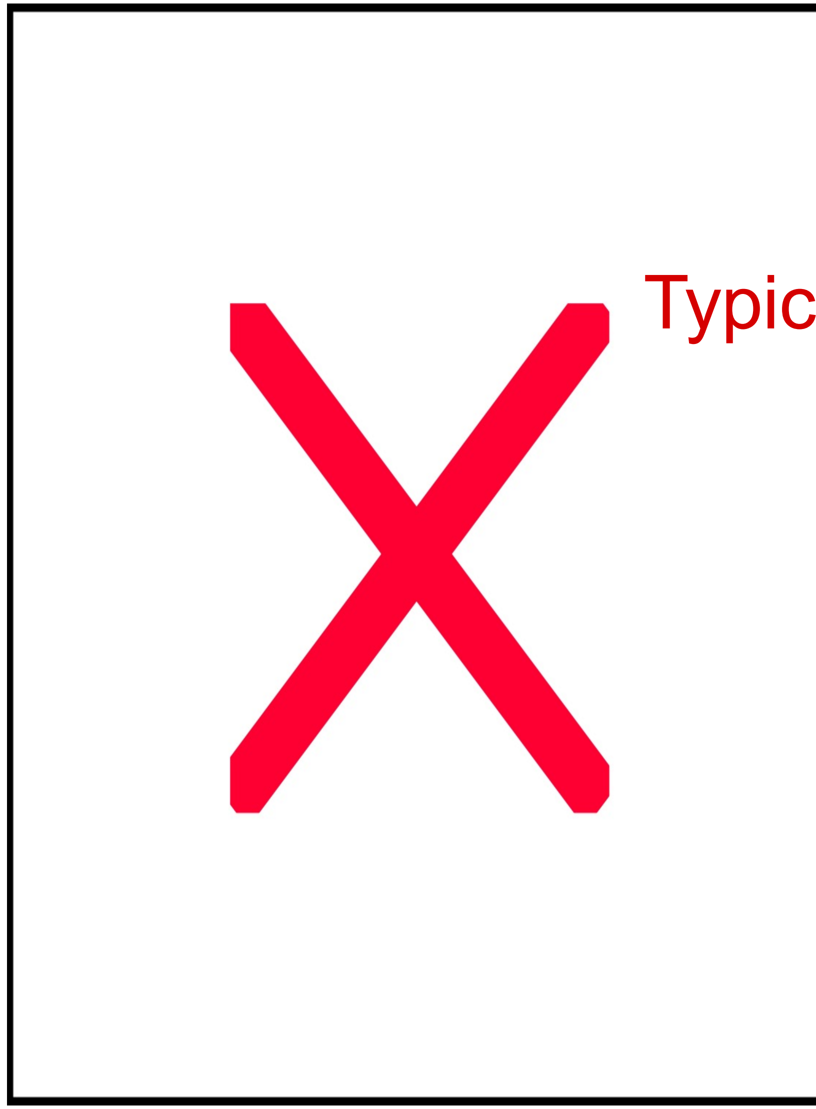
gravity

pressure fluctuations

time

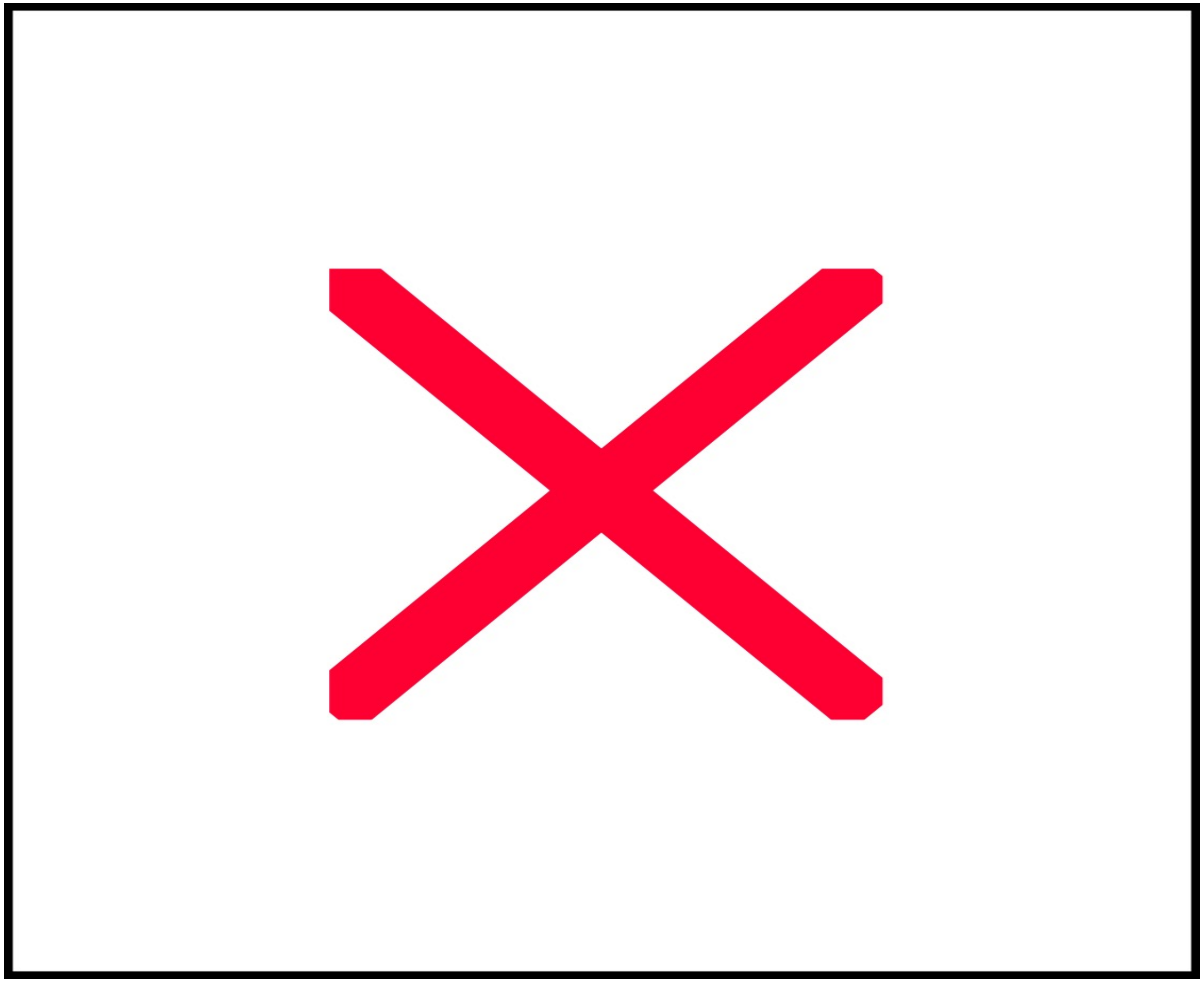
viscosity

Coriolis term

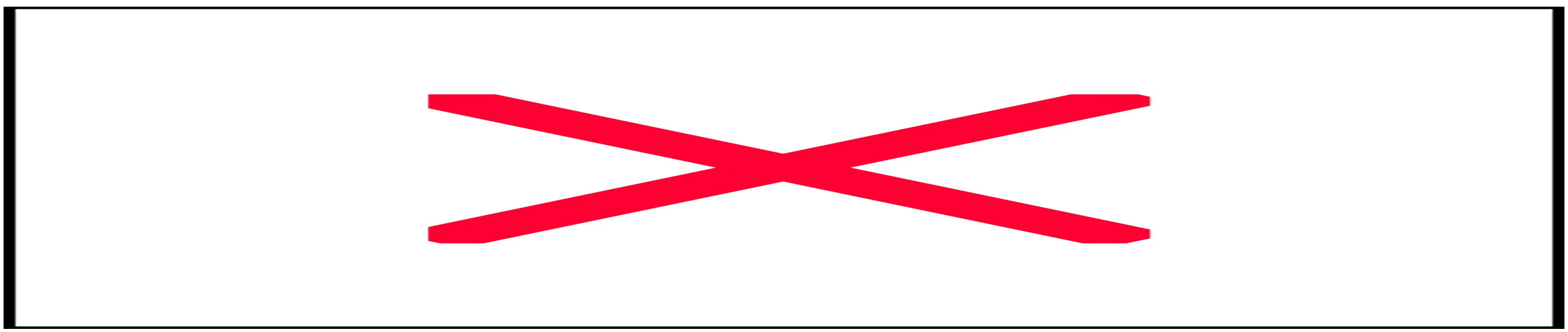


Typical scales ?

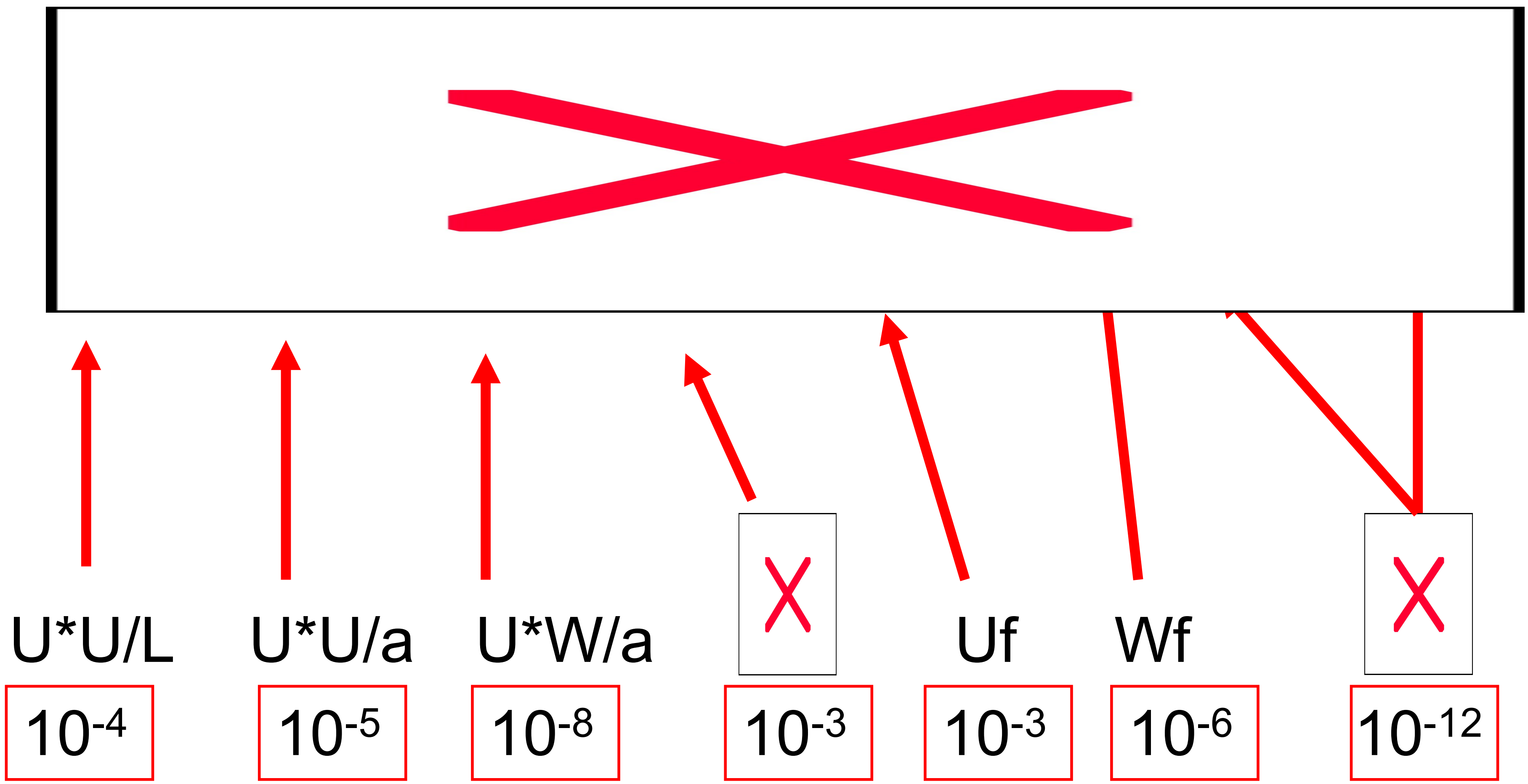
Scales for "large-scale" mid-latitude weather systems



What are the scales of the terms?
Class exercise



What are the scales of the terms?



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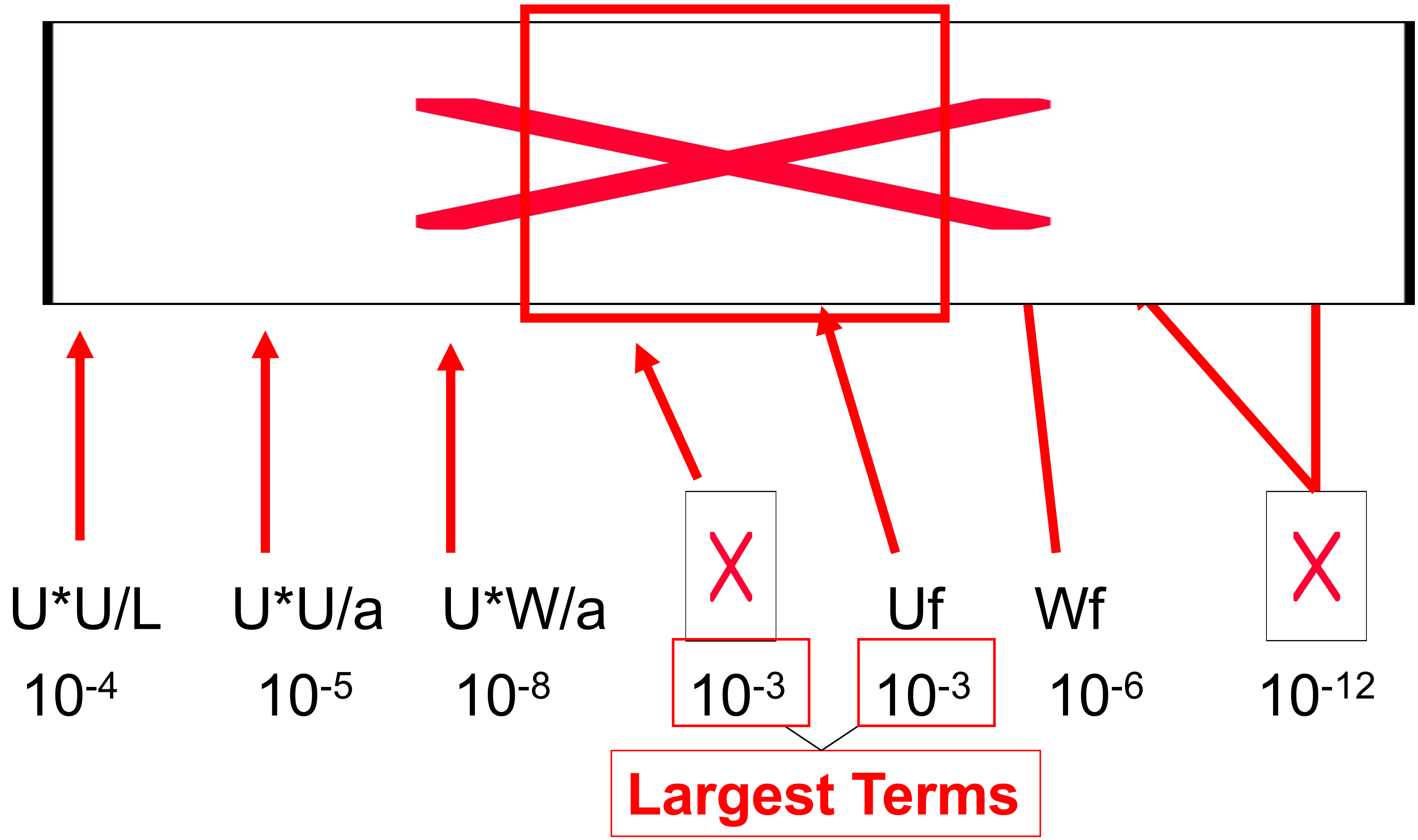
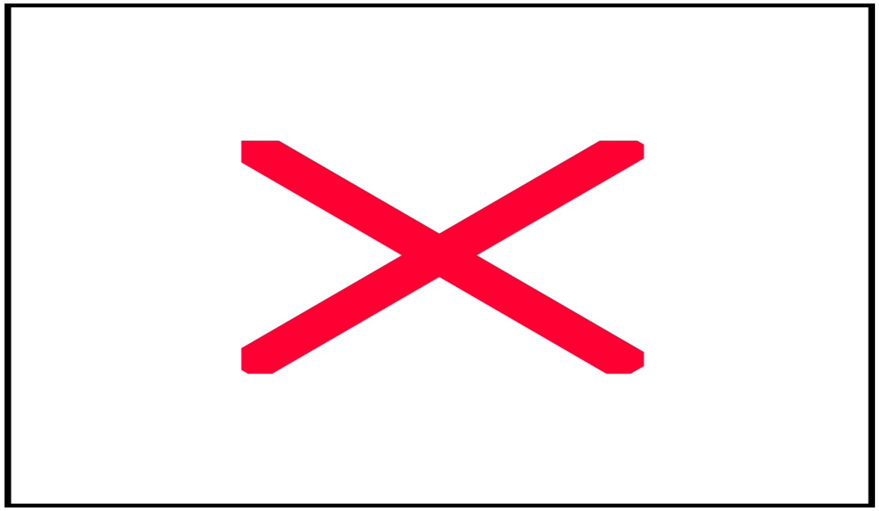


Consider only the largest terms

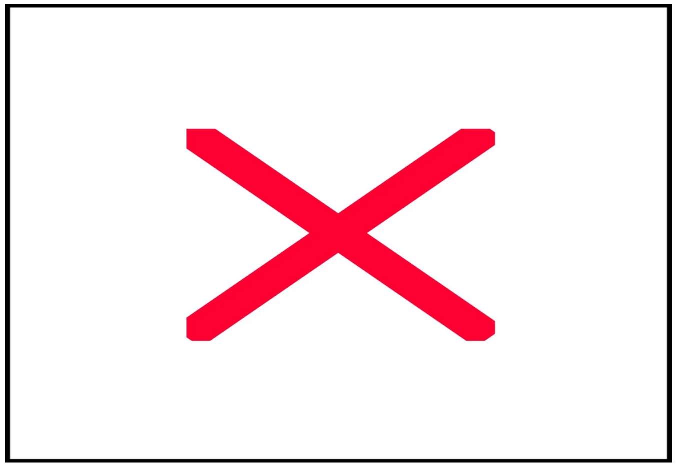


This is a dominant **balance** between the

- pressure gradient force
- Coriolis force (the dominant sin(ϕ) components of the Coriolis force)

This is the **geostrophic balance**.

Consider only the largest terms



Note: There is no D(   )/Dt term. Hence, no acceleration, no change with time. This is a balance.

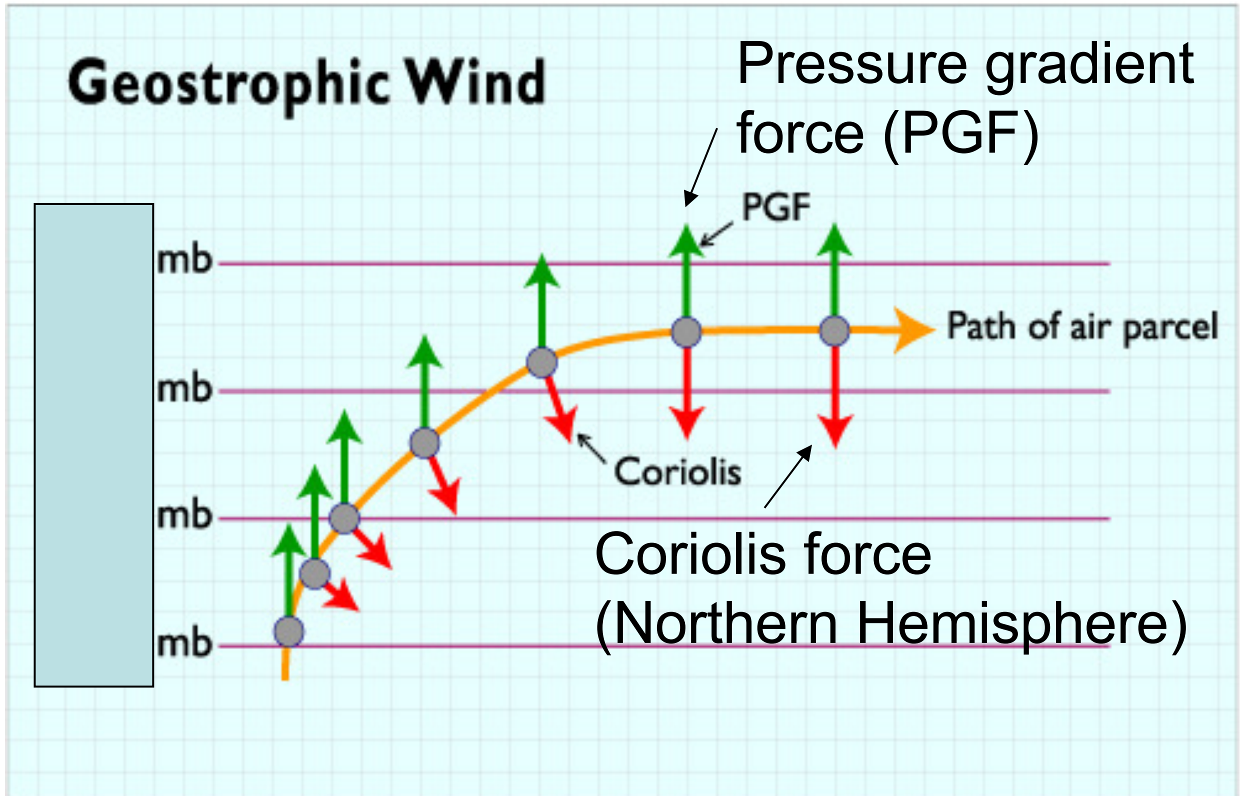
This is the **geostrophic balance**.

Geostrophic balance

Check out:
http://itg1.meteor.wisc.edu/wxwise/AckermanKnox/chap6/balanced_flow.html

Low Pressure

High Pressure



In Northern Hemisphere: Coriolis force points to the right (perdendicular) relative to the path of the air parcel

Geostrophic Wind

Component form:

Vector form:

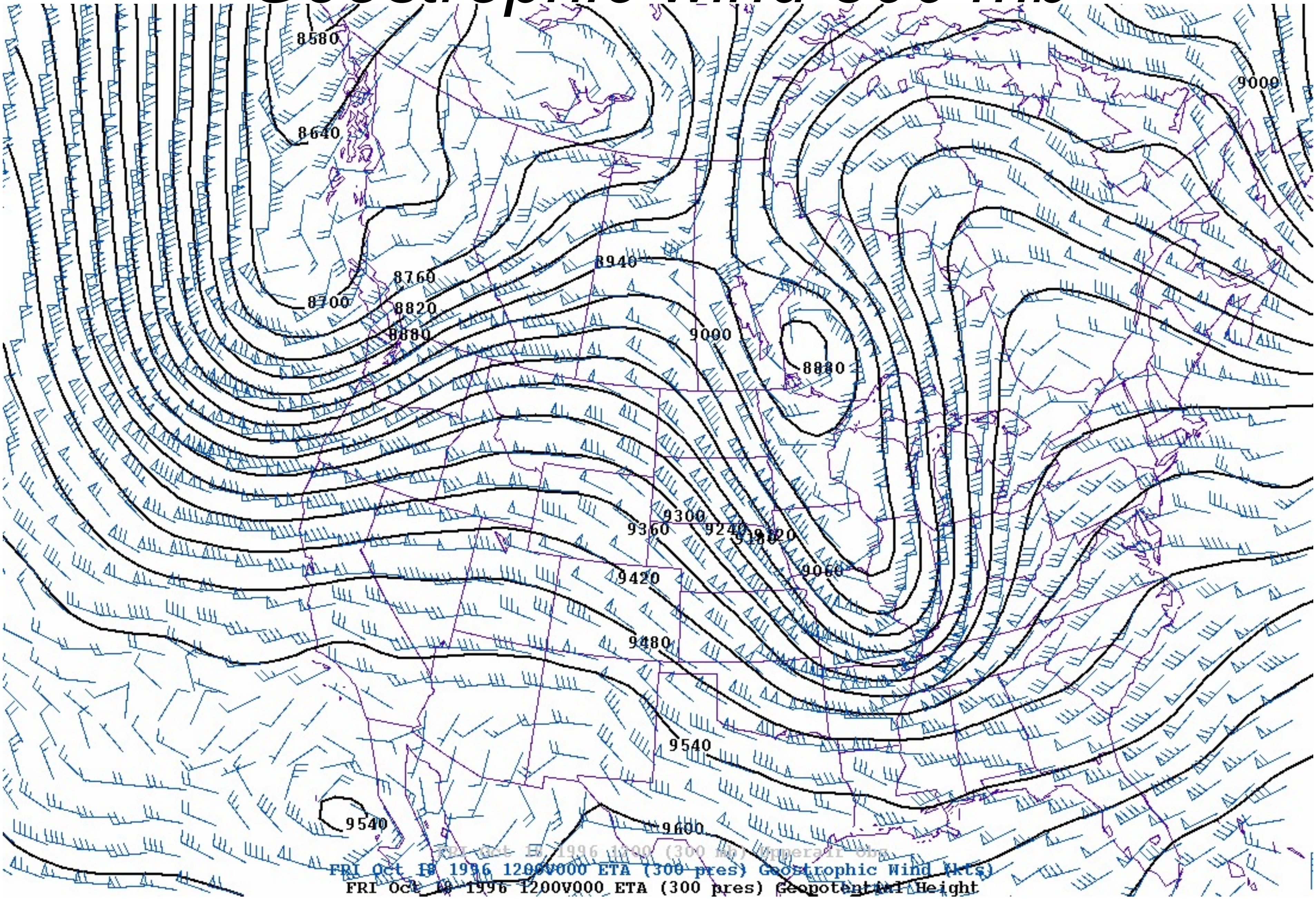
with the Coriolis parameter f = 2Ω sinϕ

Note: There is no D( )/Dt term.

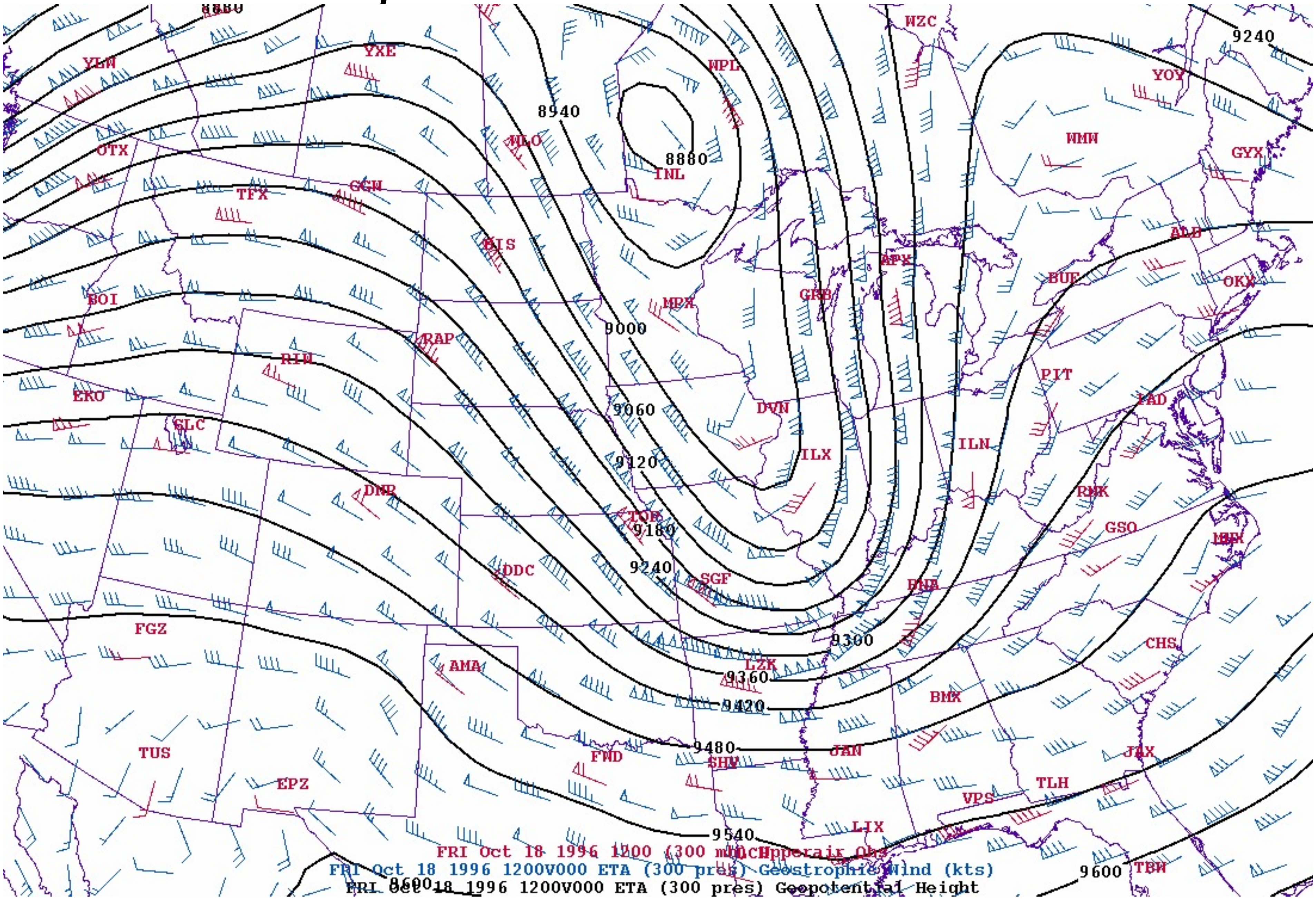
Hence, no acceleration, no change with time.

The geostrophic wind describes the dominant balance between the pressure gradient force and the Coriois force.

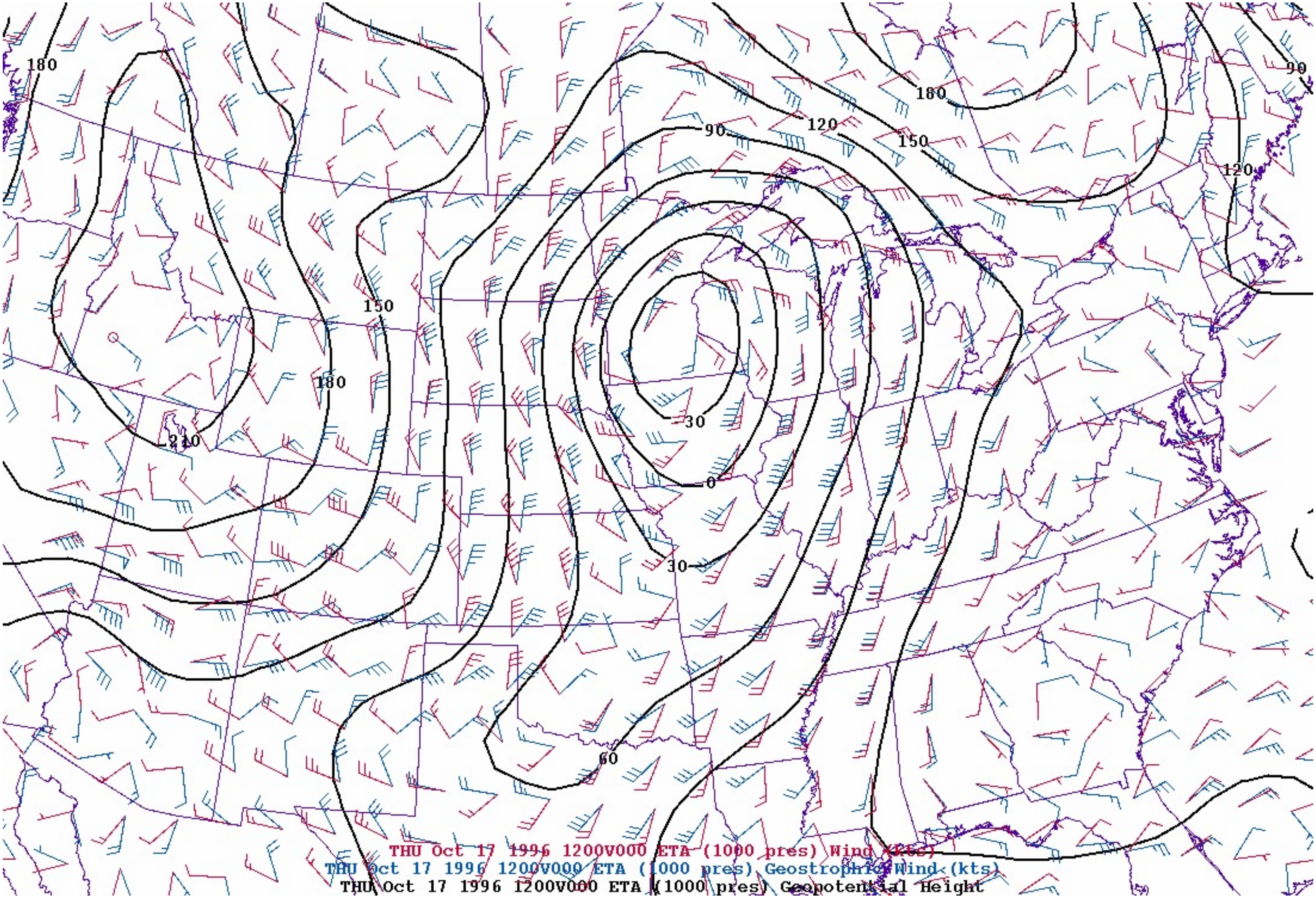
Geostrophic wind 300 mb



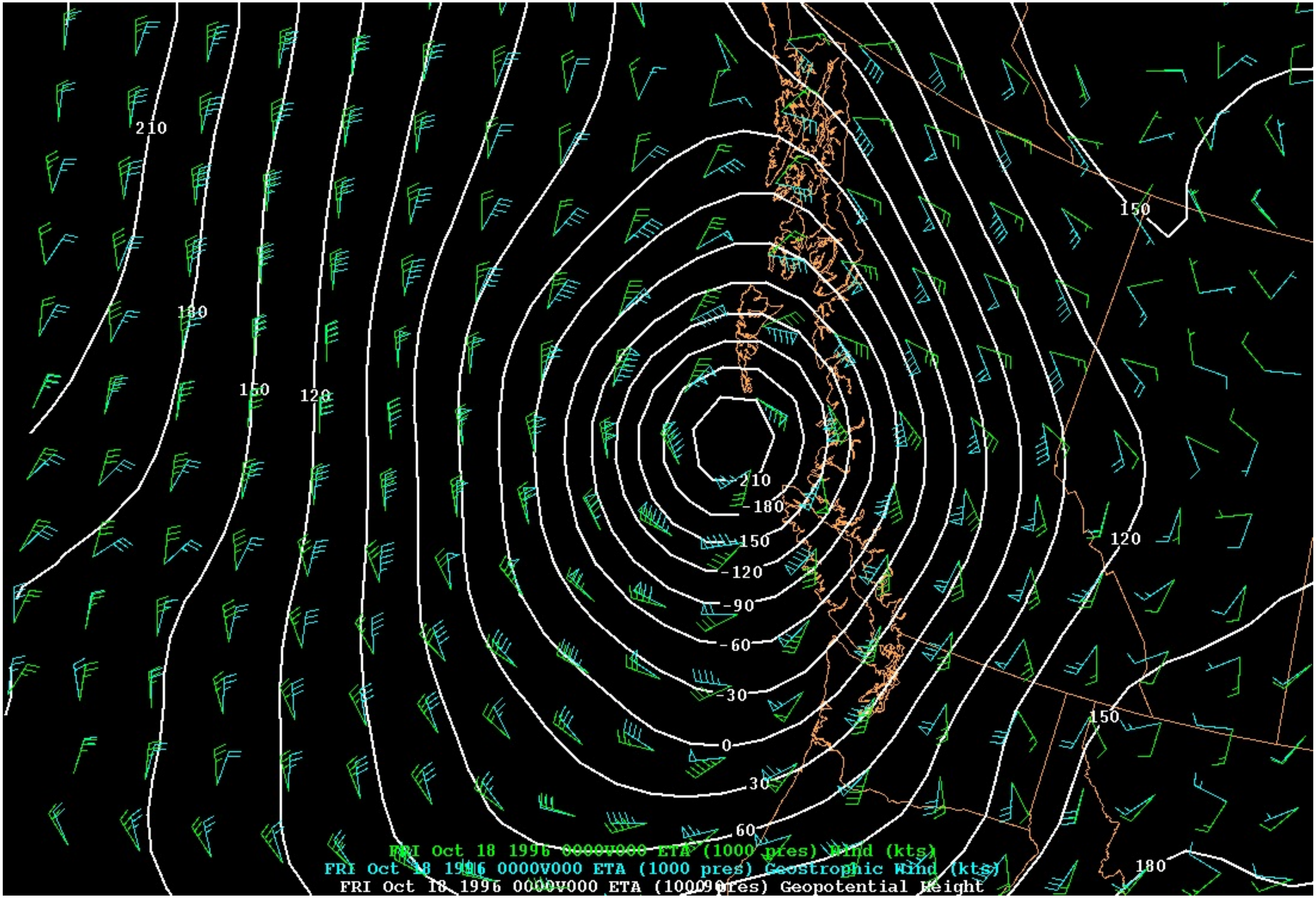
Geostrophic & observed wind 300 mb



Geostrophic and observed wind 1000 mb (land)



Geostrophic and observed wind 1000 mb (ocean)



Geostrophic wind

- The direction of the geostrophic wind is parallel to the isobars.
- The geostrophic wind $\mathbf{v}_g$ is a good approximation of the real horizontal wind vector, especially over oceans and at upper levels. Why?
- The closer the isobars are together, the stronger the magnitude of the geostrophic wind $| \mathbf{v}_g |$ (isotachs increase).

Geostrophic Wind

Component form:

Vector form:

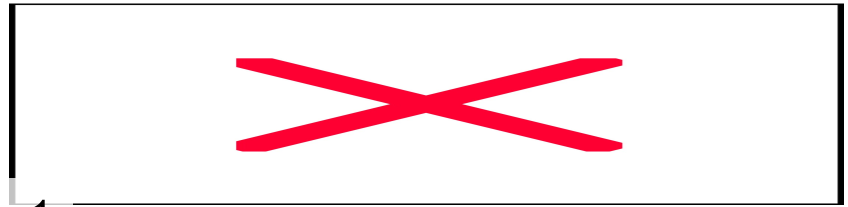
with the Coriolis parameter $f = 2\Omega \sin\phi$

Note: There is no D( )/Dt term.

Hence, no acceleration, no change with time.

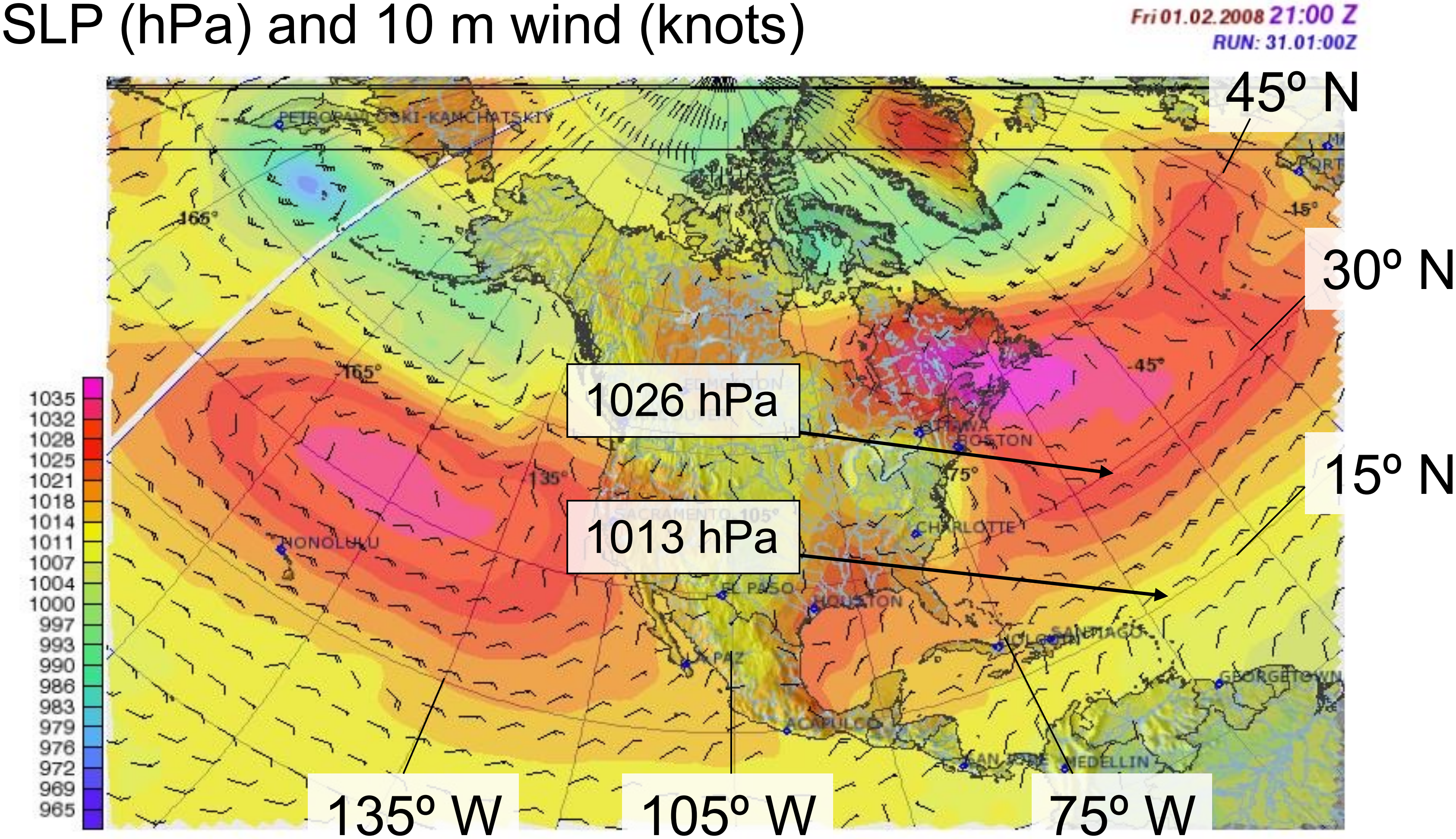
This is a **DIAGNOSTIC** equation

Geostrophic Wind



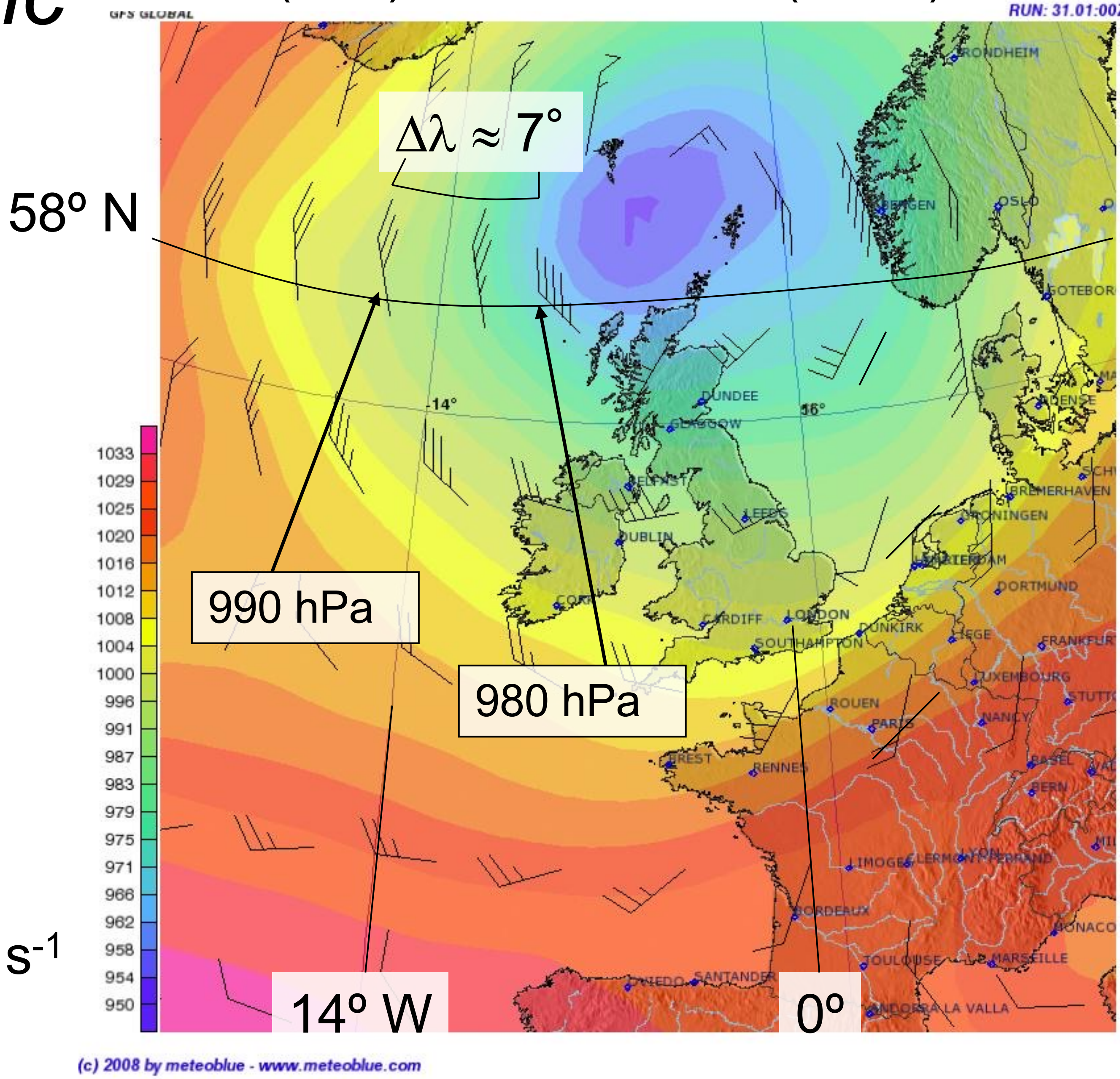
$f = 2\,\Omega \sin(\phi)$, $\Omega \approx 7.292 \times 10^{-5}\ s^{-1}$

MSLP (hPa) and 10 m wind (knots)



Geostrophic Wind

MSLP (hPa) and 10 m wind (knots)

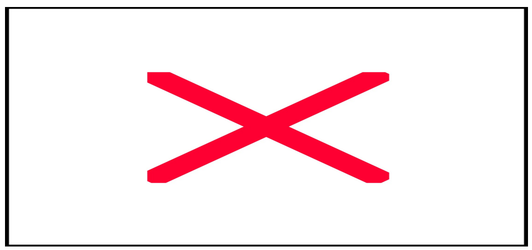


$f = 2\,\Omega \sin(\phi)$

$\Omega \approx 7.292 \times 10^{-5}\ s^{-1}$

Rossby number

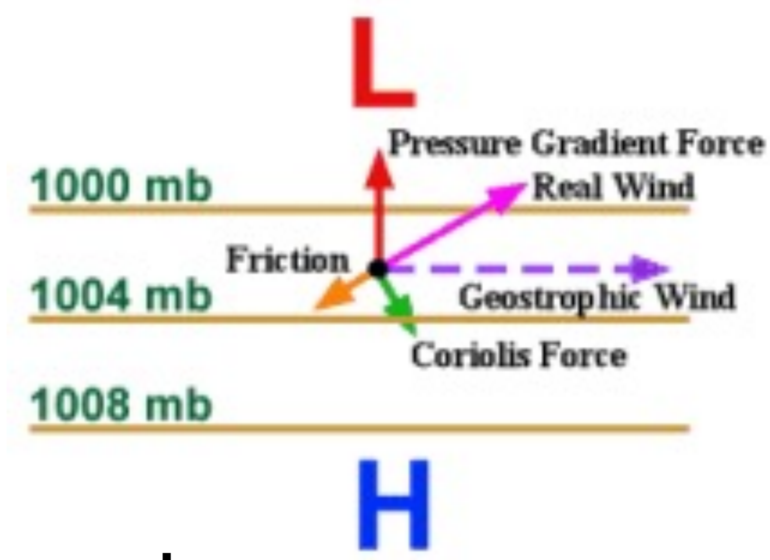
- Rossby number: A dimensionless number that indicates the importance of the Coriolis force
- Defined as the ratio of the characteristic scales for the acceleration and the Coriolis force



- Small Rossby numbers (≈ 0.1 and smaller): Coriolis force is important, the geostrophic relationship (in midlatitudes) is valid.
- What are typical Rossby numbers of midlatitudinal cyclones and tornadoes?

Impact of friction

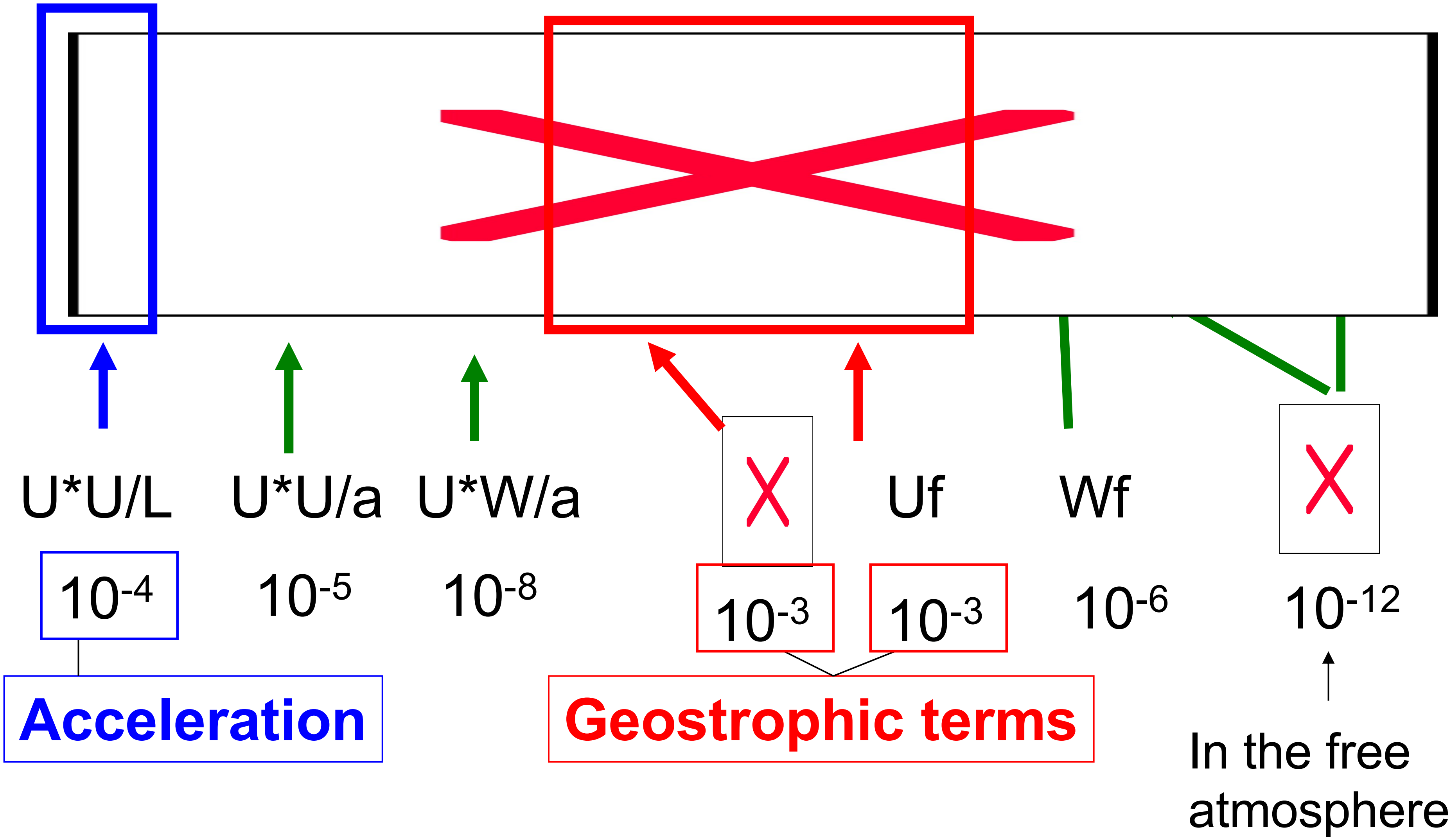
- http://ww2010.atmos.uiuc.edu/(Gh)/guides/mtr/fw/fric.rxml
- Friction (especially near the Earth's surface) changes the wind direction and slows the wind down, thereby reducing the Coriolis force.
- The pressure gradient force becomes more dominant. As a result, the total wind deflects slightly towards lower pressure.
- The (ageostrophic) wind reduces the differences between high and low pressure!



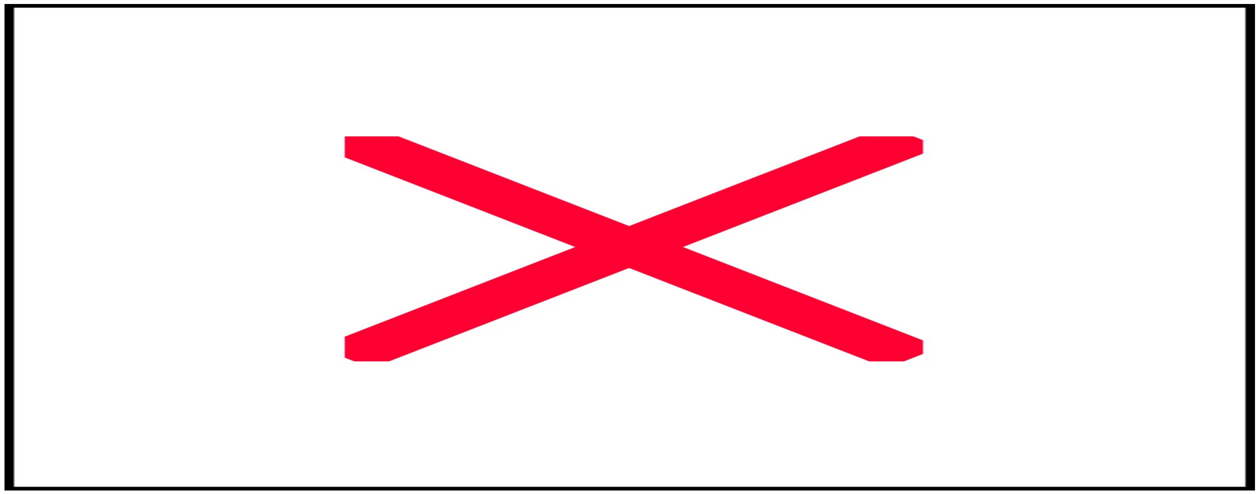
Diagnostic and Prognostic

- In order to predict what is going to happen next, we need to know the rate of change with time. We need D ( )/Dt. This is known as a **prognostic** equation.
- Scale analysis shows that for middle latitude large scale motion (1000 km spatial), 1 day (temporal)) the acceleration term is an order of magnitude smaller than the geostrophic terms.
- Hence, for much of the atmosphere we can think of the geostrophic balance as some notion of an equilibrium state, and we are interested in determining the difference from this equilibrium state.
- **Ageostrophic is this difference from geostrophic.**

What are the scales of the terms?



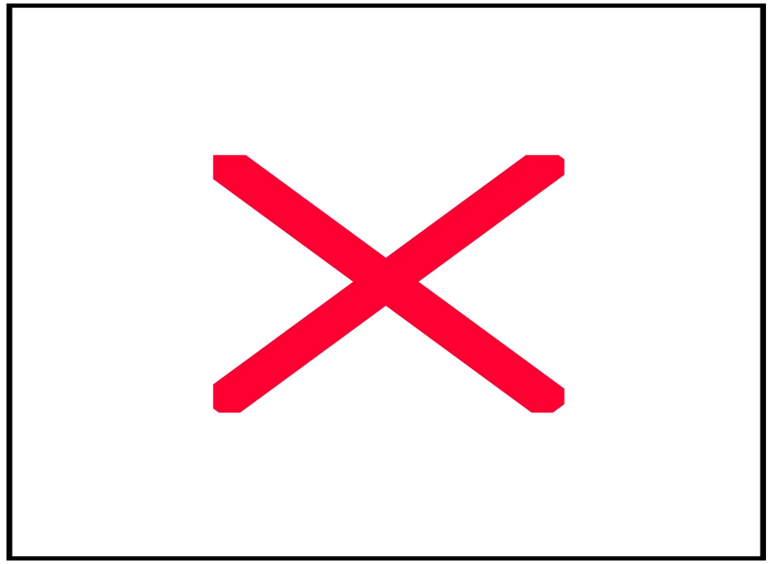
A simple prognostic equation



$\mathbf{v}_g$: geostrophic wind vector, not the real wind. Within 10-15% of real wind in middle latitudes, large-scale.

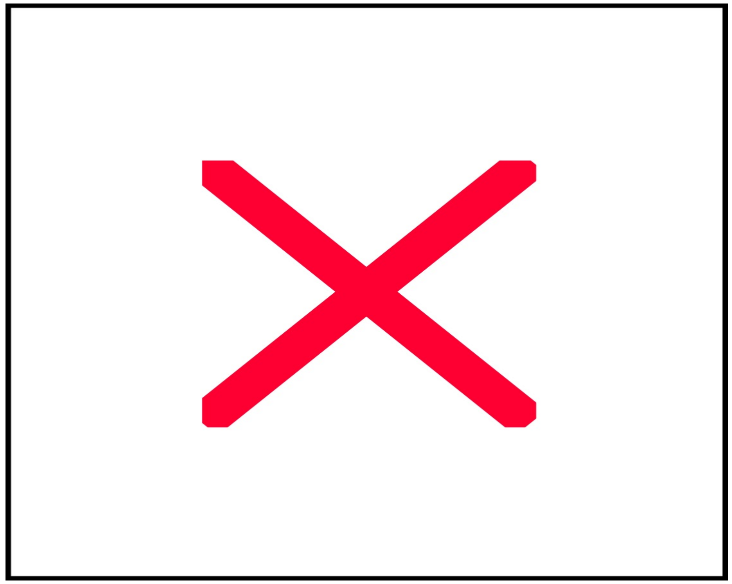
$\mathbf{v}_a$: ageostrophic wind vector, difference between actual (real) wind and geostrophic wind.

A simple prognostic equation



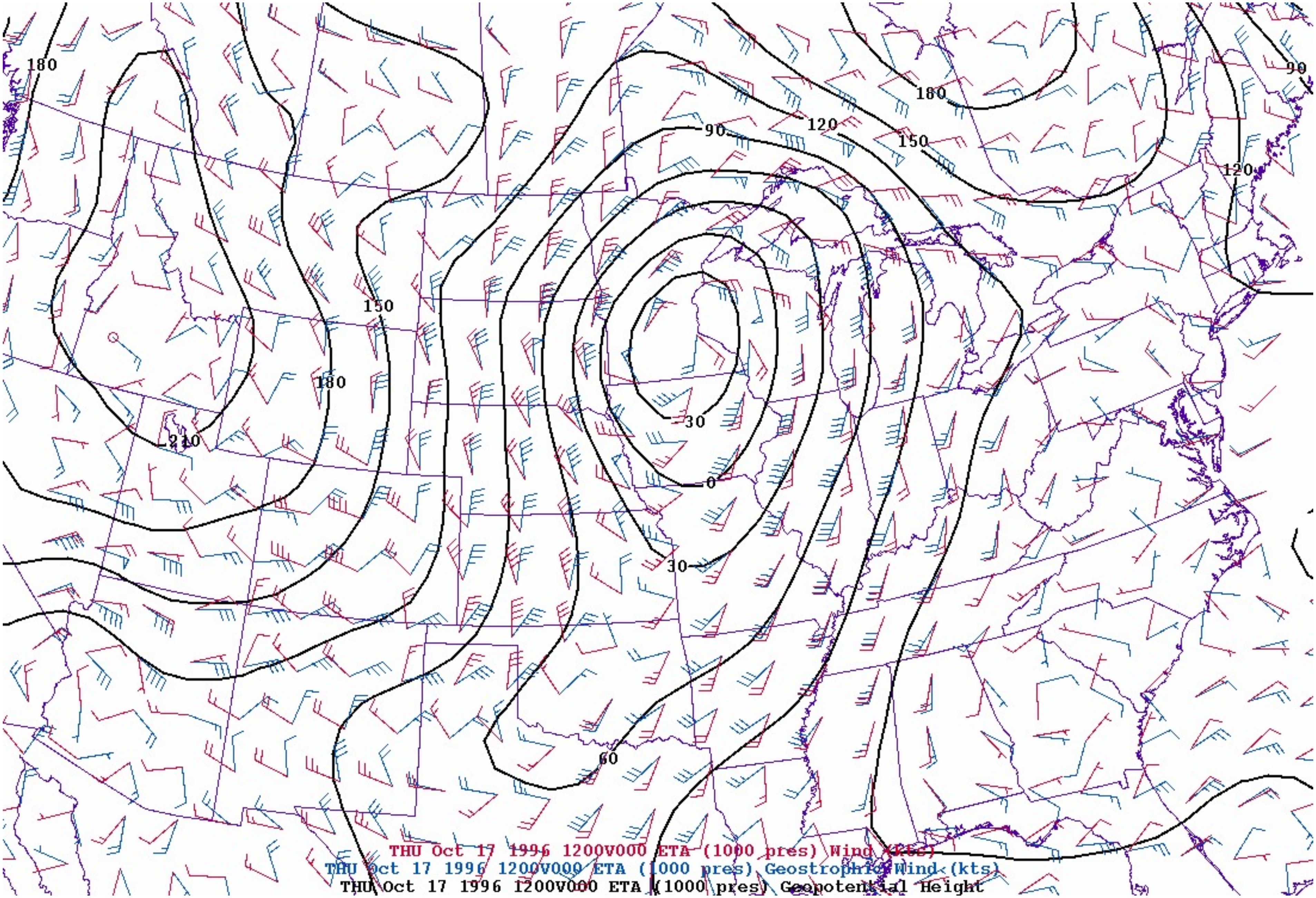
This shows, explicitly, that the acceleration, the prognostic attribute of the atmosphere, is related to the difference from the geostrophic balance.

A simple prognostic equation

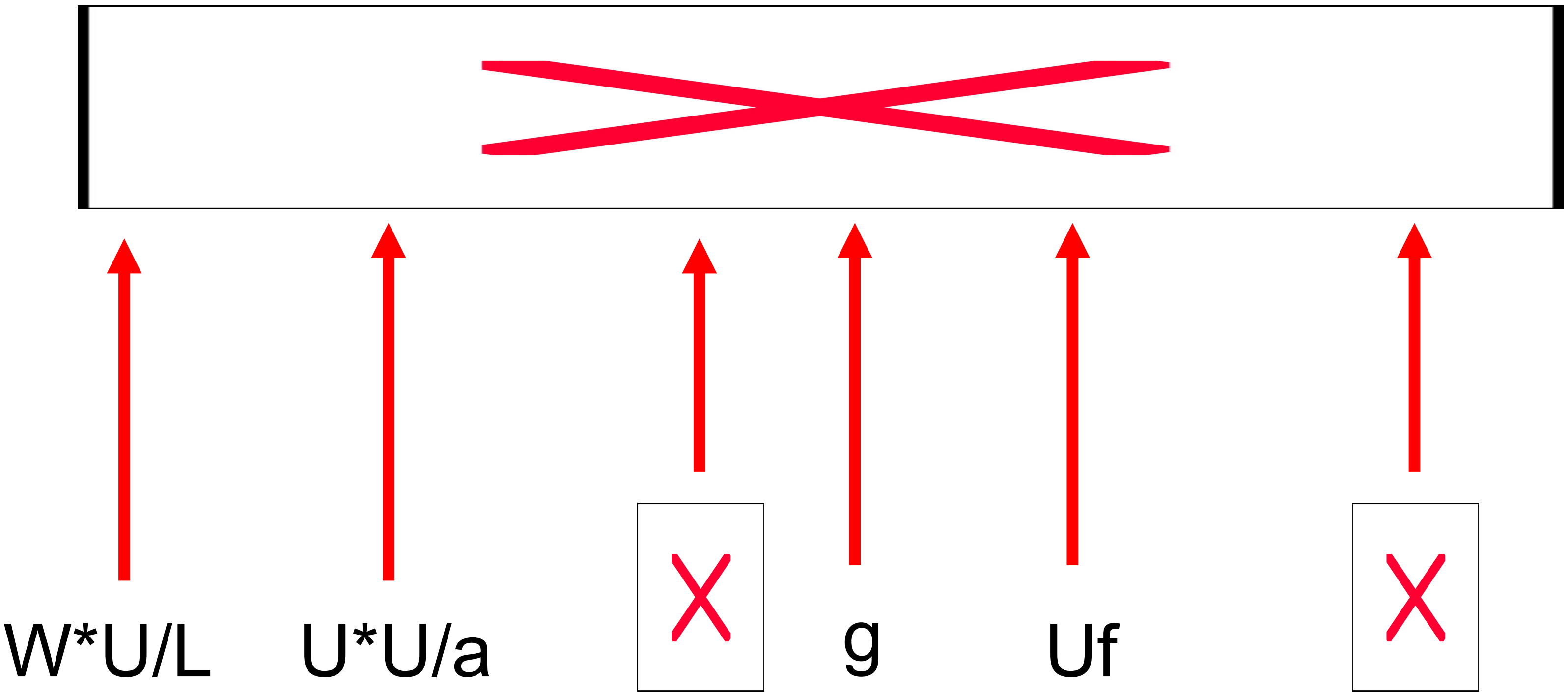


Looks like we are getting towards two variables u and v, but we have buried the pressure and density in geostrophic balance. This links, directly, the mass field and velocity field. And what about w?

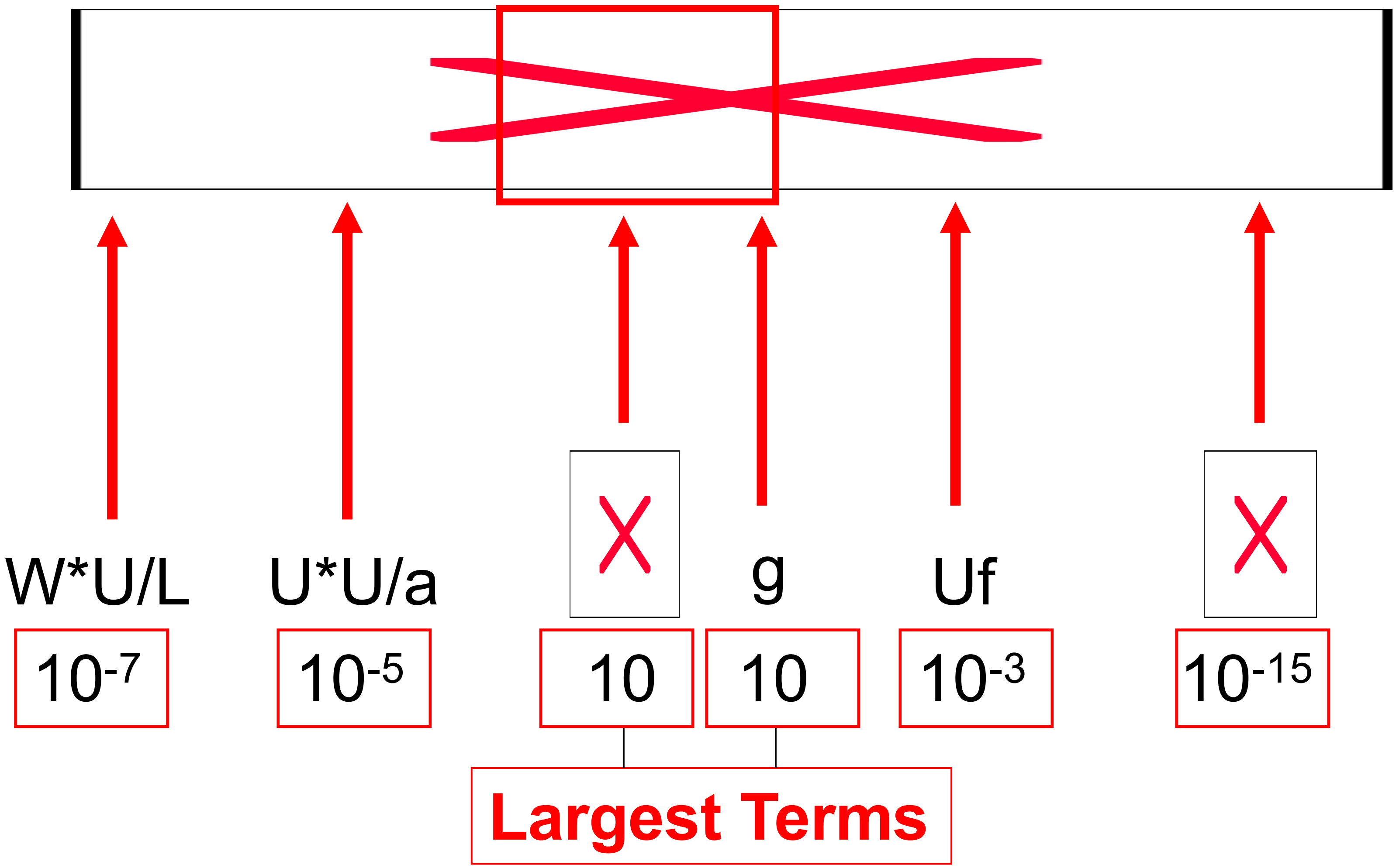
Geostrophic and observed wind 1000 mb (land)



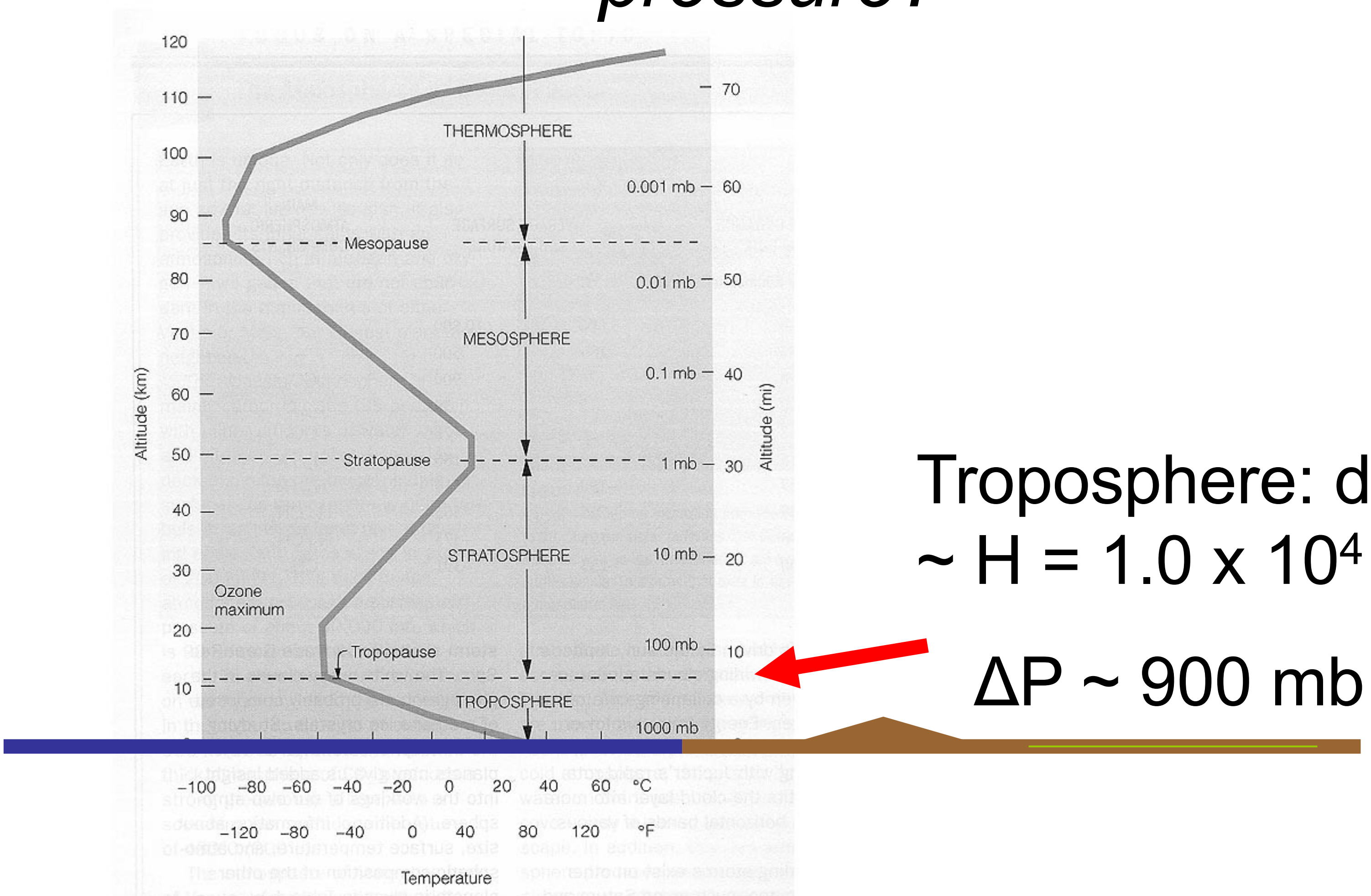
Scale analysis of the vertical momentum equation



Scale analysis of the vertical momentum equation



How did we get that vertical scale for pressure?

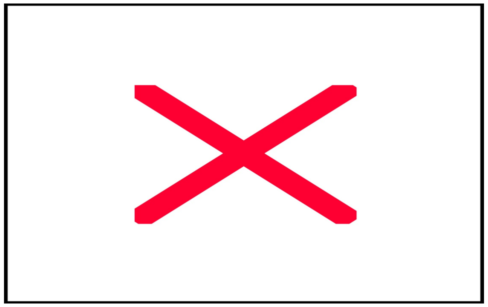


Troposphere: depth
~ H = 1.0 x 10^4 m

ΔP ~ 900 mb

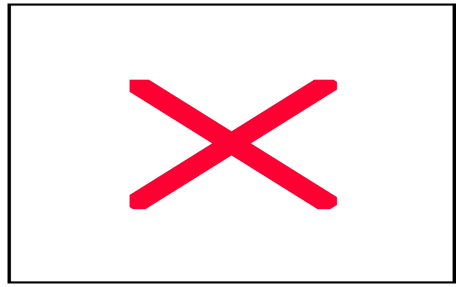
This scale analysis tells us that the troposphere is thin relative to the size of the Earth and that mountains extend half way through the troposphere.

Hydrostatic relation



And the vertical acceleration Dw/Dt is **8 orders of magnitude smaller** than this balance.
So the ability to use the vertical momentum equation to estimate w is essentially nonexistent.

Hydrostatic relation



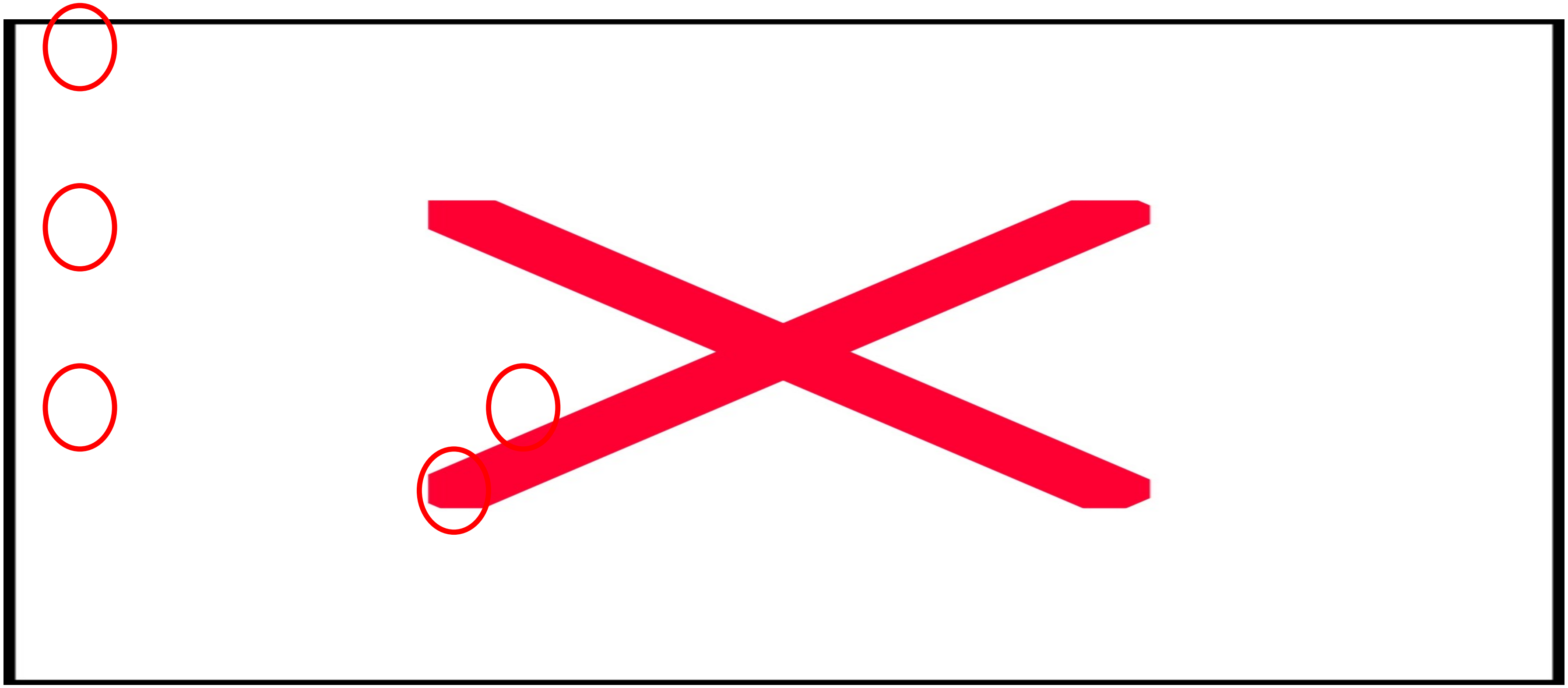
Hence:

w must be "diagnosed" from some balance.

exception: small scales: thunderstorms, tornadoes

Non-hydrostatic scale of ~ 10000 m, 10 km.

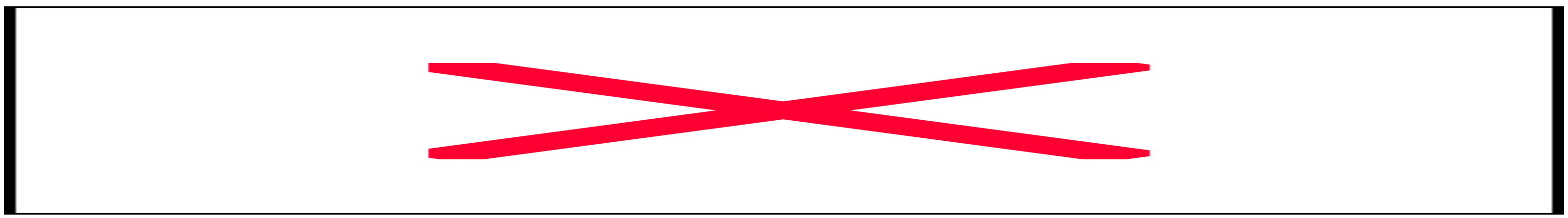
Momentum equation



Currently, we have 5 independent variables
u, v, w, ρ, p and 3 momentum equations.
Can we solve this system of equations? Are we done?

Are we done?

- Currently, 5 unknowns and 3 equations
- But wait, we also discussed the **ideal gas law** (also called **Equation of State**)



- This adds another unknown 'T' to the system
- Now we have 6 unknowns (u, v, w, ρ, p, T) and 4 equations. Are we done?
- Recall what we said about conservation laws
  - Momentum
  - Energy
  - Mass