AOSS 321, Winter 2009
Earth System Dynamics

Lecture 3
1/15/2009

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Class News

- Class web site: https://ctools.umich.edu/portal
- Homework #1 posted today, due on Thursday (1/22) in class
- Our current grader is Kevin Reed (kareed@umich.edu)
- Office Hours
  - Easiest: contact us after the lectures
  - Prof. Jablonowski, 1541B SRB: Tuesday after class 12:30-1:30pm, Wednesday 4:30-5:30pm
  - Prof. Hetland, 2534 C.C. Little, office hour TBA

Today's lecture

- Ideal gas law for dry air
- Pressure
- Hydrostatic equation
- Scale height
- Geopotential, geopotential height
- Hypsometric equation: Thickness
- Layer-mean temperature
- Mathematical tools

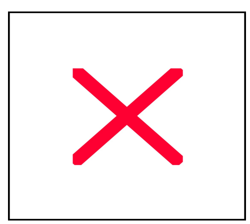
Pressure, temperature, density

- Unit 3: Pressure (online resource) http://www.atmos.washington.edu/2005Q1/101/CD/MAIN3.swf
- Consider a hot air balloon

- What is the pressure difference between the inside and outside of the hot air balloon?
- What is the temperature difference between the inside and outside of the balloon?
- What is the density difference?

Pressure, temperature, density
The ideal gas law

- Pressure is proportional to temperature times density
- Temperature is proportional to the average kinetic energy (mass times velocity squared) of the molecules of a gas
- Density is the mass divided by the volume:



Ideal gas law

- One form of the ideal gas law is:

  $$p\,V = n\,R^*\,T$$

  where the number of moles *n* in mass *m* (in grams) of a substance is given by

  $$n = m / M$$

- M is the molecular weight of a substance, e.g. for nitrogen (N_2) in the gas phase: M = 28.0116 g / mol
- p, V, T: pressure, volume and temperature
- R^* = 8.3145 J/(K mol): universal gas constant

Another form of the ideal gas law

- Another form of the ideal gas law for dry air:

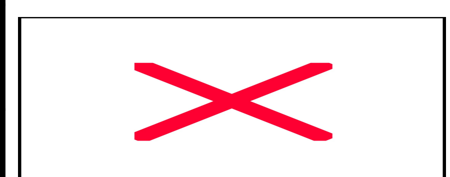
  $$p = \rho R_d T$$

  where R_d is the gas constant for 1 kg of dry air

- ρ is the dry air density, units are kg/m^3

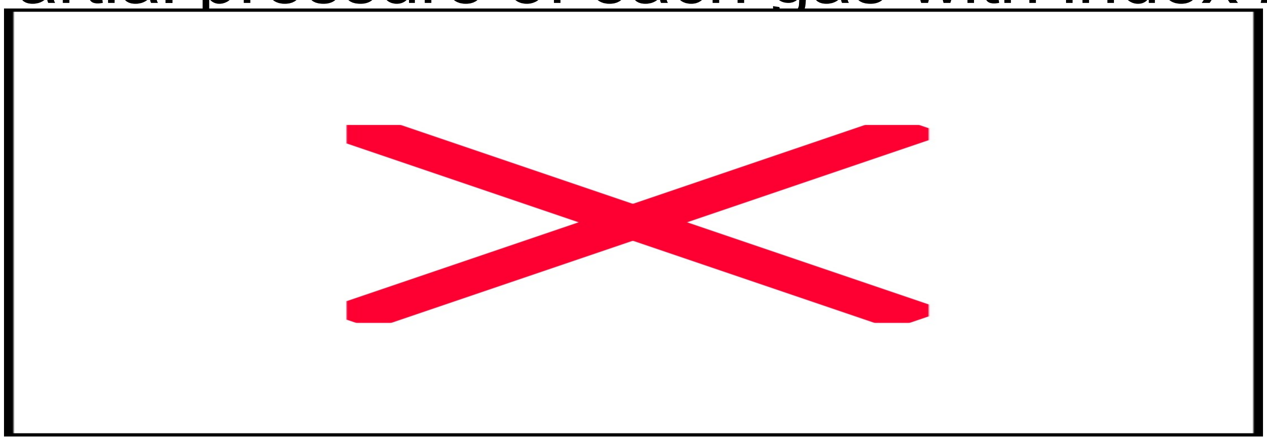
- Dry air is a mix of different gases, mainly nitrogen N_2, oxygen O_2, argon (Ar), carbon dioxide CO_2

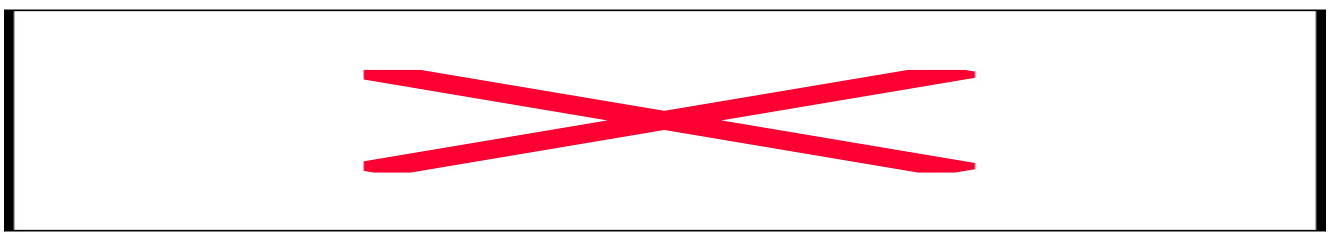
Gas_i	M_i (g/mol)	Volume ratio	Molecular mass of air
N_2	28.016	0.7809	21.88
O_2	32.000	0.2095	6.704
Ar	39.444	0.0093	0.373
CO_2	44.010	0.0003	0.013



Ideal gas law: Conversions

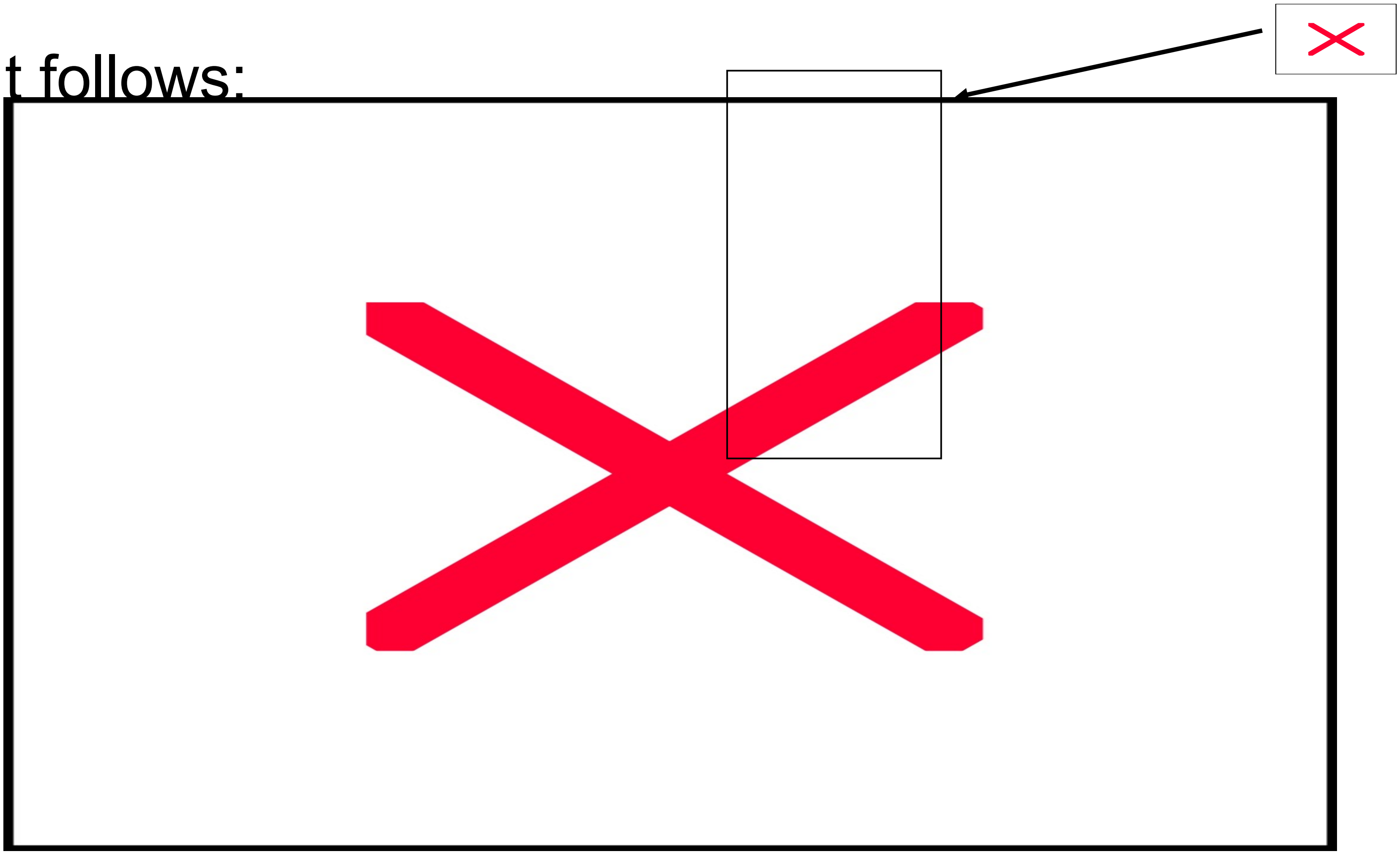
- Dalton's law applies:
  - each gas completely occupies the whole volume
  - each gas obeys its own pressure law
  - total dry air pressure = Σ partial pressures = $\Sigma\, p_i$
- Partial pressure of each gas with index *i* is:



- Using 

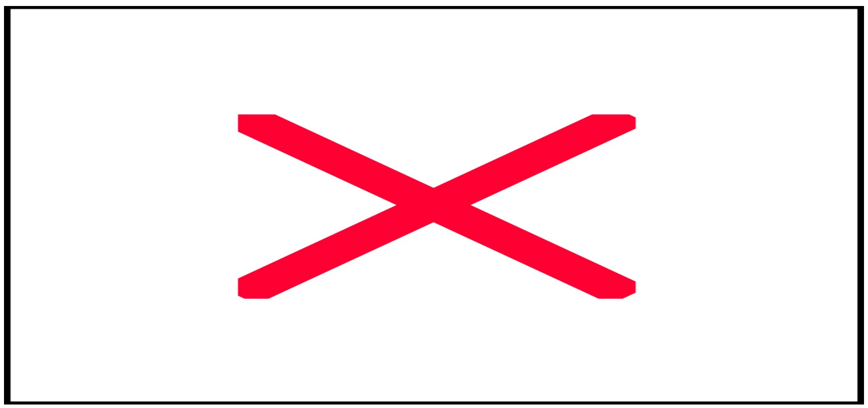
Ideal gas law: Conversions

- It follows:

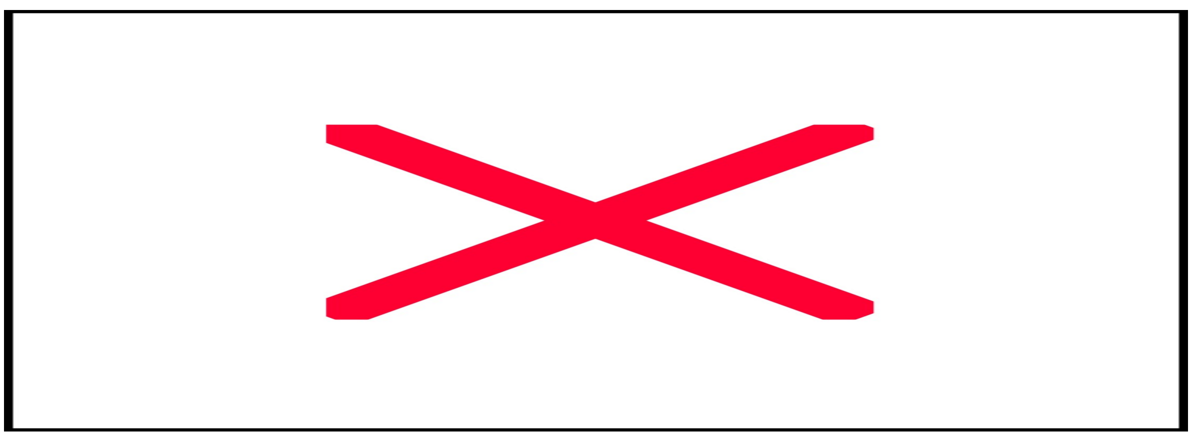


Conversions

- with M_d: apparent molecular weight of dry air



- Gas constant for dry air:



Ideal gas law
(used in atmospheric dynamics)

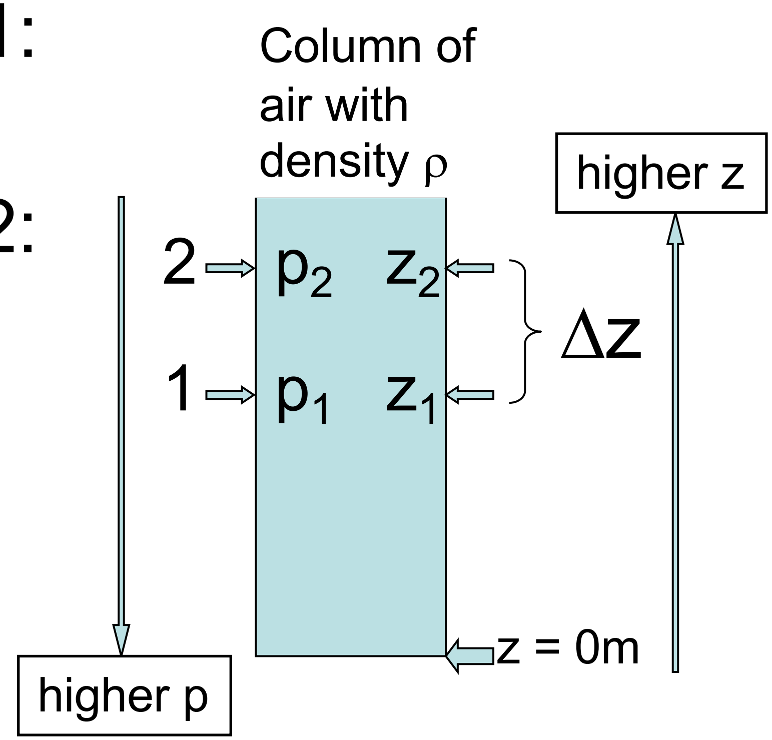
- Ideal gas law for dry air:

$$p = \rho R_d T$$

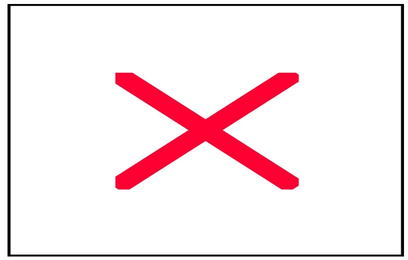
- p: pressure in Pa
- ρ: density in kg/m^3
- R_d: gas constant for 1 kg of dry air
  $R_d = 287.0$ J /(K kg)
- T: temperature in K

Hydrostatic Equation: Derivation

- Pressure (per 1 m^2) at point 1:
- Pressure (per 1 m^2) at point 2:
- Compute pressure difference between layer 1 and 2
-
- Rearrange:



Hydrostatic equation



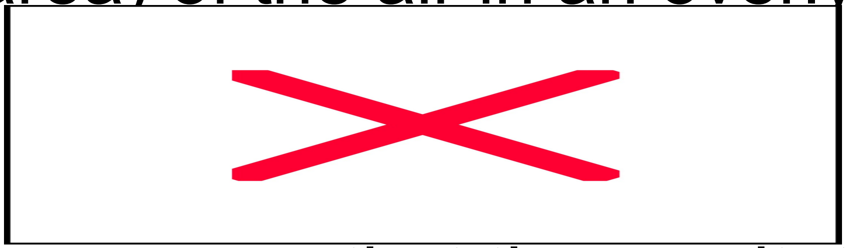
- p: pressure in Pa
- z: height in m
- ρ: density in kg/m^3
- g: gravitational acceleration of the Earth, approximately constant, $g = 9.81\ m/s^2$

Pressure

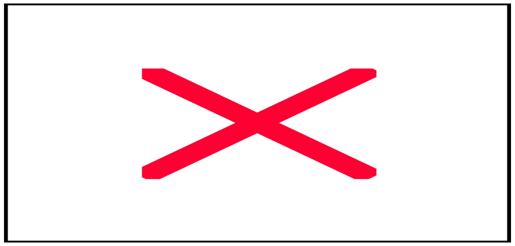
- Pressure decreases with height in the atmosphere
  - Why?
- Pressure increases with depth in the ocean
  - Why?
- Atmosphere exerts a downward force on the underlying surface due to the Earth's gravitational acceleration
- Downward force (here: the weight force) of a unit volume (1 m^3) of air with density ρ is given by $F/V = \rho\ g$

Pressure and mass

- The atmospheric pressure due to the weight (per unit area) of the air in an overlying column

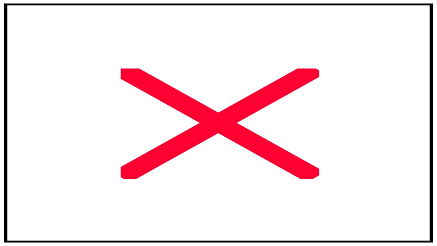


- Usually we assume that the gravitational acceleration g is constant: $g = g_0$
- m: vertically integrated mass per unit area of an overlying column of air at height z:

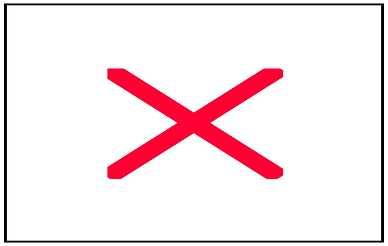


Surface Pressure p_s

- At the surface

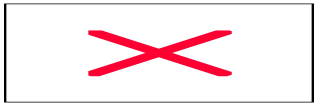


- With m:

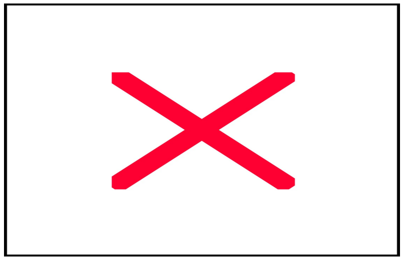


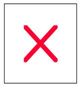
vertically integrated mass per unit area (!) of an overlying column of air at the surface, units:kg/m^2

- We get:

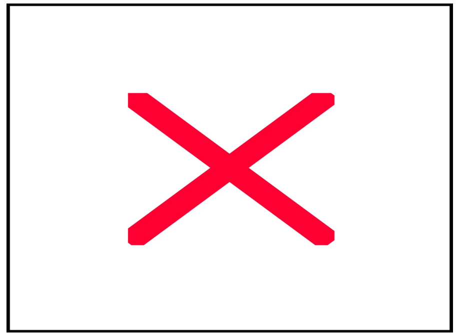


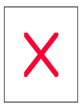
1st Exercise

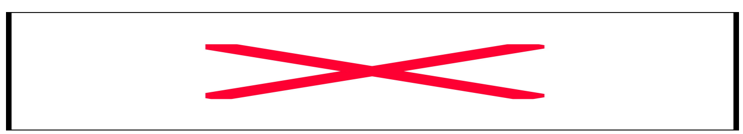


- Compute the pressure profile p(z) with respect to the height z for an isothermal (constant) atmosphere 
- Assume a flat Earth (no mountains): The surface height is z_0 = 0 m
- Assume the surface pressure $p_0 = p(z_0)$ is known

Scale Height



- : average (constant) temperature
- Scale height of the atmosphere: around 8km
- This is the height where the pressure of the atmosphere is reduced by a factor of 1/e
- For isothermal atmosphere with $z_0 = 0$ m (as seen before):

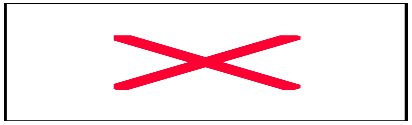


2nd Exercise

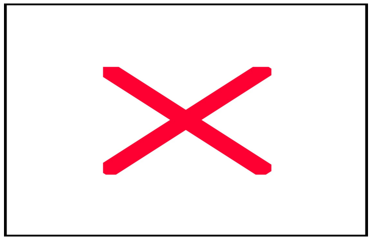
- Compute the pressure profile $p(z)$ with respect to the height z for an atmosphere whose temperature varies with height like:
  $T = T_0 - \Gamma z$
- $T_0 = T(z_0)$ is the known surface temperature at the surface height z_0
- Γ is a known, constant vertical temperature gradient (lapse rate) $\Gamma = -\partial T/\partial z$, units K/m
- Assume a flat Earth (no mountains): $z_0 = 0$ m
- Assume the surface pressure $p_0 = p(z_0)$ is known

Some definitions

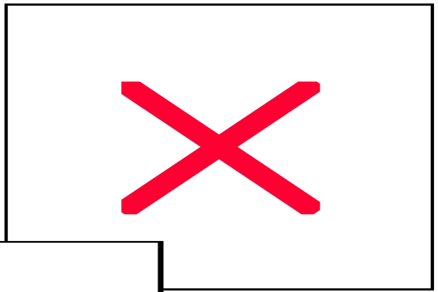
- Ideal gas law for dry air:



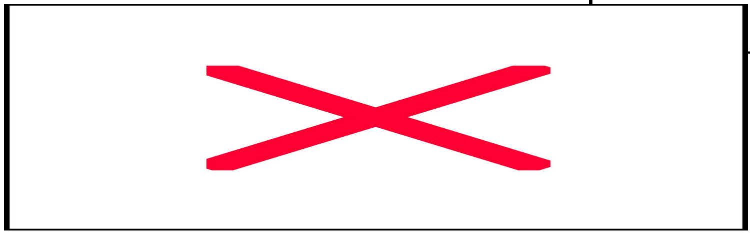
- Hydrostatic equation



- Define the **geopotential:**

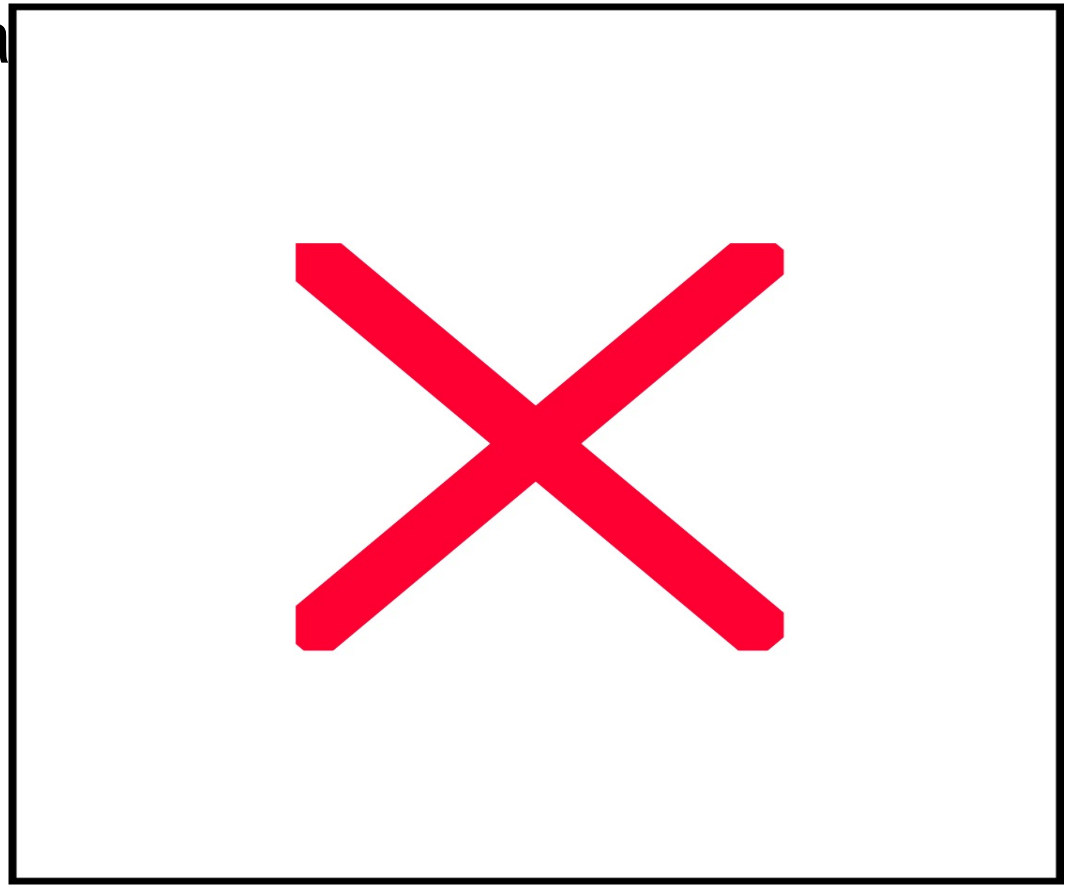


- Note that



Hypsometric equation: Derivation

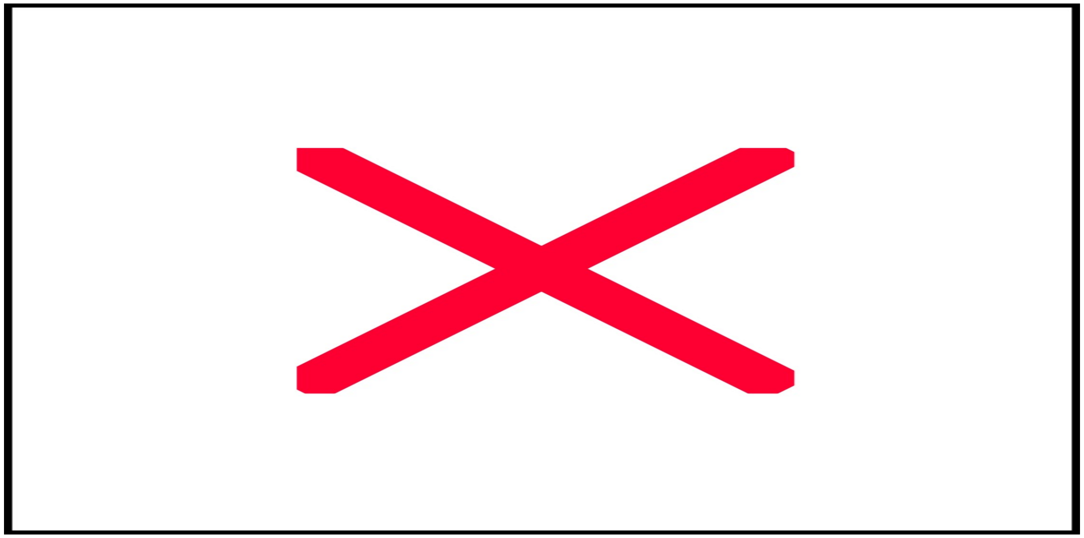
- Express the hydrostatic equation in terms of the geopotentia



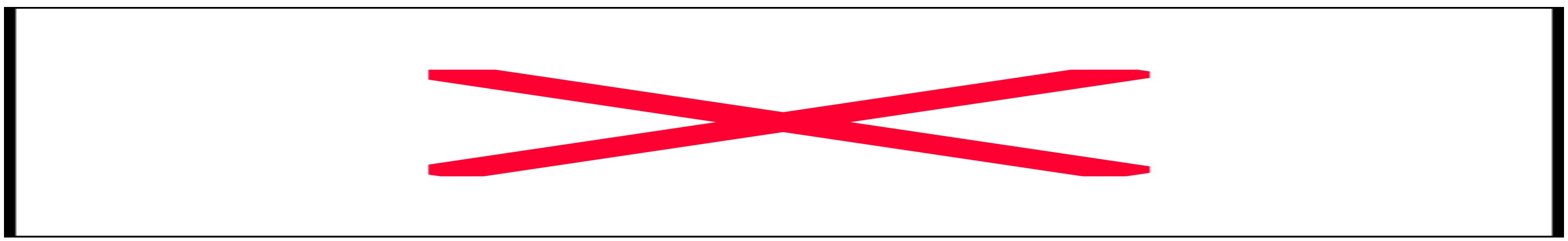
- The variation of the geopotential with respect to pressure only depends on the temperature

Hypsometric equation: Derivation

- Integration in the vertical gives:

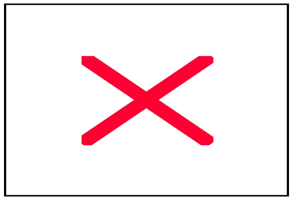


- Hypsometric equation:



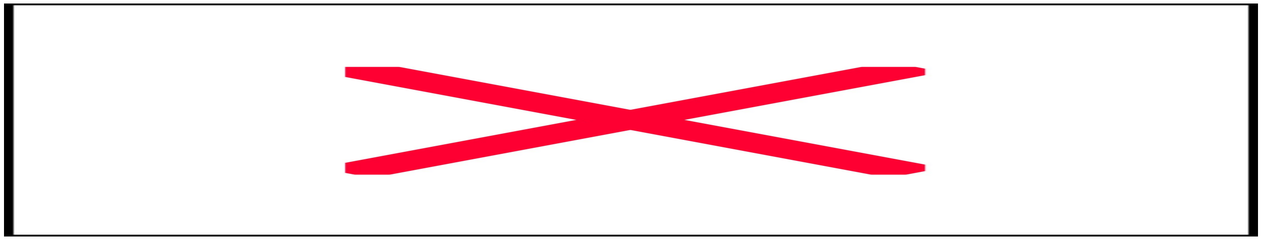
Hypsometric equation

- With the definition of the **geopotential height:**



(with constant g)

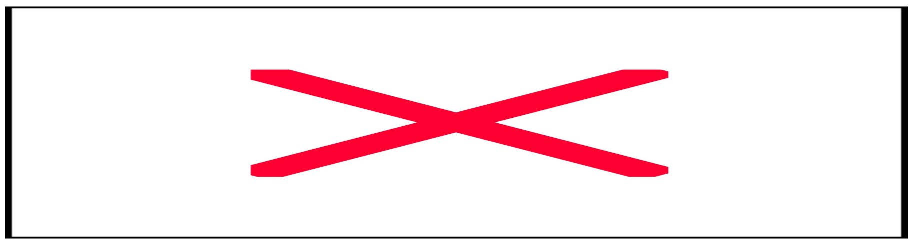
- the **hypsometric equation** becomes:



- **Z_T: Thickness** of an atmospheric layer (in m) between the pressure surfaces p_2 and p_1

Hypsometric equation for isothermal atmospheres

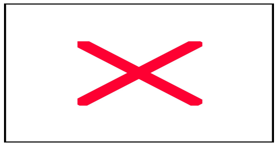
- If the atmosphere is **isothermal with $T=T_0$** the hypsometric equation becomes:



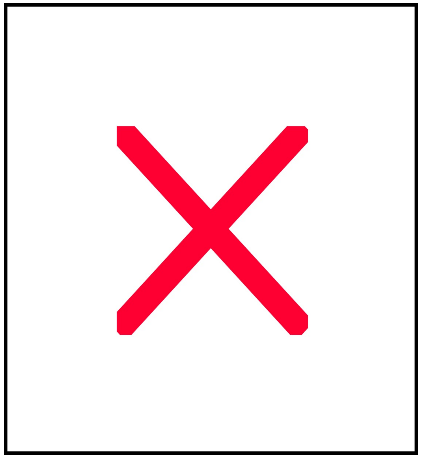
- with scale height $H = R_d T_0/g$, $g = 9.81 m\ s^{-2}$
- We see: the **thickness** of a layer bounded by two isobaric pressure surfaces is **proportional to** the **mean temperature** (here constant T_0) of the layer

Layer mean temperature

- In an **isothermal atmosphere** with T_0 the layer mean temperature is

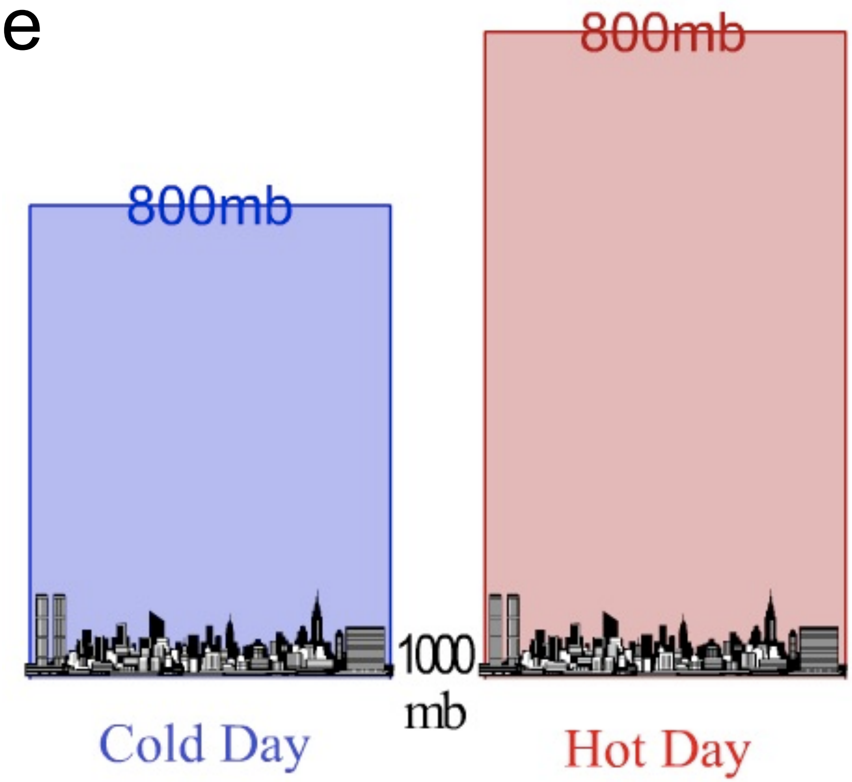


- More general definition of the layer mean temperature (for non-isothermal atmospheres):



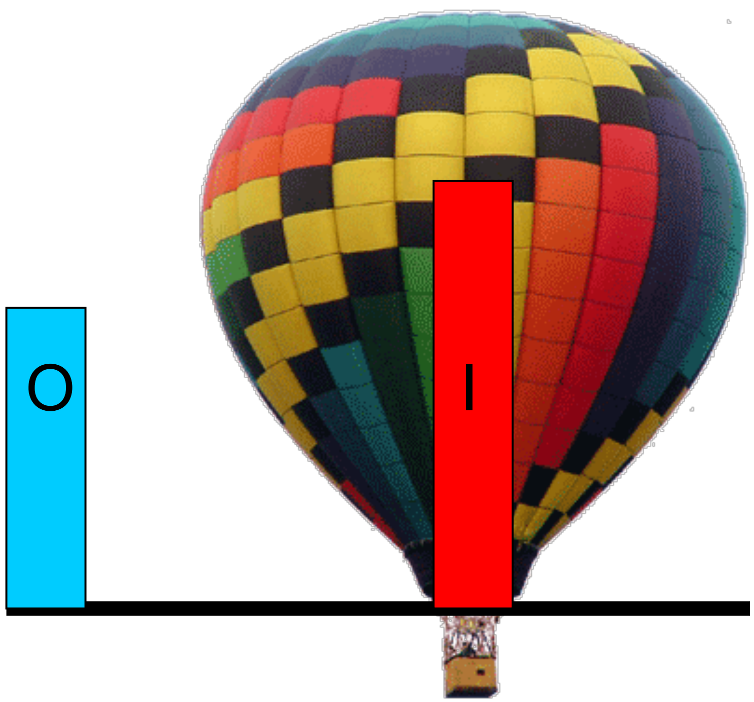
Examples

- Unit 3 (online resource), frame 20: Dependence on the 800 hPa height on temperature http://www.atmos.washington.edu/2005Q1/101/CD/MAIN3.swf
- Assess the height of the 800 hPa surface on a cold and hot day:



Source: Dale Durran, University of Washington

Hot air balloon revisited



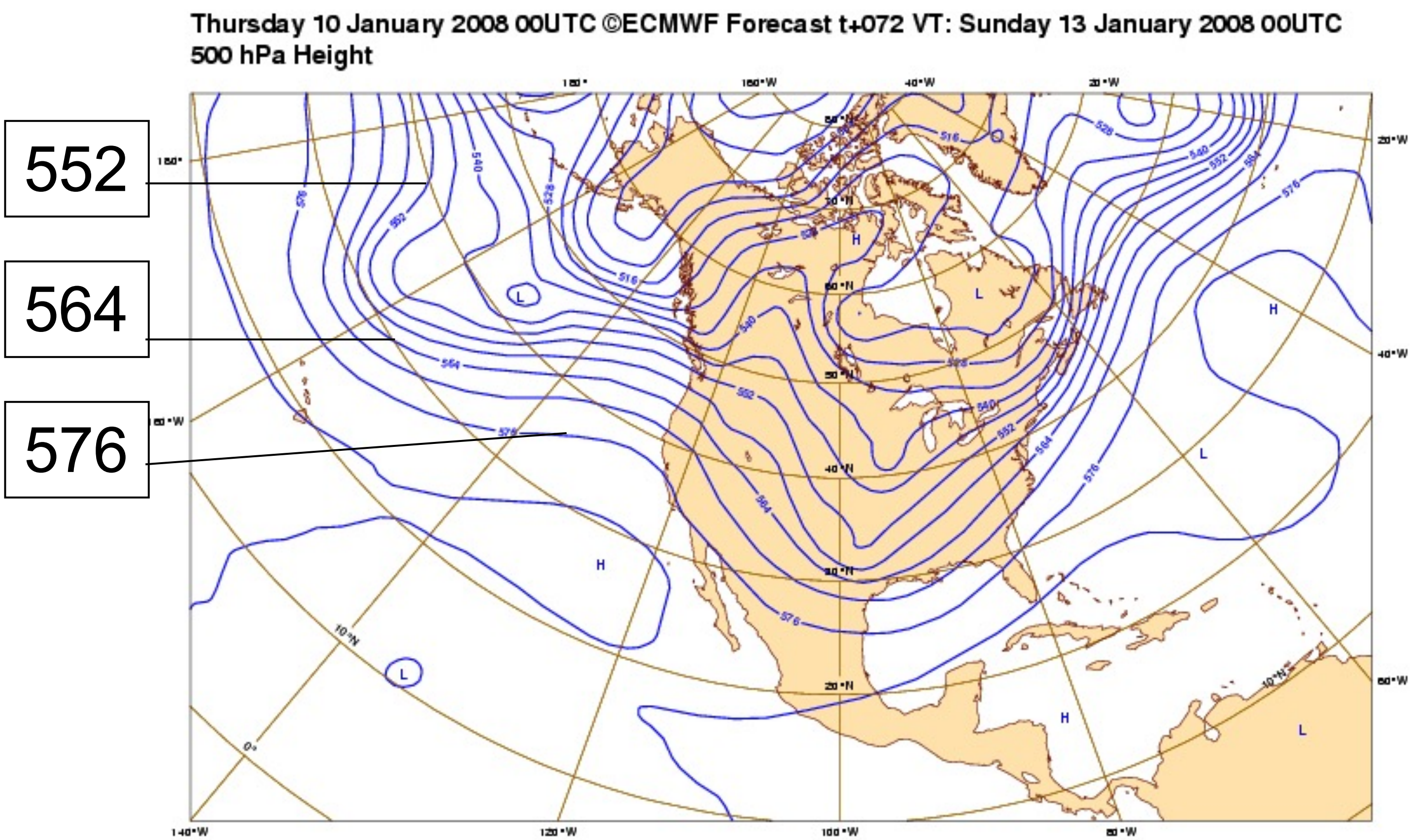
- Pressure is the same at the top of the two columns. hotter (red) column is taller than cold (blue) column

- How does the pressure at point O compare to the pressure at point I ?

- Pressure is the same inside and outside at the bottom of the balloon

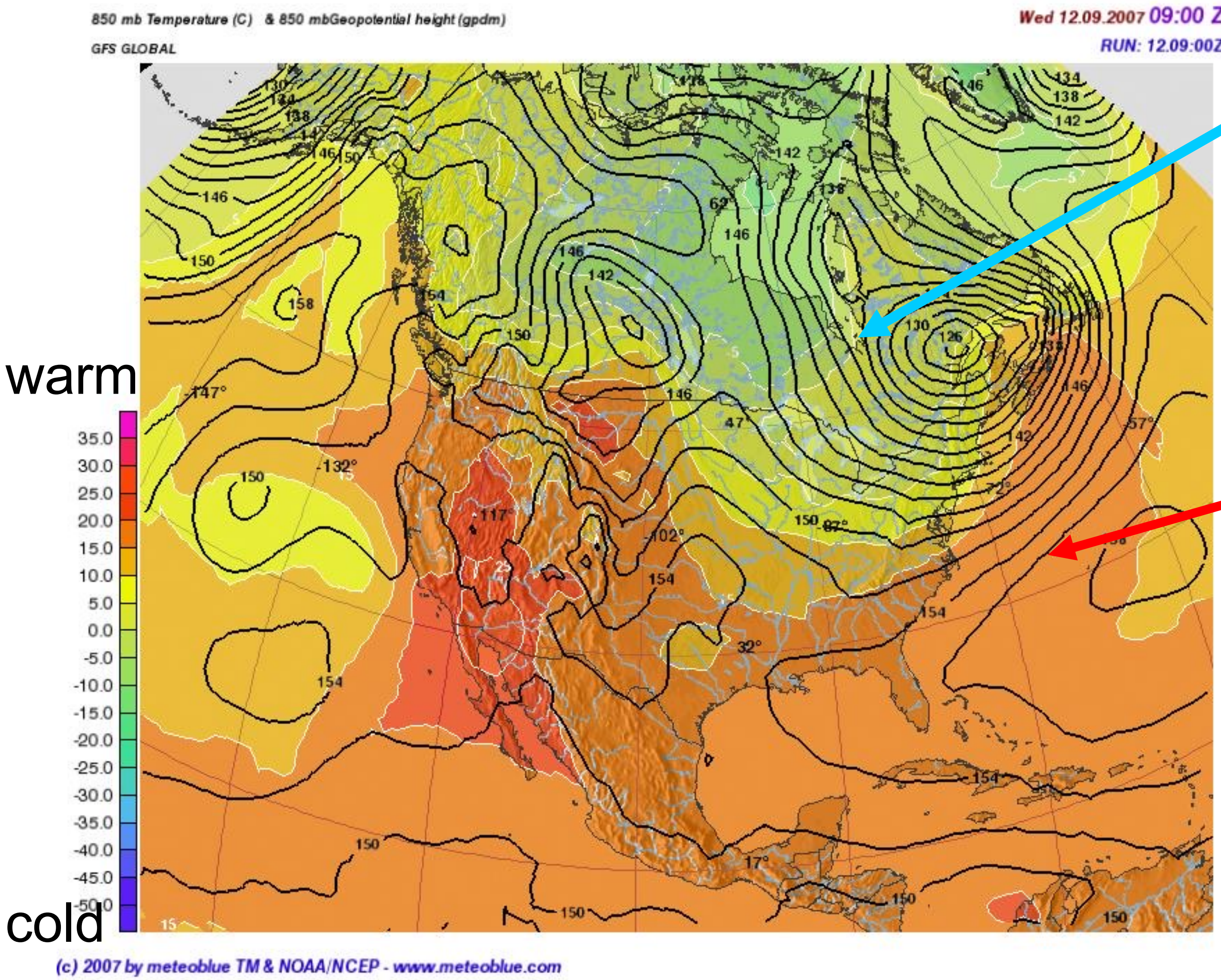
Example: Weather maps

- 500 hPa geopotential height map
- Blue: isolines that connect equal geopotential height values (in dm= 10 m)



Weather maps:http://www.rap.ucar.edu/weather/model/

Real weather situations: 850 hPa temperature and geopot. thickness



cold air, low geopotential height values

warm air, high geopotential height values

Mathematical tools: We use

- Logarithms and exponential functions
- Integrals
- Derivatives:
  - Partial derivatives
  - Chain rule, product rule
- Sine, cosine
- Vectors and vector calculus
- Operators:
  - Gradient
  - Divergence
  - Vector (cross) product
- Spherical coordinates

Logarithms and exponentials

- Natural logarithm, manipulations
  - $\ln(x)$ for $x > 0$
  - $\ln(x/y) = \ln(x) - \ln(y)$
  - $\ln(x*y) = \ln(x) + \ln(y)$
  - $a * \ln(x) = \ln(x^a)$
  - Values: $\ln(1) = 0$,
  - $\ln(\exp(x)) = x$,
- $\exp(\ln(x)) = x$
- $x^a * x^b = x^{a+b}$
- $(x^a)^b = x^{a*b}$
- $(x\ y)^a = x^a\ y^a$

Integrals

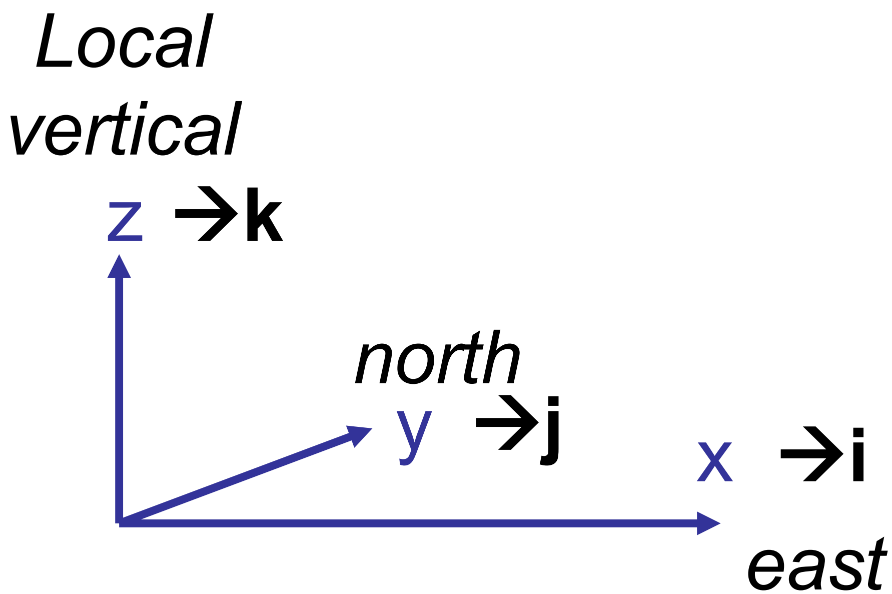
- Integrals without limits, a ≠0 is a constant:

Derivatives, partial derivatives

- Derivatives, a ≠0 is a constant:

- Partial derivatives: treat the variables that are not differentiated as constants, e.g. f(x,y,z)=xyz

Cartesian Coordinates



Velocity vector **v** = (u**i** + v**j** + w**k**)

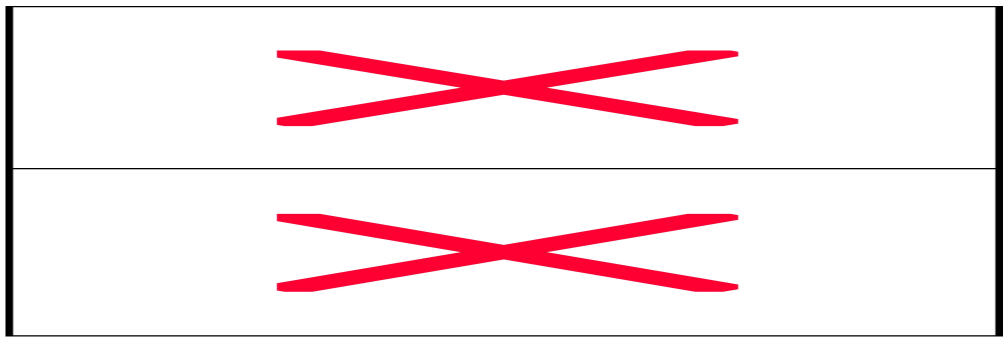
i: unit vector in x direction

j: unit vector in y directon

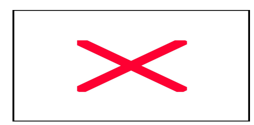
k: unit vector in z direction

Vector notation

Define two vectors, **A** and **B**, in a Cartesian coordinate system:



A$=(A_x,A_y,A_z)^T$ is a vector

other symbol: **A**$=(A_x,A_y,A_z)$, also 

i, j, k are unit vectors (here in Cartesian coordinates), normalized, orthogonal

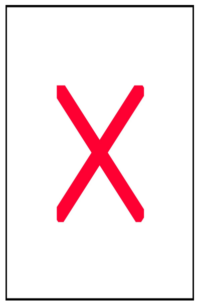
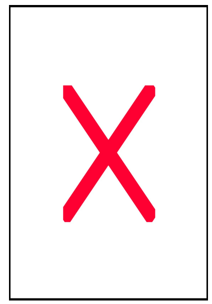
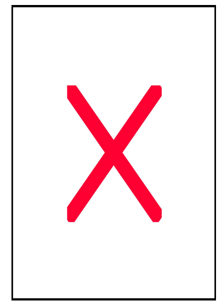
Vector calculus

Are you comfortable with:

- c * **A**, c is a scalar constant
- Magnitude of a vector **A: |A|**
- **A + B, A - B**
- Scalar product of two vectors:
- Vector product:
- Gradient of a scalar, e.g. T:
- Divergence:
- Curl:

Unit vectors

Unit vectors in Cartesian coordinates

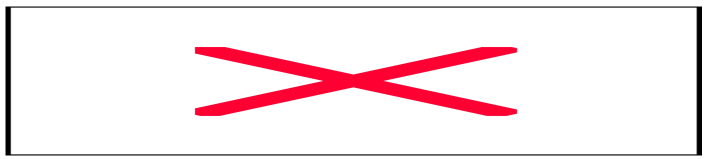
Unit vectors are normalized: |**i**| = |**j**| = |**k**| = 1

i, **j**, **k** are orthogonal:

Example: **i•i** = 1, **i•j** = 0, **i•k** = 0

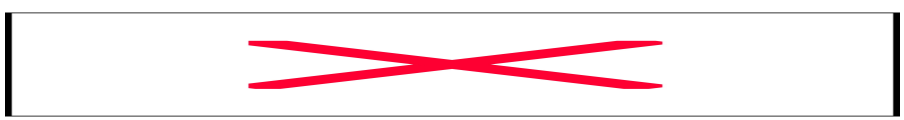
Magnitude and dot product (Cartesian coordinates)

- Magnitude of a vector: |**A**|



- Scalar (dot) product: **A•B = |A| |B|** cos(α)

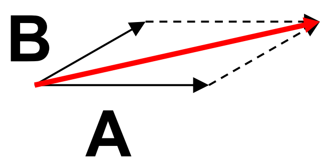
  α: angle between **A** and **B**



Sum/Subtraction of two vectors (Cartesian coordinates)

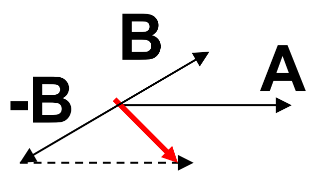
- Sum of two vectors:
  $\mathbf{A} + \mathbf{B} = (A_x+B_x)\mathbf{i} + (A_y+B_y)\mathbf{j} + (A_z+B_z)\mathbf{k}$

Sum:



- Subtraction of two vectors:
  $\mathbf{A} - \mathbf{B} = (A_x-B_x)\mathbf{i} + (A_y-B_y)\mathbf{j} + (A_z-B_z)\mathbf{k}$

Subtraction:



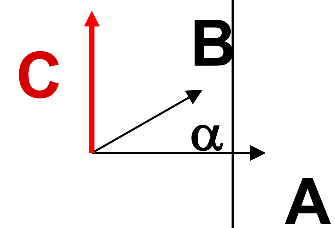
Vector (cross) product (Cartesian coordinates)

The picture can't be displayed.

- Vector product:

determinant

- In component form:
  $\mathbf{C} = \mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} + (A_zB_x - A_xB_z)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$



- The resultant vector **C** is in the plane that is perpendicular to the plane that contains **A** and **B** (direction: right hand rule)

Gradient (Cartesian coordinates)

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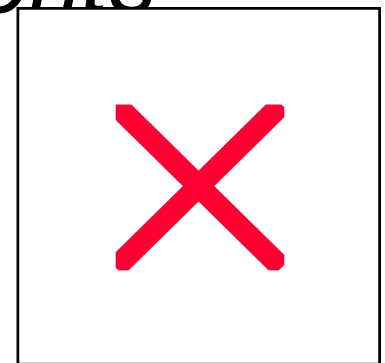
- Gradient of a scalar, e.g. temperature T:

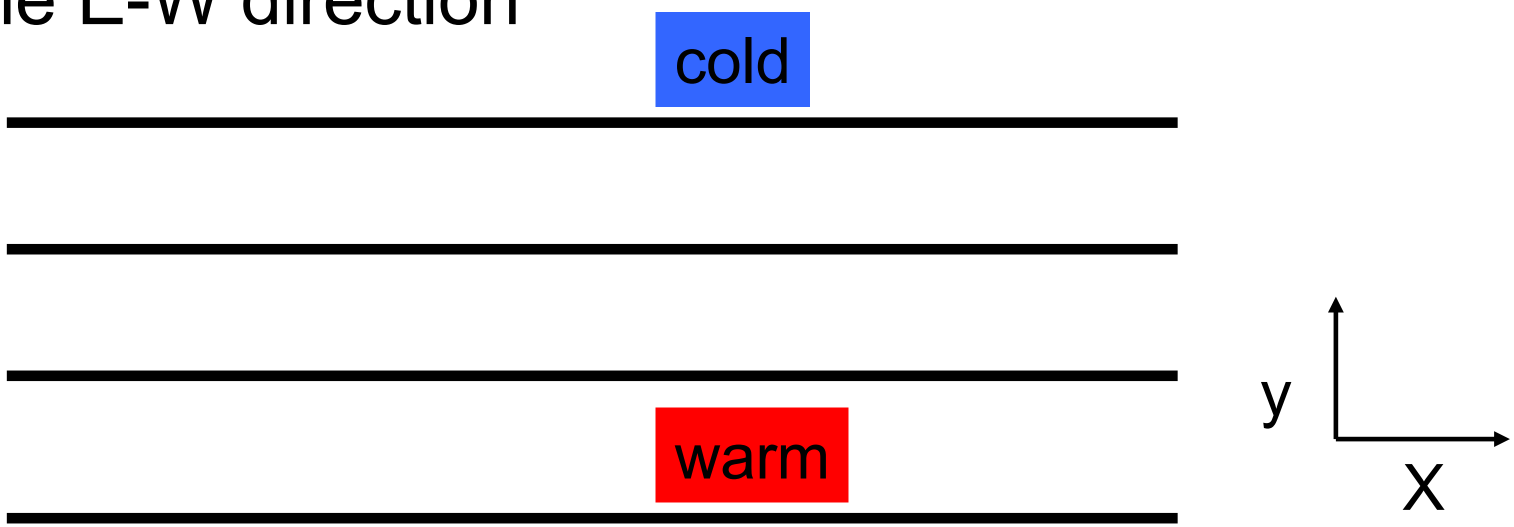
- Resulting vector quantity, use of partial derivatives

Horizontal temperature gradients

- Horizontal temperature gradient vector:



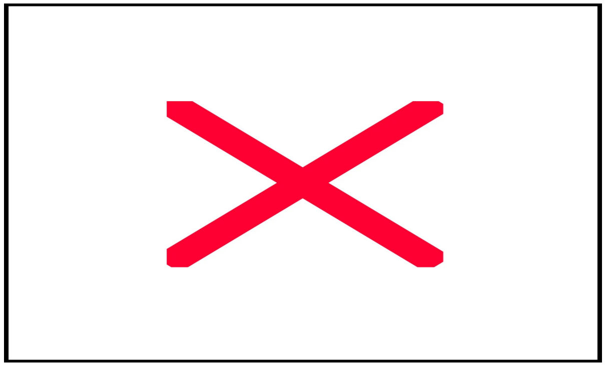
- Imagine the isotherms (solid lines)are oriented in the E-W direction

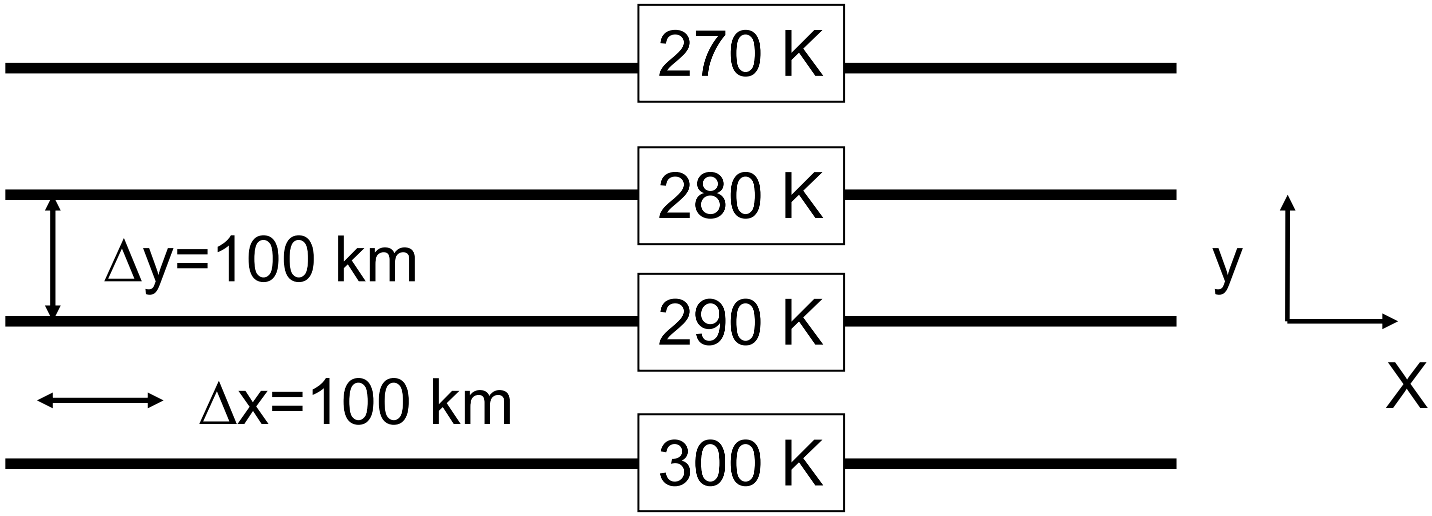


- Draw the horizontal temperature gradient vector.

Horizontal temperature gradients

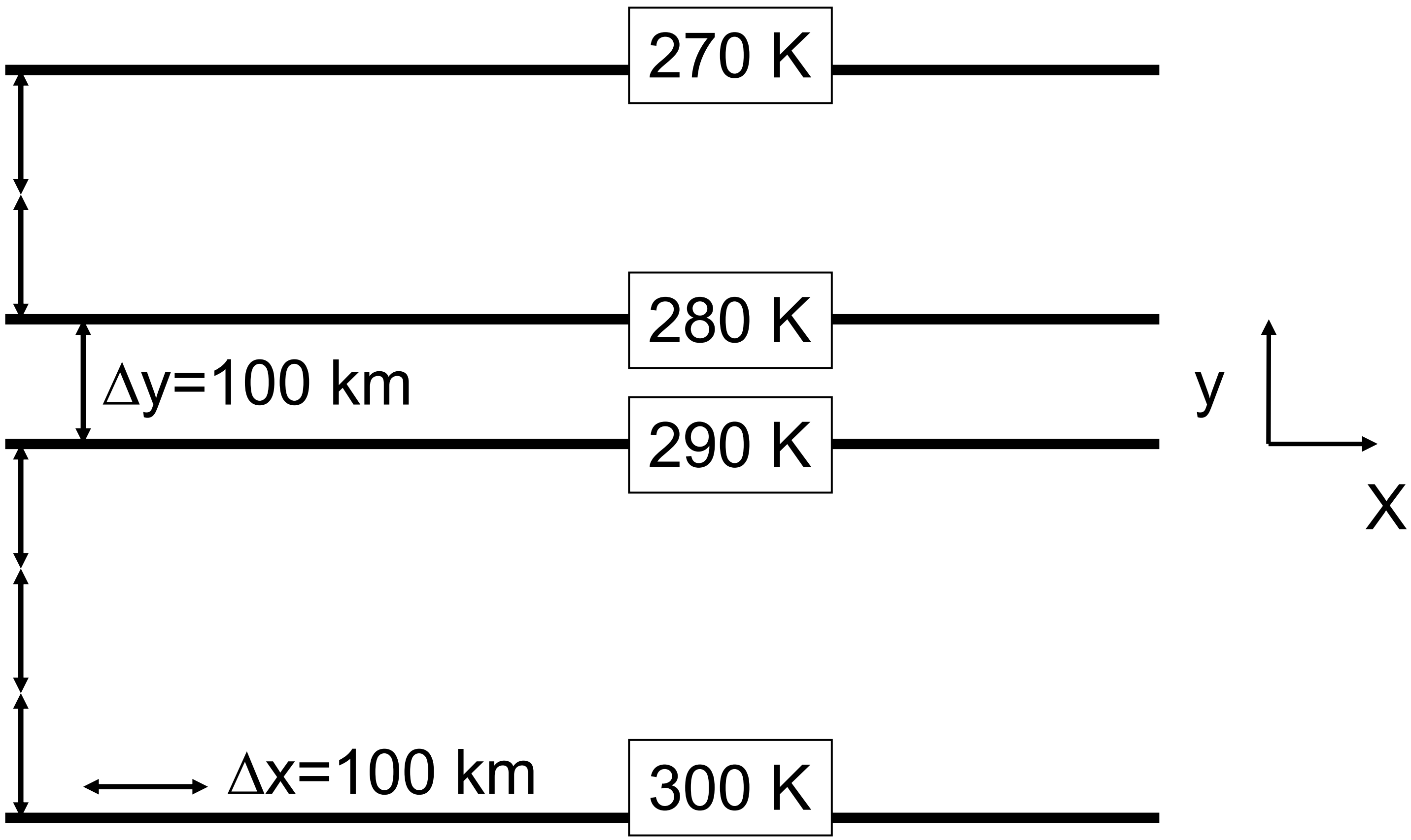
- Solid lines are isotherms
- Compute the components of the 2D temperature gradient vector:





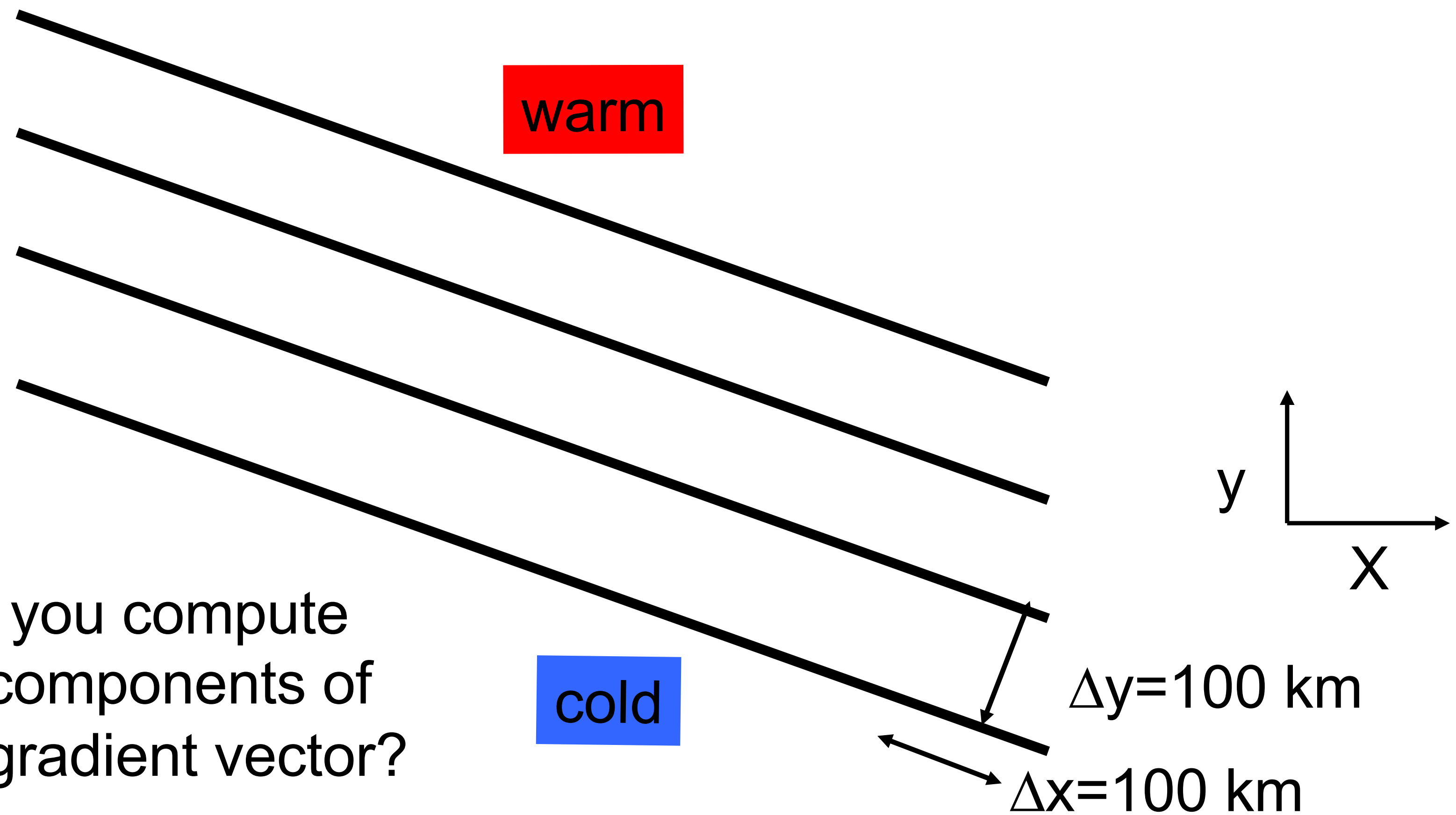
Horizontal temperature gradients

- How does the situation change below?
- Compute the components of the 2D temperature gradient vector:



Horizontal temperature gradients

- Draw the horizontal temperature gradient vector.
- Solid lines are isotherms, they are 10 K apart.



Can you compute the components of the gradient vector?