AOSS 321, Winter 2009
Earth System Dynamics

Lecture 10
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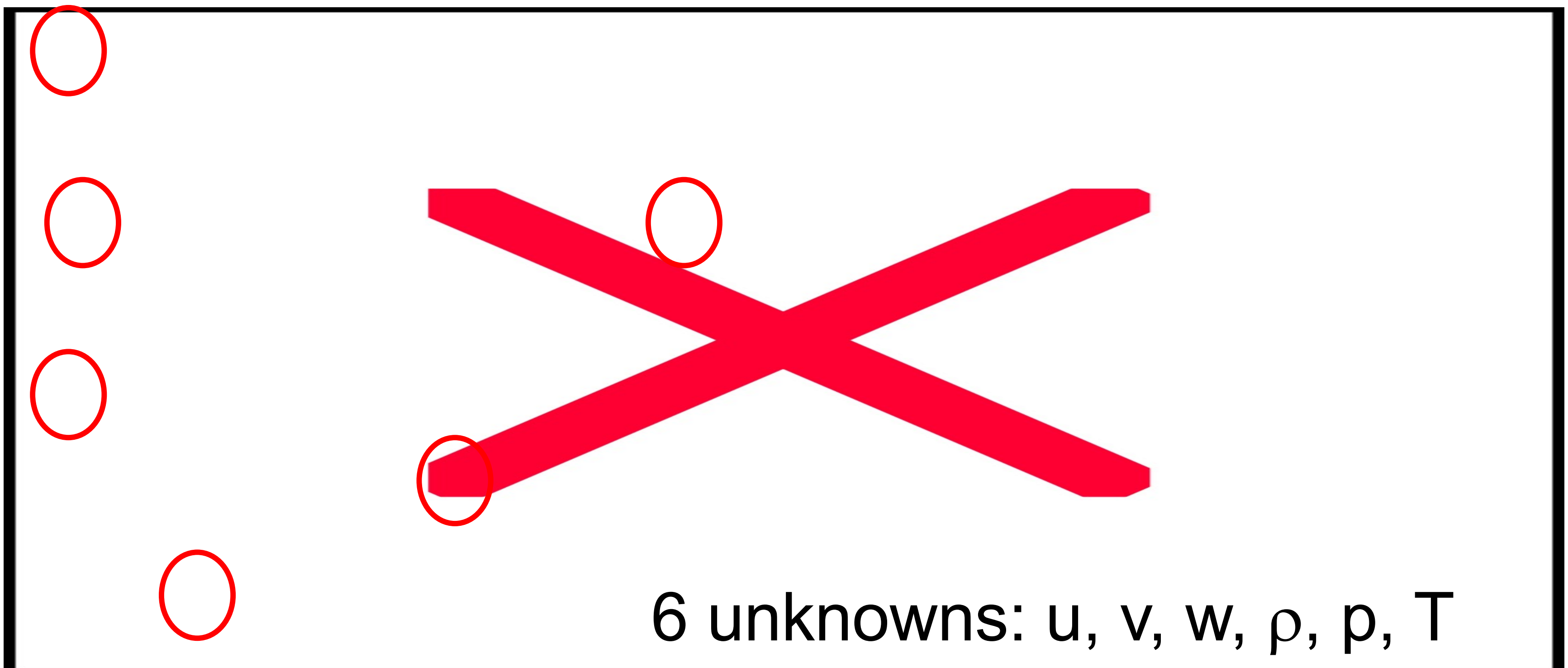
Today's class

- Conservation principles:
  - Conservation of mass: Continuity equation
  - Thermodynamic equation
  - Conservation of total energy

Leads to

- The equations of motion for the atmosphere

So far: Equations of motion



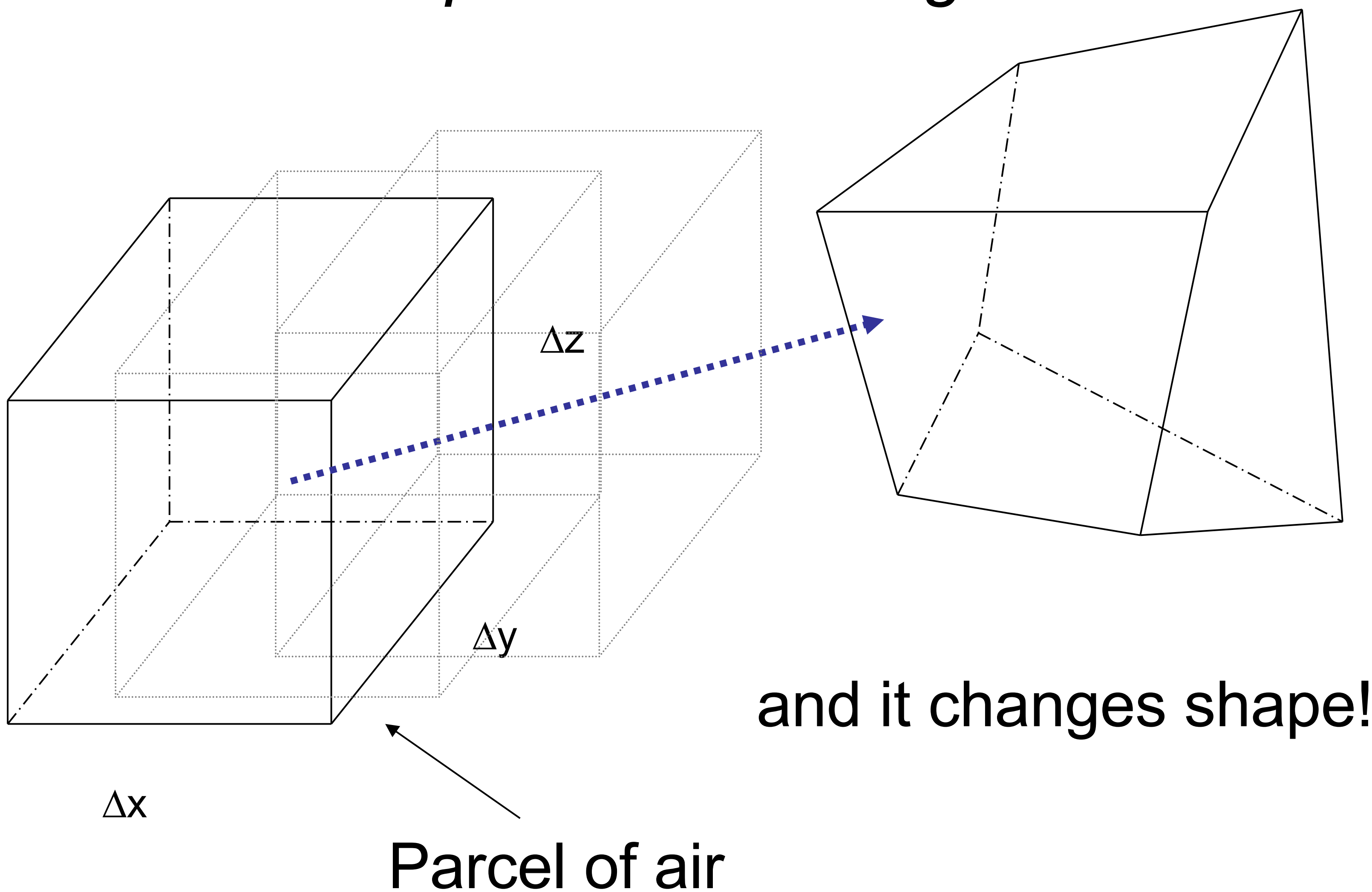
6 unknowns: u, v, w, ρ, p, T
4 equations

We need two more equations!

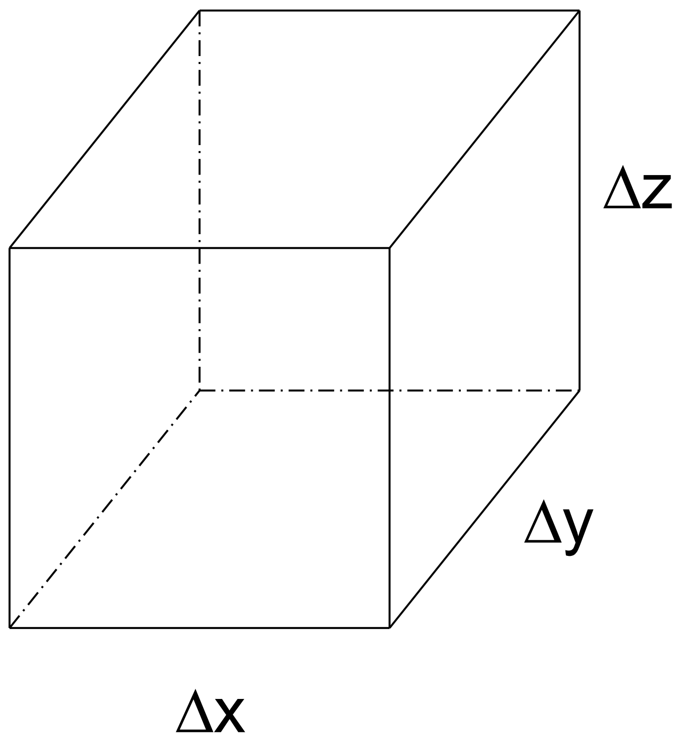
Recall: Eulerian Point of View

- This point of view, where is observer sits at a point and watches the fluid go by, is known as the Eulerian point of view.
  - Useful for developing theory
  - Looks at the fluid as a field.
  - Requires considering only one coordinate system for all parcels
  - A value for each point in the field – no gaps or bundles of “information.”

Recall: Lagrangian point of view is that the parcel is moving.



The Eulerian point of view: our parcel is a fixed volume and the fluid flows through it.



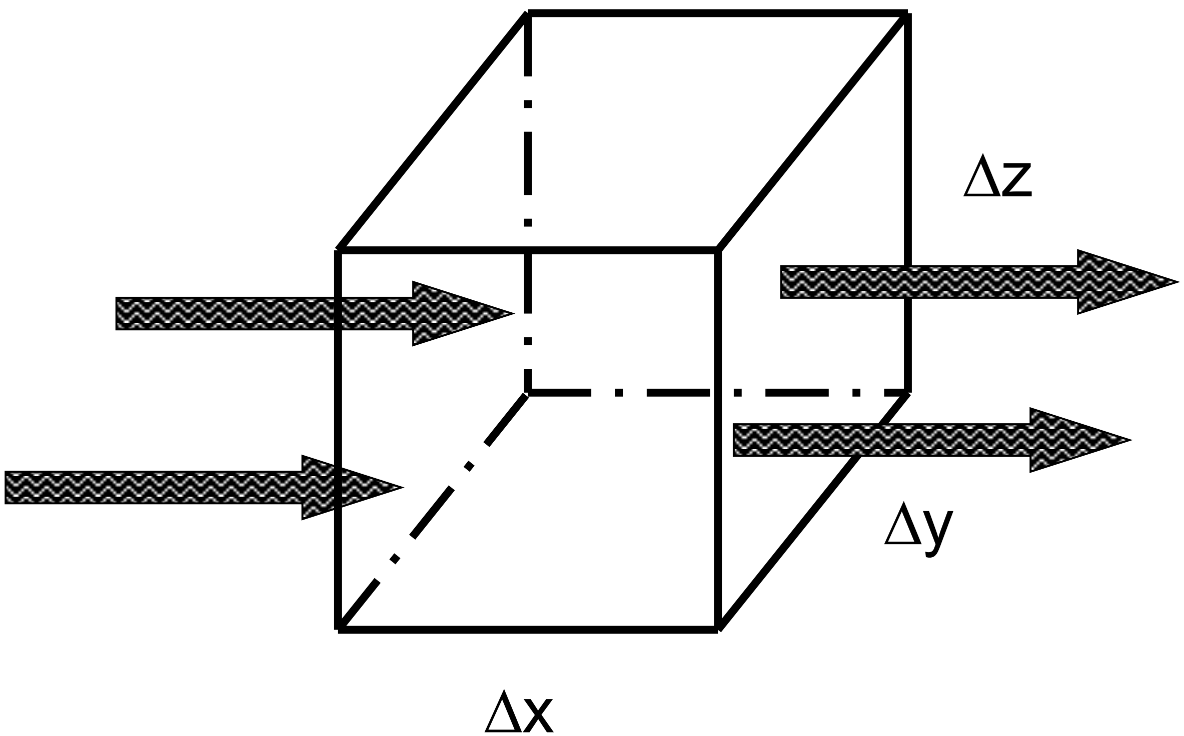
$\rho \equiv$ density = mass per unit volume (ΔV)

$\Delta V = \Delta x \Delta y \Delta z$

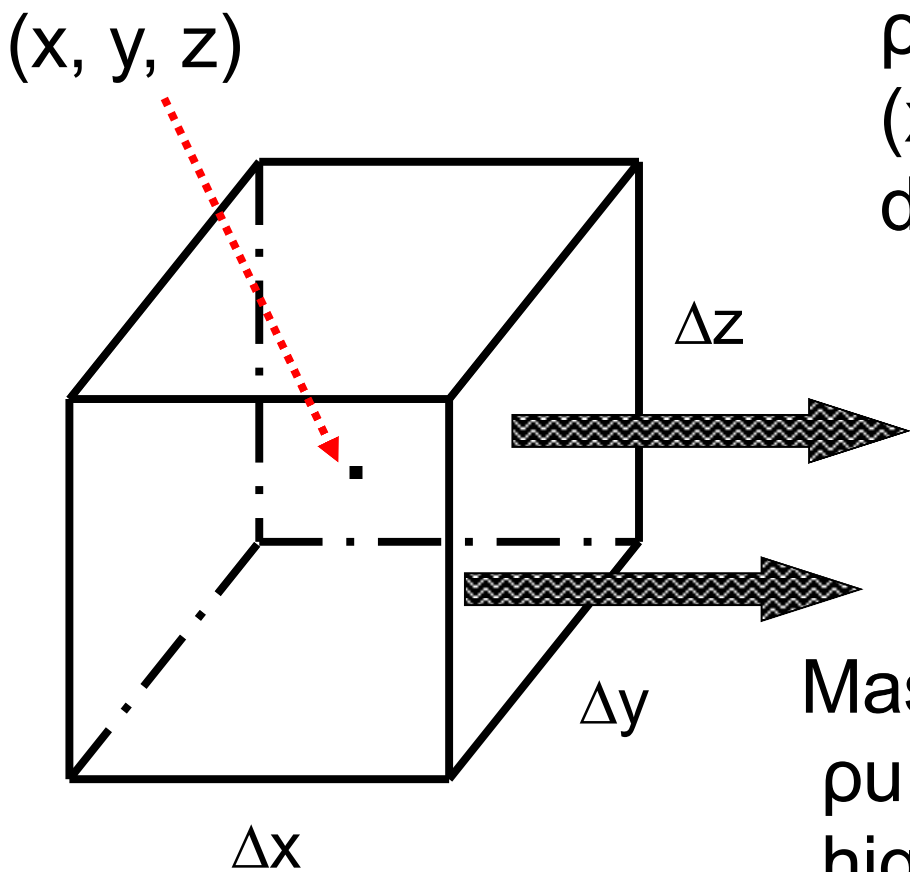
$m = \rho \Delta x \Delta y \Delta z$

$p \equiv$ pressure = force per unit area acting on the particle of atmosphere

The Eulerian point of view: our parcel is a fixed volume and the fluid flows through it.



*Introduce mass flux, ρ**u***



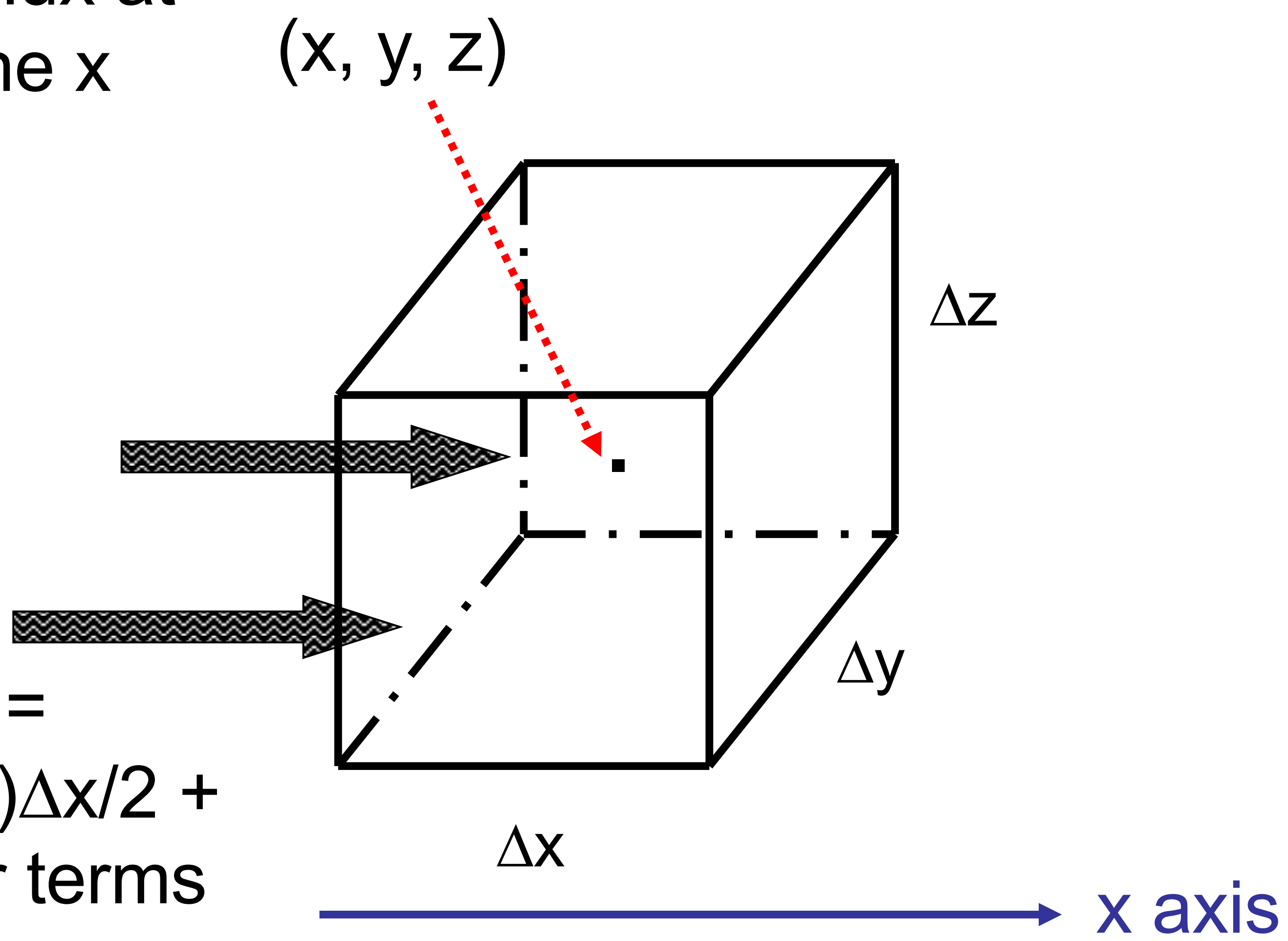
ρu = mass flux at (x, y, z) in the x direction.

Mass flux **out** = ρu + (∂ρu/∂x)Δx/2 + higher order terms

x axis

*Introduce mass flux, ρ**u***

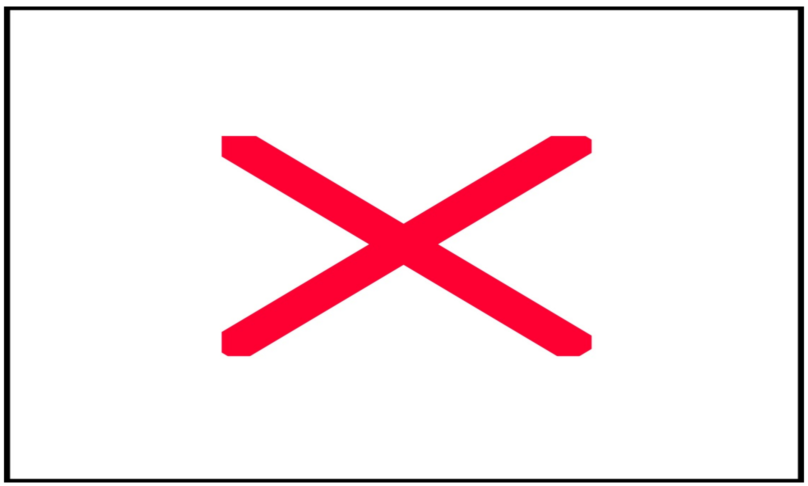
ρu = mass flux at (x, y, z) in the x direction.



Mass flux **in** =
ρu - (∂ρu/∂x)Δx/2 + higher order terms

What is mass into and out of the fixed volume?

- Mass flux times the area of the face of the box which it is flowing through.

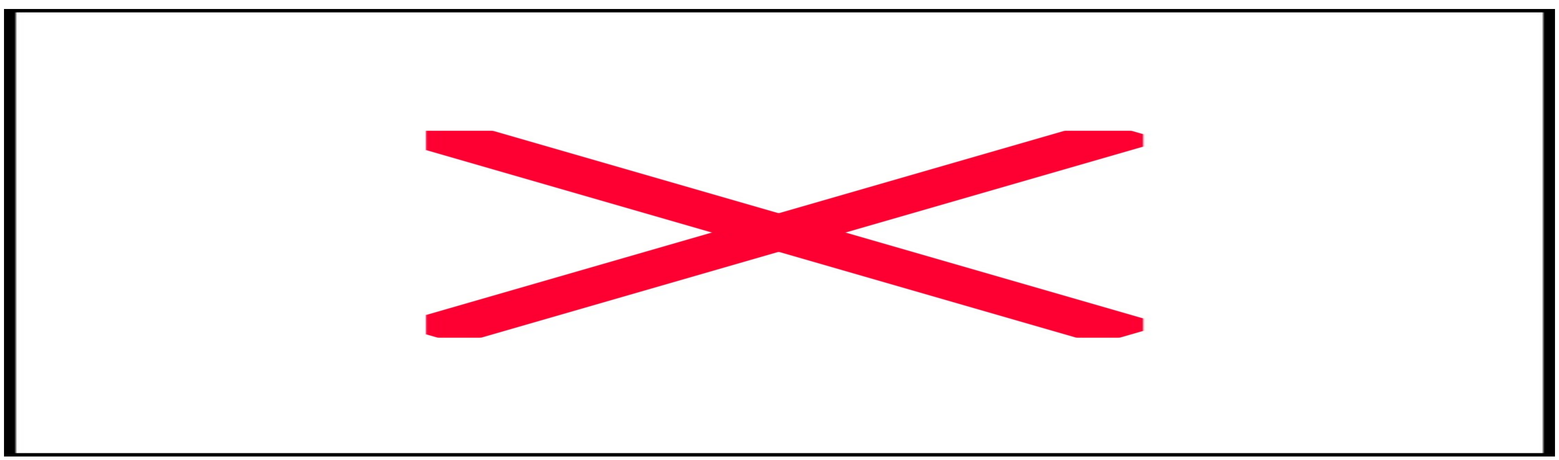


Mass out right (downstream) face

Mass in left (upstream) face

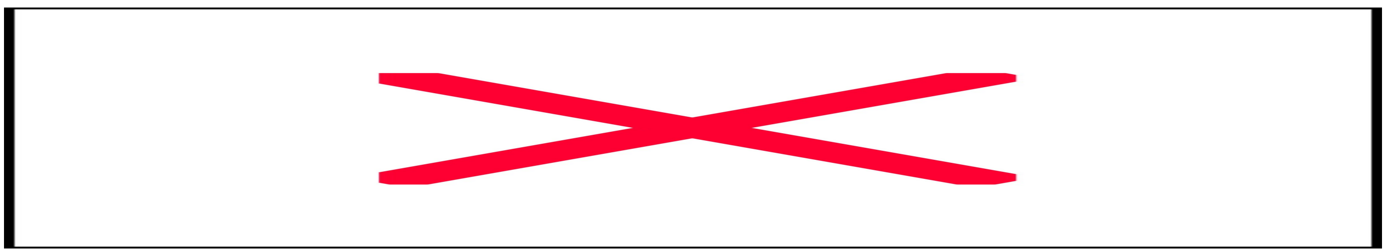
Conservation of mass

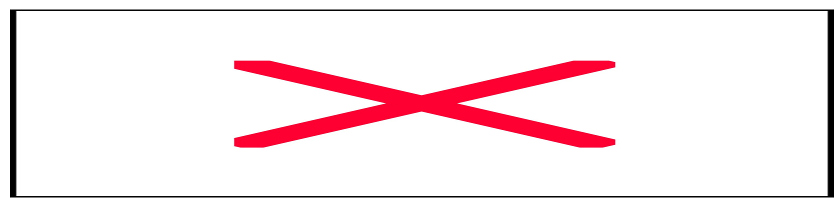
- The change of mass in the box is equal to the mass that flows into the box minus the mass that flows out of the box.

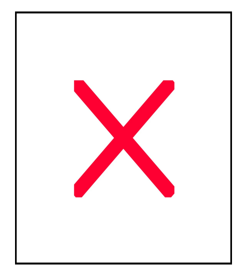


Extend to 3-Dimensions

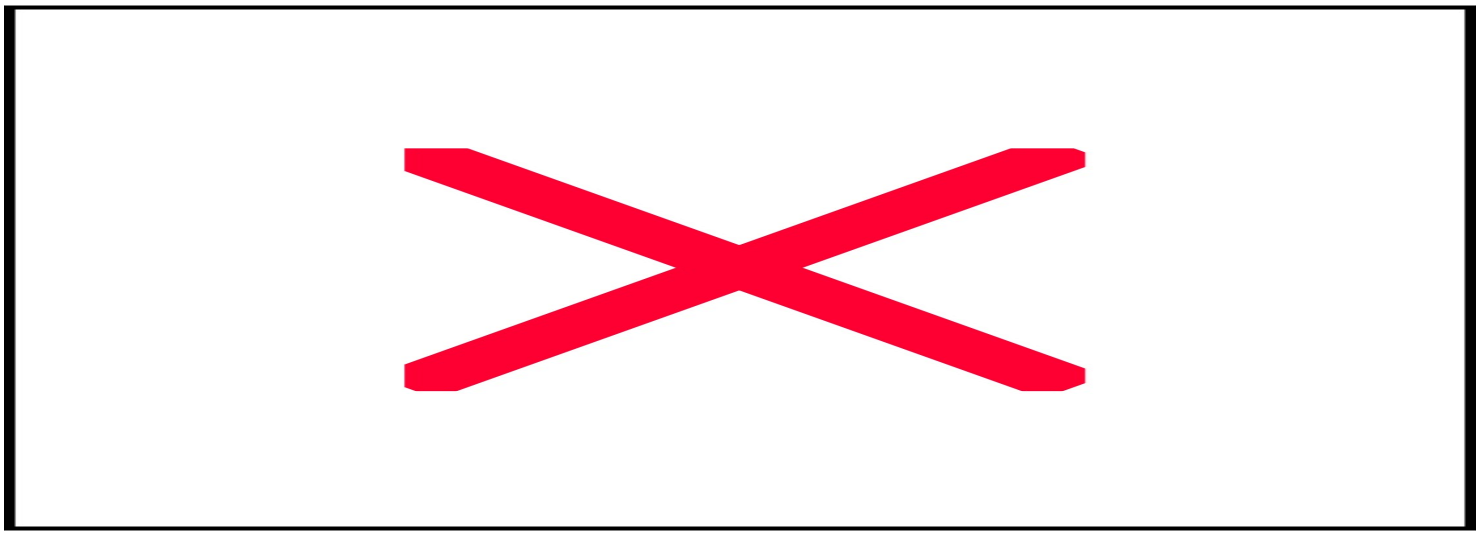
- The change of mass in the box is equal to the mass that flows into the box minus the mass that flows out of the box.



Mass flux per unit volume: 

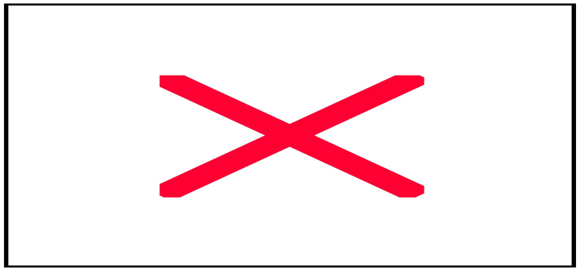
This is equal to the local rate of change 

Conservation of Mass: The continuity equation



Conservation of Mass: The continuity equation

Can you show that the continuity equation can also be written as



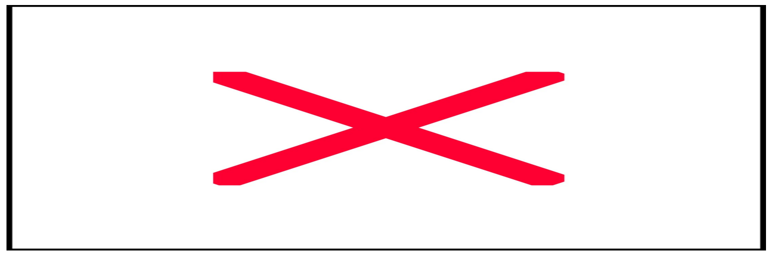
So far: Equations of motion

6 unknowns: u, v, w, ρ, p, T
5 equations
Almost closed system!
One more equation is missing.

Conservation of thermodynamic energy: The thermodynamic equation

- Change in internal energy is equal to the difference between the heat added to the system and the work done by the system.
- Internal energy is due to the kinetic energy of the molecules.
- Total thermodynamic energy is the internal energy plus the energy due to the parcel moving.

Thermodynamic Equation



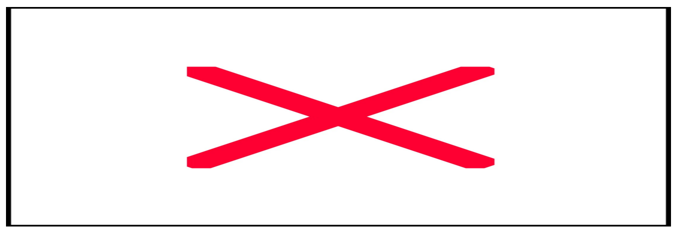
J is the source or sink of heating which are –
radiation, latent heat release, thermal conductivity, frictional heating

$\alpha = 1/\rho$ is the specific volume

$c_v = 717\ \mathrm{J\ K^{-1}\ kg^{-1}}$ is the specific heat of dry air at constant *volume*

Another form of the Thermodynamic Equation

Can be derived via the ideal gas law (show):



c_p = 1004 J K^{-1} kg^{-1} is the specific heat of dry air at constant *pressure*

Note: $c_p = c_v + R_d$ = 1004 J K^{-1} kg^{-1}
R_d = 287 J K^{-1} kg^{-1} gas constant for dry air

Full equations of motion

(Navier-Stokes equations)

6 unknowns: u, v, w, ρ, p, T

6 equations

Closed system!

These are our equations of motions in the atmosphere!

Conservation of total energy

- The law of conservation of energy states that the sum of all energies in the universes is constant.
- There are many forms of energy in the atmosphere, e.g.: kinetic energy, potential energy, latent heat energy, radiant energy, …
- Radiant energy from the Sun is the source of nearly all of the total energy in the atmosphere/ocean system.
- When solar energy is absorbed at the Earth's surface it appears as internal energy (noticeable as temperature changes).
- One of the major challenges in atmospheric science is determining how this internal energy is converted into the other forms of energy.

Derivation of the total energy equation

- Let’s gain some insights into the nature of the energies in the atmosphere.
- Start by taking the dot (scalar) product of the acceleration vector with the velocity vector
- Equivalent to multiplying the components of the momentum equations with their respective component velocities (u,v,w).

Derivation of the total energy equation

- Multiplication leads to

- Sum all three expressions:

Metric terms and Coriolis forces cancel!

Derivation of the total energy equation

frictional effects

Rate of change of kinetic energy per unit mass

Pressure advection divided by density

Measure of the work required to raise a unit mass a distance z

- Leads to

Sum of the kinetic and potential energies per unit mass of an atmospheric parcel

Derivation of the total energy equation

- Also referred to as mechanical energy equatio

- Adding the fir hodynamics

l

- Use:

Total Energy Equation

- Rewriting leads to the energy equation:

- Relationship implies that if the flow is *frictionless* , *adiabatic* (J=0) and *steady-state* , then the quantity

is **constant (conserved).**