AOSS 321, Winter 2009
Earth System Dynamics

Lecture 4
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Today's lecture

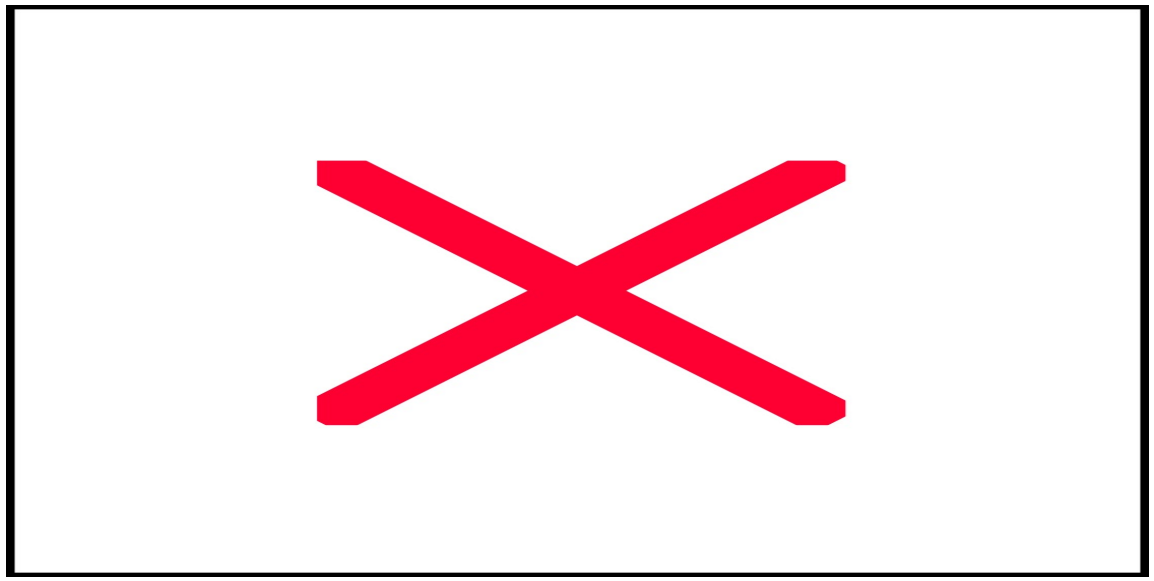
We will discuss

- Divergence / Curl / Laplace operator
- Spherical coordinates
- Vorticity / relative vorticity
- Divergence of the wind field
- Taylor expansion

Divergence (Cartesian coordinates)

The picture can't be displayed.

- Divergence of a vector **A**:

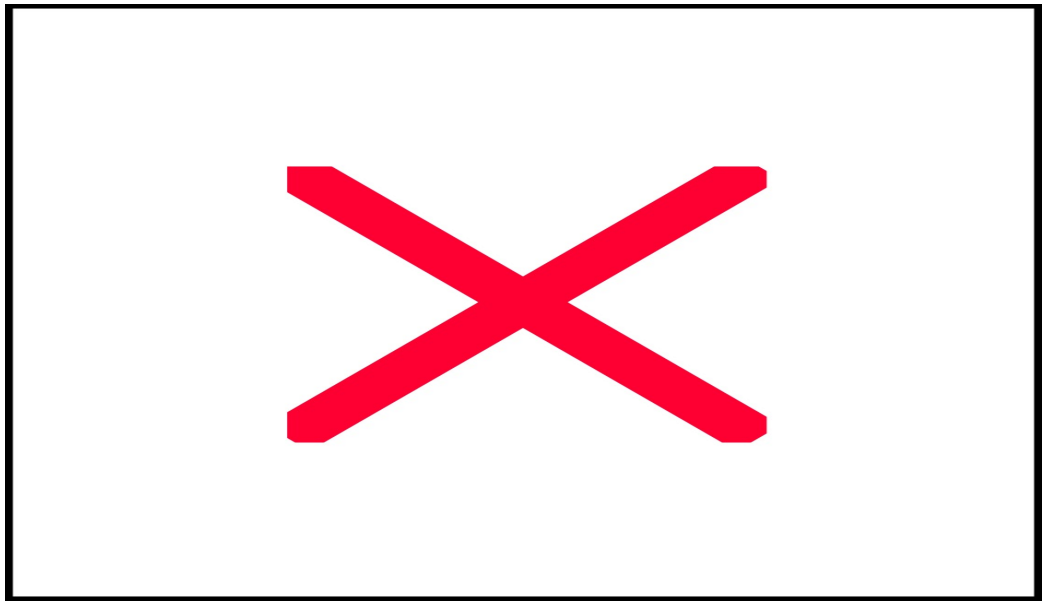


- Resulting scalar quantity, use of partial derivatives

Curl
(Cartesian coordinates)

The picture can't be displayed.

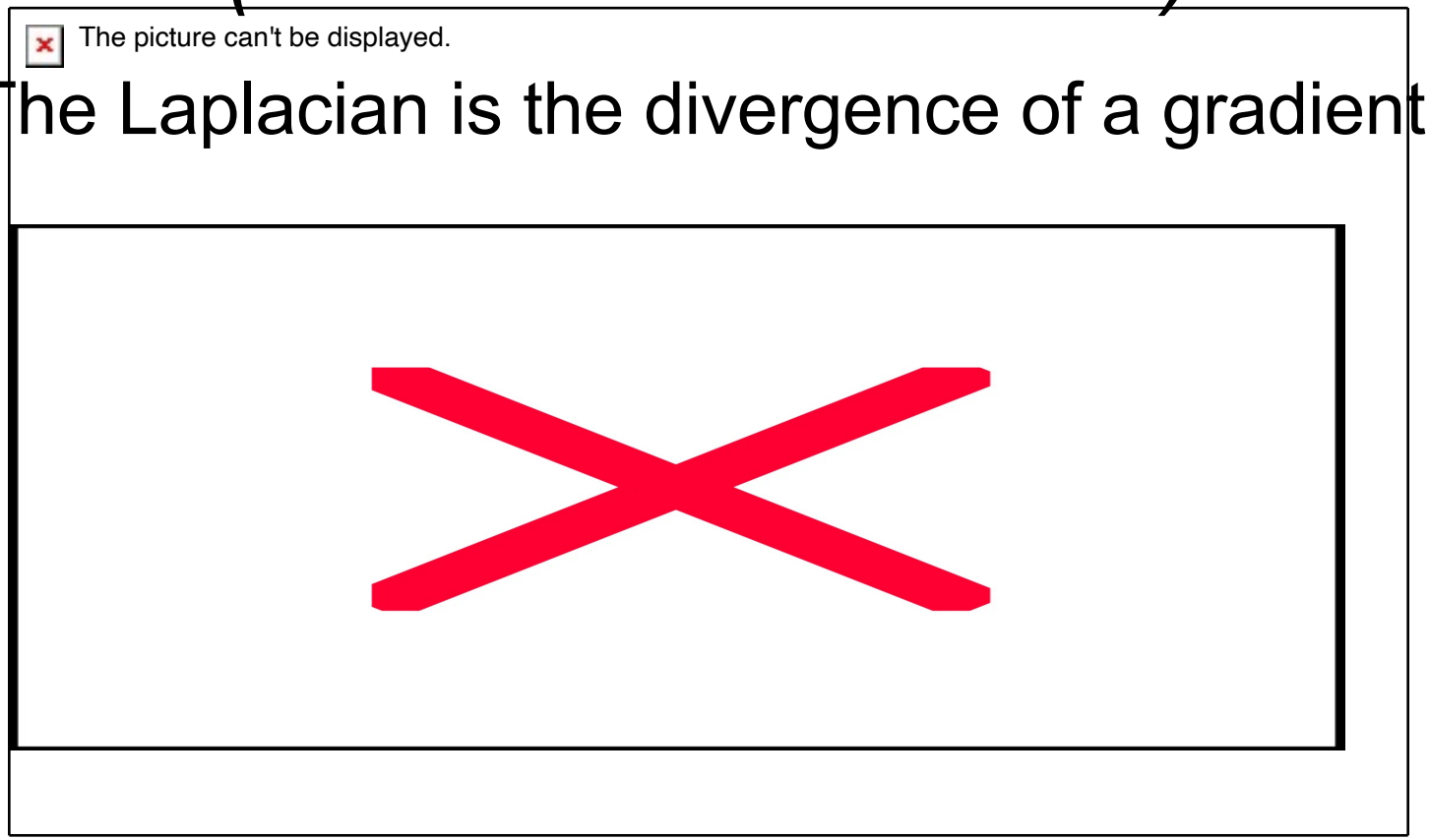
- Curl of a vector **A**:



- Resulting vector quantity, use of partial derivatives

Laplace operator (Cartesian coordinates)

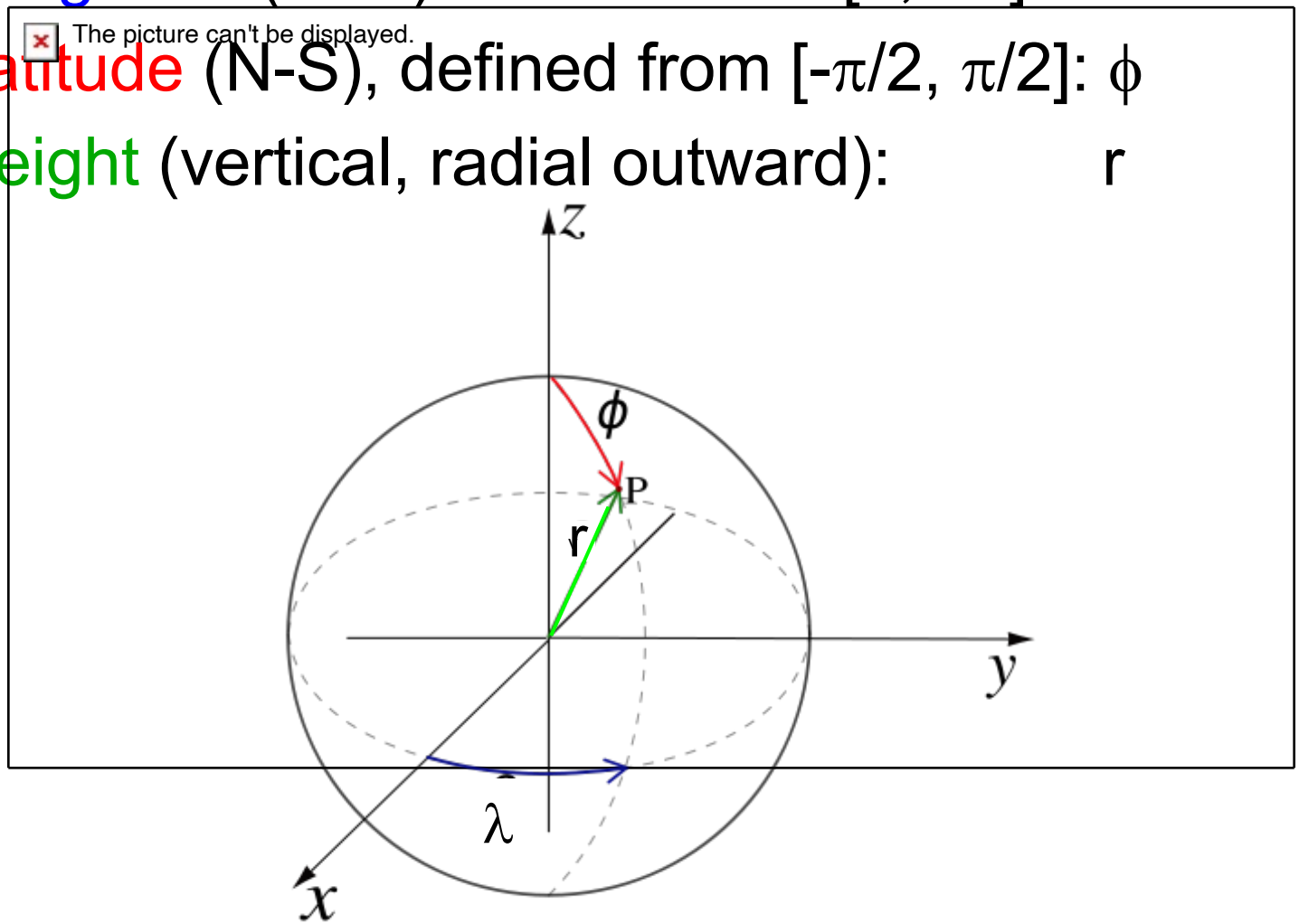
- The Laplacian is the divergence of a gradient



- Resulting scalar quantity, use of second-order partial derivatives

Spherical coordinates

- Longitude (E-W) defined from [0, 2π]: λ
- Latitude (N-S), defined from [-π/2, π/2]: ϕ
- Height (vertical, radial outward): r



Spherical velocity components

- Spherical velocity components:

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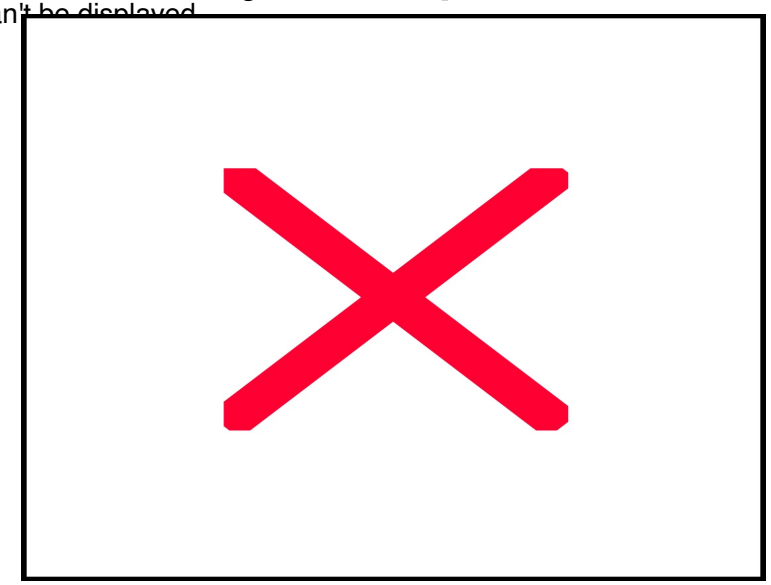
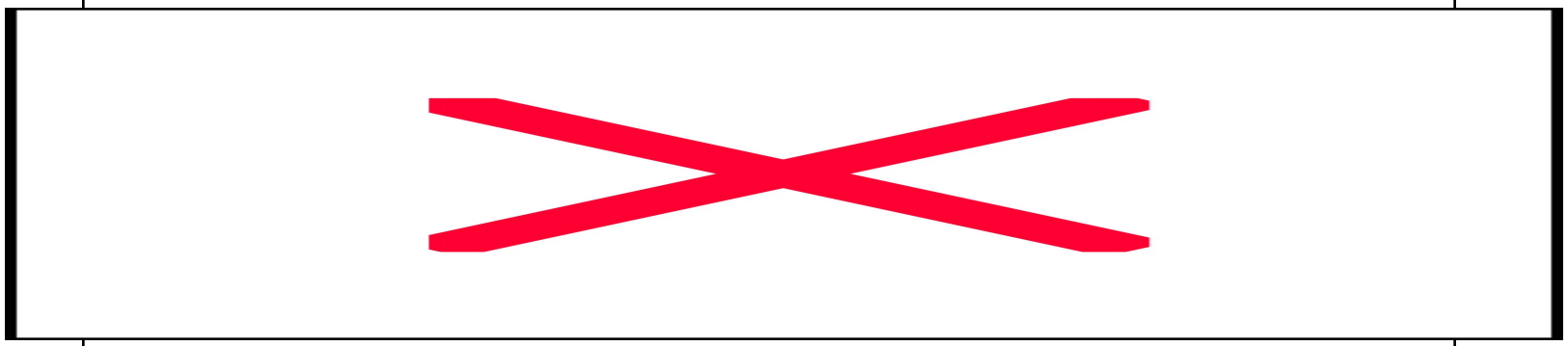


- r is the distance to the center of the earth, which is related to z by r = a + z
- a is the radius of the earth (a=6371 km), z is the distance from the Earth's surface

Unit vectors in spherical coordinates

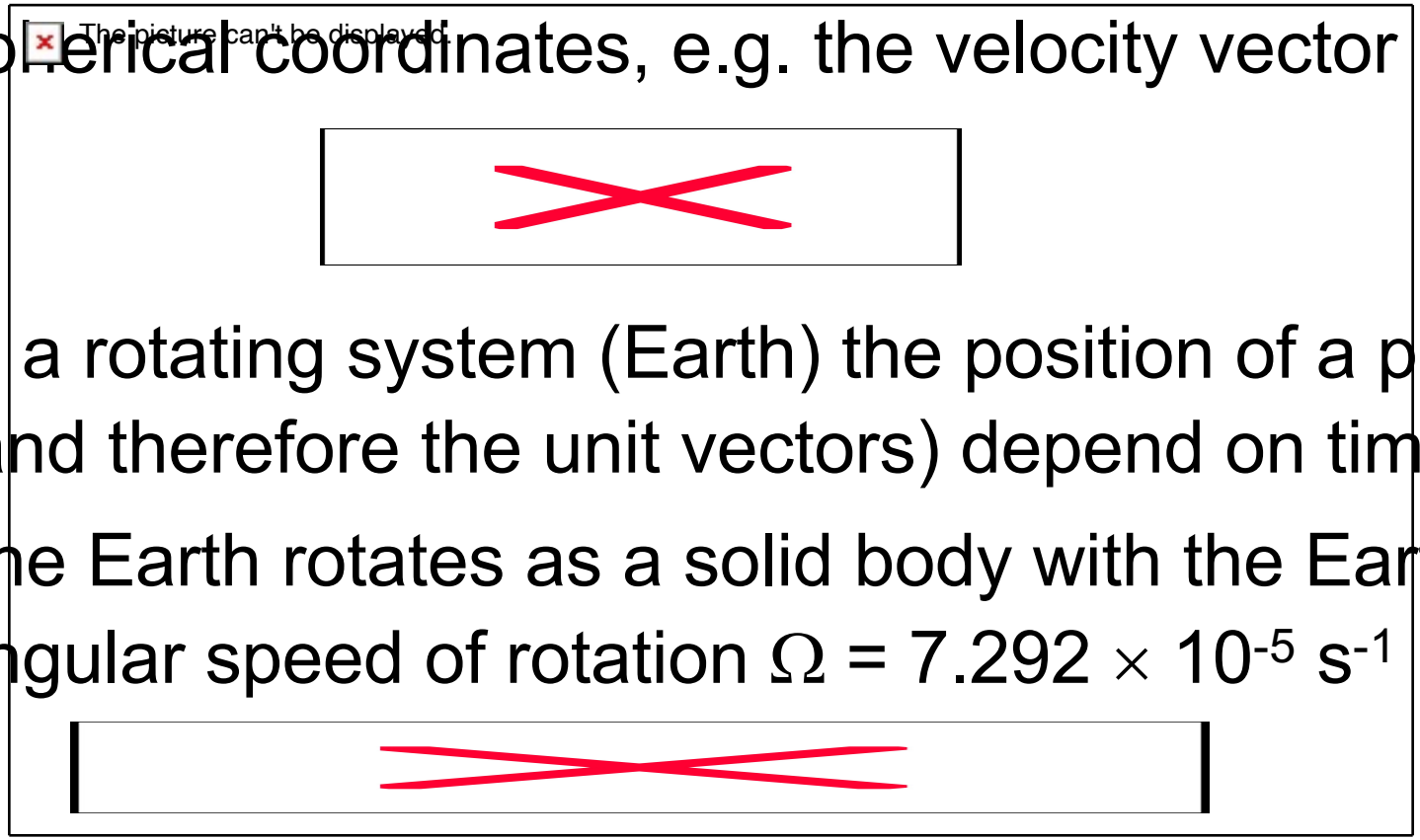
- Unit vectors in longitudinal, latitudinal and vertical direction:



- Unit vectors in the spherical coordinate system depend on the location (λ, ϕ)
- As in the Cartesian system: They are orthogonal and normalized

Velocity vector in spherical coordinates

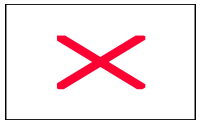
- Use the unit vectors to write down a vector in spherical coordinates, e.g. the velocity vector

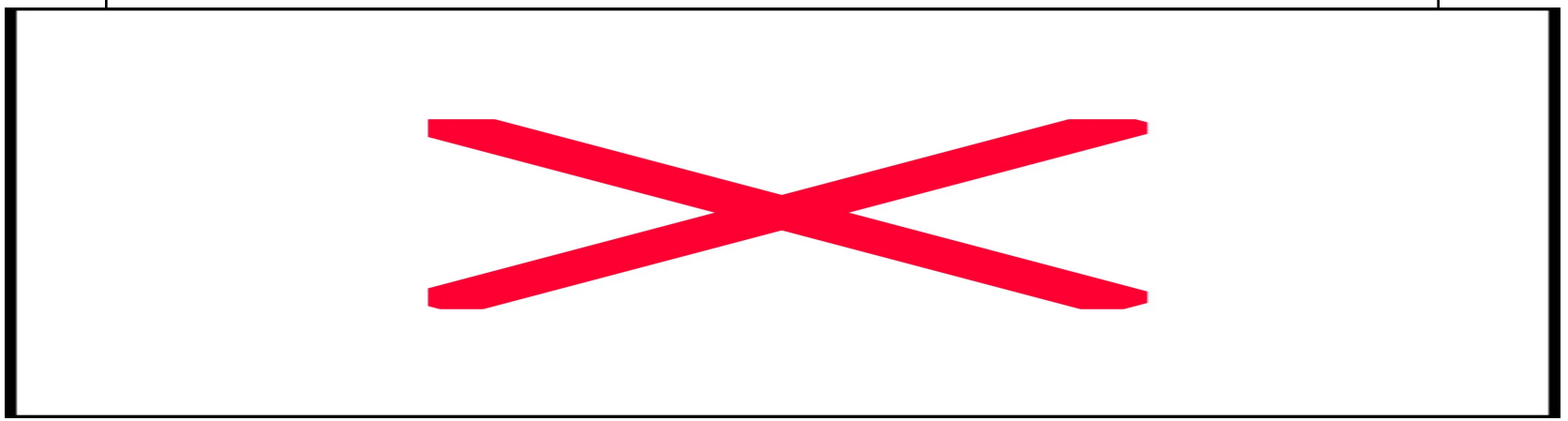


- In a rotating system (Earth) the position of a point (and therefore the unit vectors) depend on time
- The Earth rotates as a solid body with the Earth's angular speed of rotation $\Omega = 7.292 \times 10^{-5}\ s^{-1}$
- Leads to

Time derivative of the spherical velocity vector

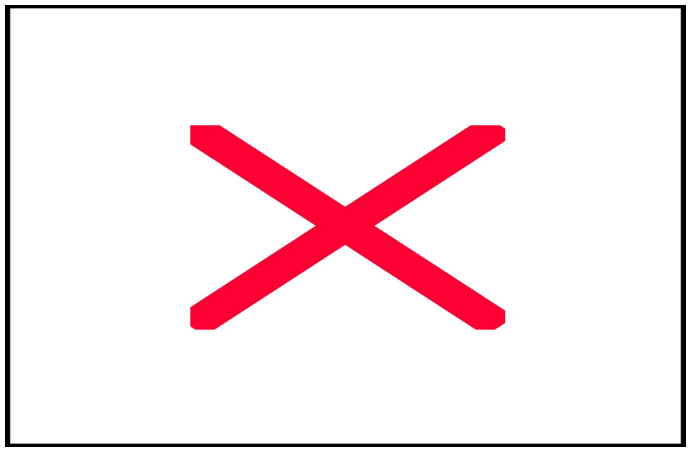
The picture can't be displayed.

- Spherical unit vectors  depend on time
- This means that a time derivative of the velocity vector in spherical coordinates is more complicated in comparison to the Cartesian system:



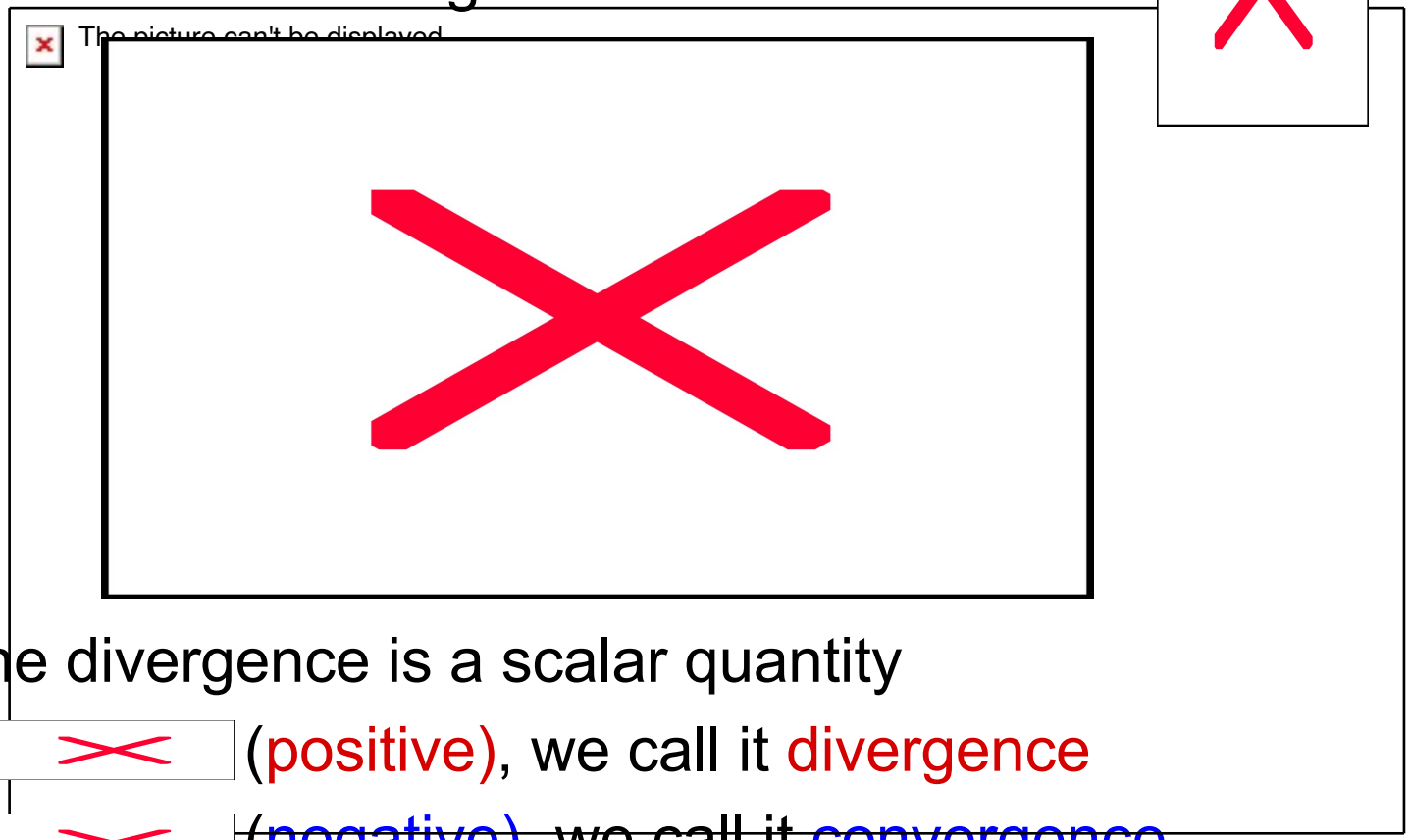
Spherical coordinates λ, ϕ, r

- Transformations from spherical coordinates to Cartesian coordinates:



Divergence of the wind field

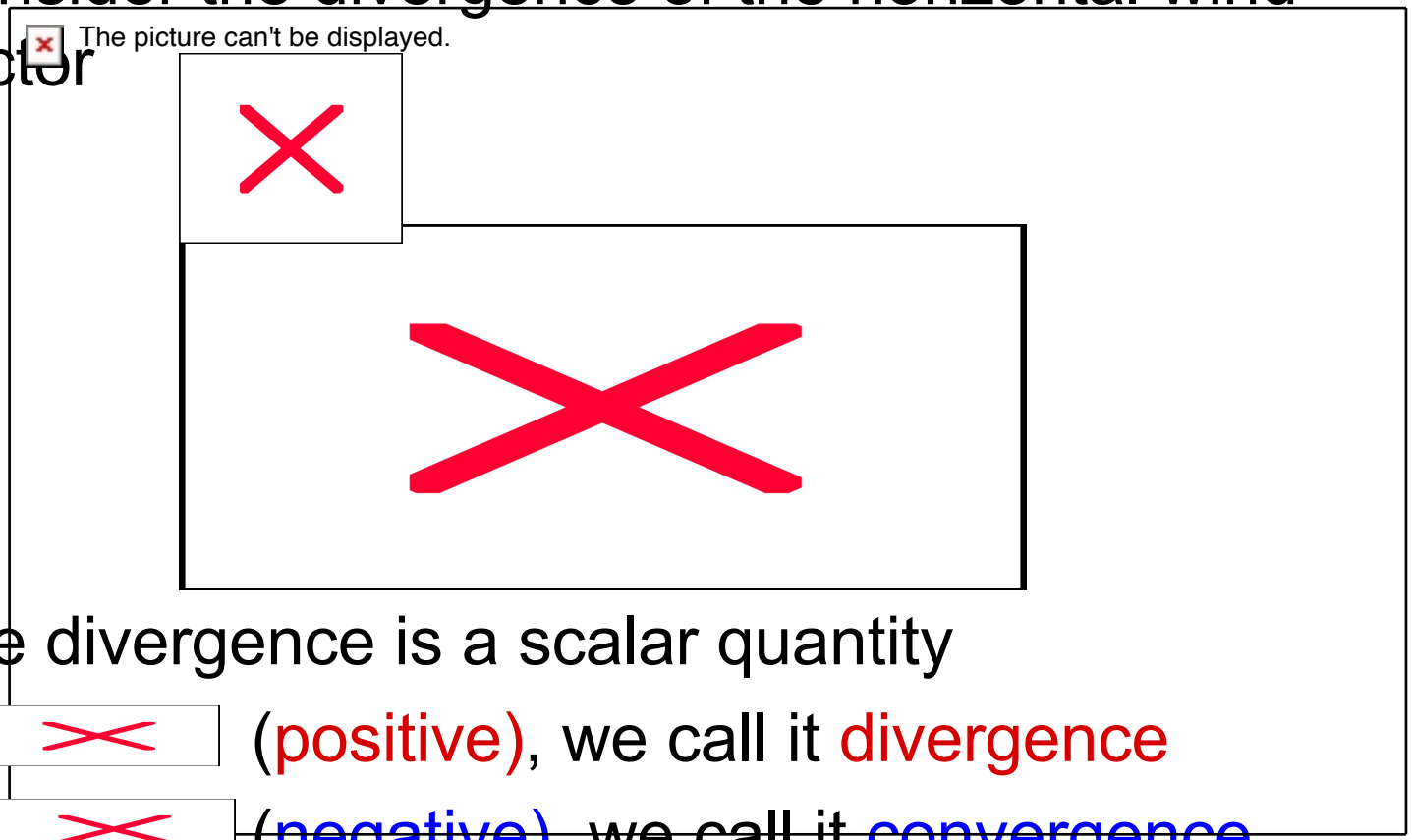
- Consider the divergence of the wind vector

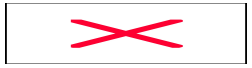


- The divergence is a scalar quantity
- If [ ] (positive), we call it divergence
- If [ ] (negative), we call it convergence
- If [ ] the flow is **nondivergent**
- Very important in atmospheric dynamics

Divergence of the wind field (2D)

- Consider the divergence of the horizontal wind vector

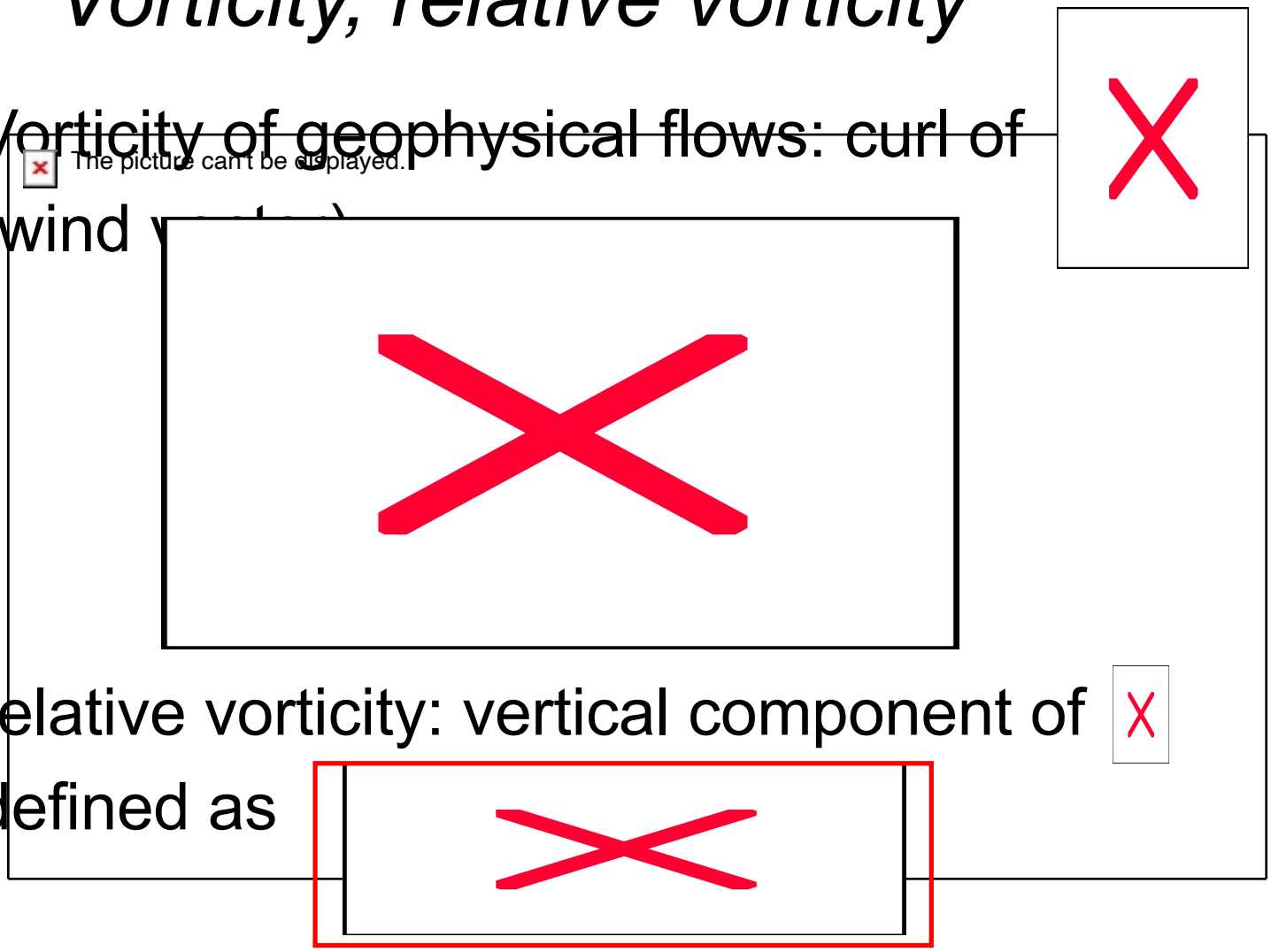


- The divergence is a scalar quantity
- If (positive), we call it divergence
- If (negative), we call it convergence
- If  the flow is **nondivergent**

Unit 3, frame 17: http://www.atmos.washington.edu/2005Q1/101/CD/MAIN3.swf

Vorticity, relative vorticity

- Vorticity of geophysical flows: curl of (wind)

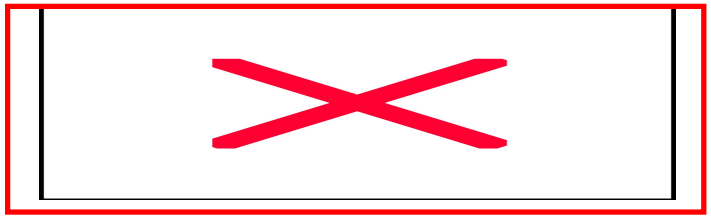


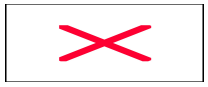
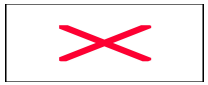
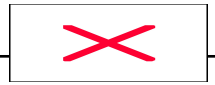
- relative vorticity: vertical component of defined as

- Very important in atmospheric dynamics

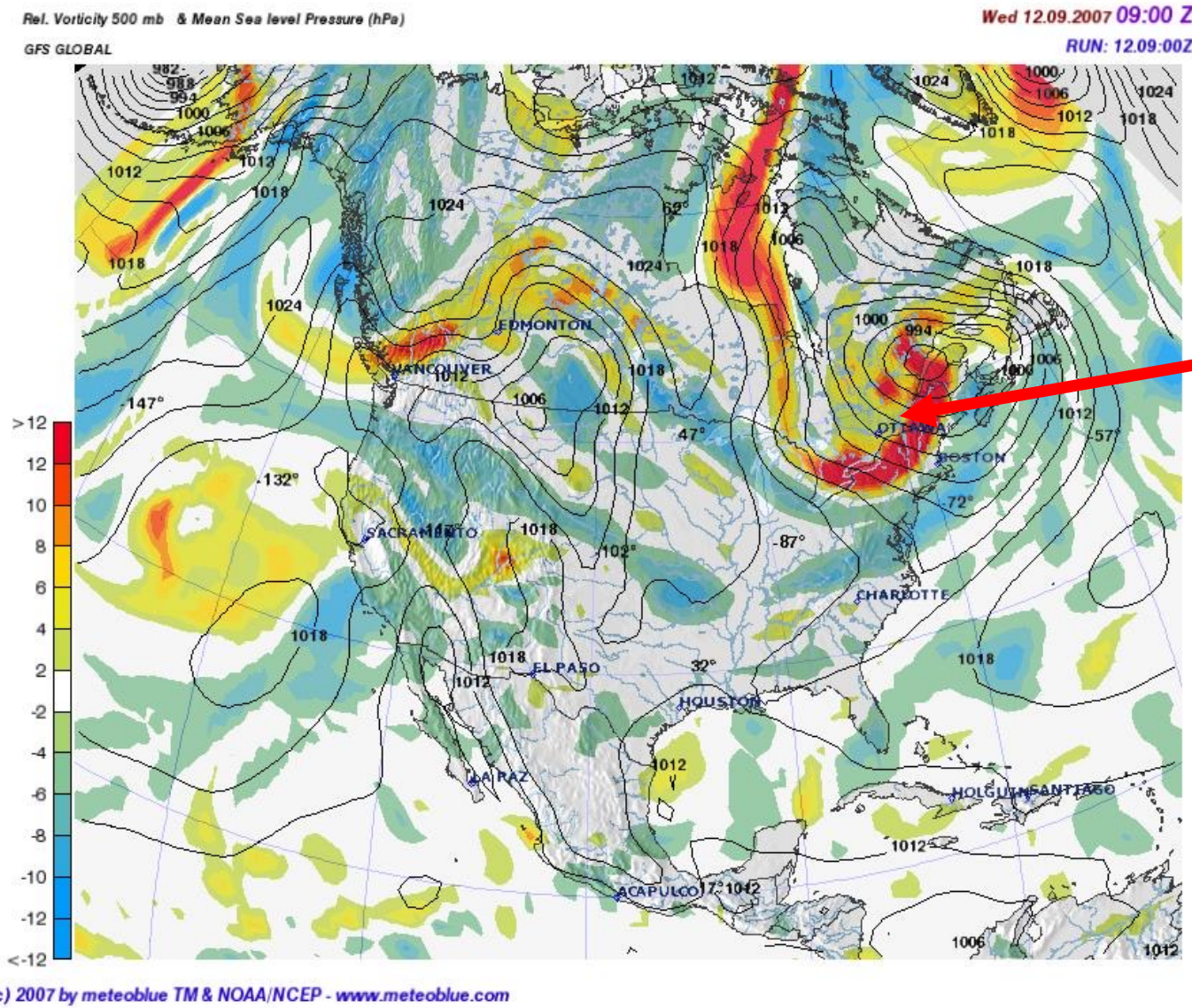
Relative vorticity

- Vertical component of 



- Relative vorticity is
  -  (positive) for counterclockwise rotation
  -  (negative) for clockwise rotation
  -  for irrotational flows
- Great web page: http://my.Meteoblue.Com/my/

Real weather situations
500 hPa rel. vorticity and mean SLP

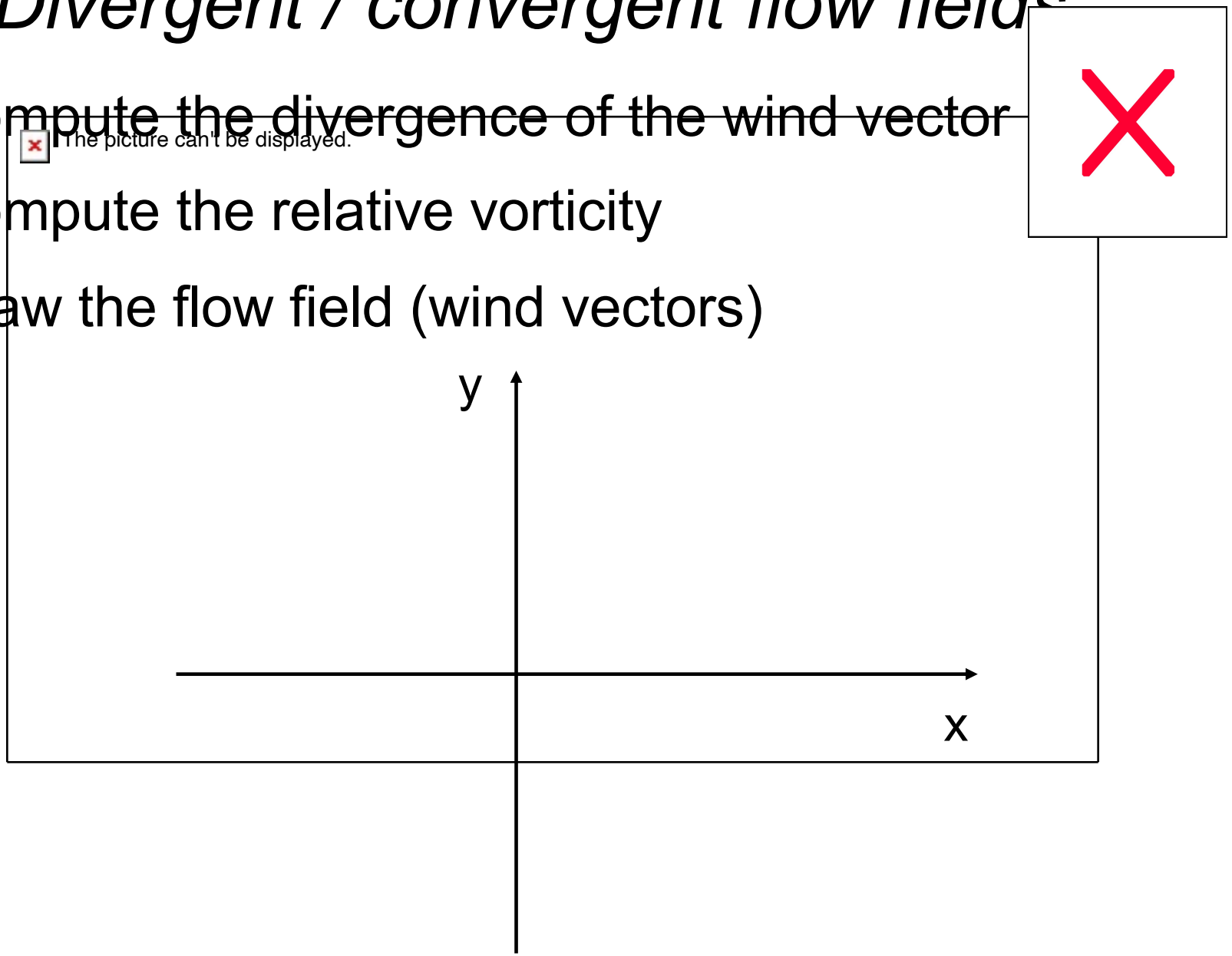


sea level pressure

Positive rel. vorticity, counterclock-wise rotation, in NH: low pressure system

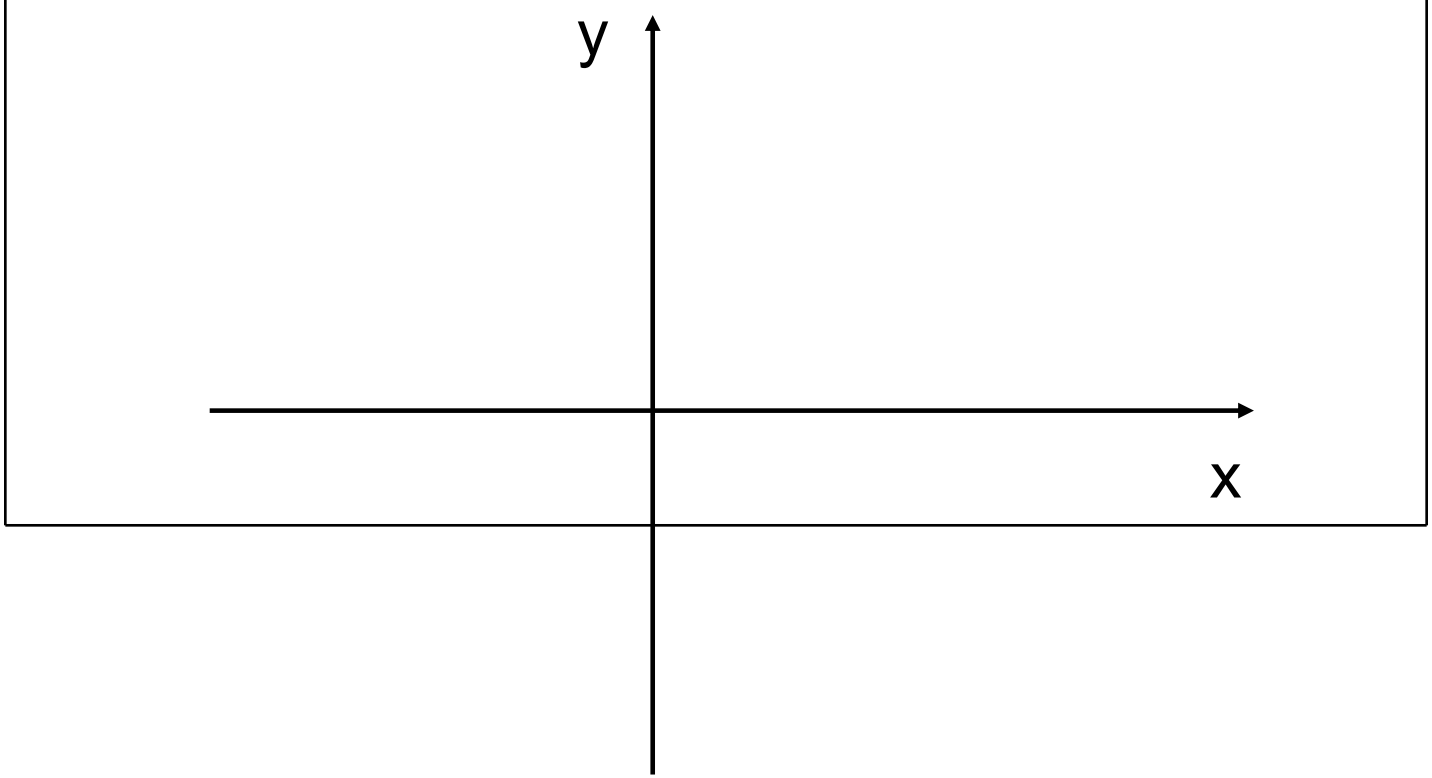
Divergent / convergent flow fields

- ~~Compute the divergence of the wind vector~~
- Compute the relative vorticity
- Draw the flow field (wind vectors)



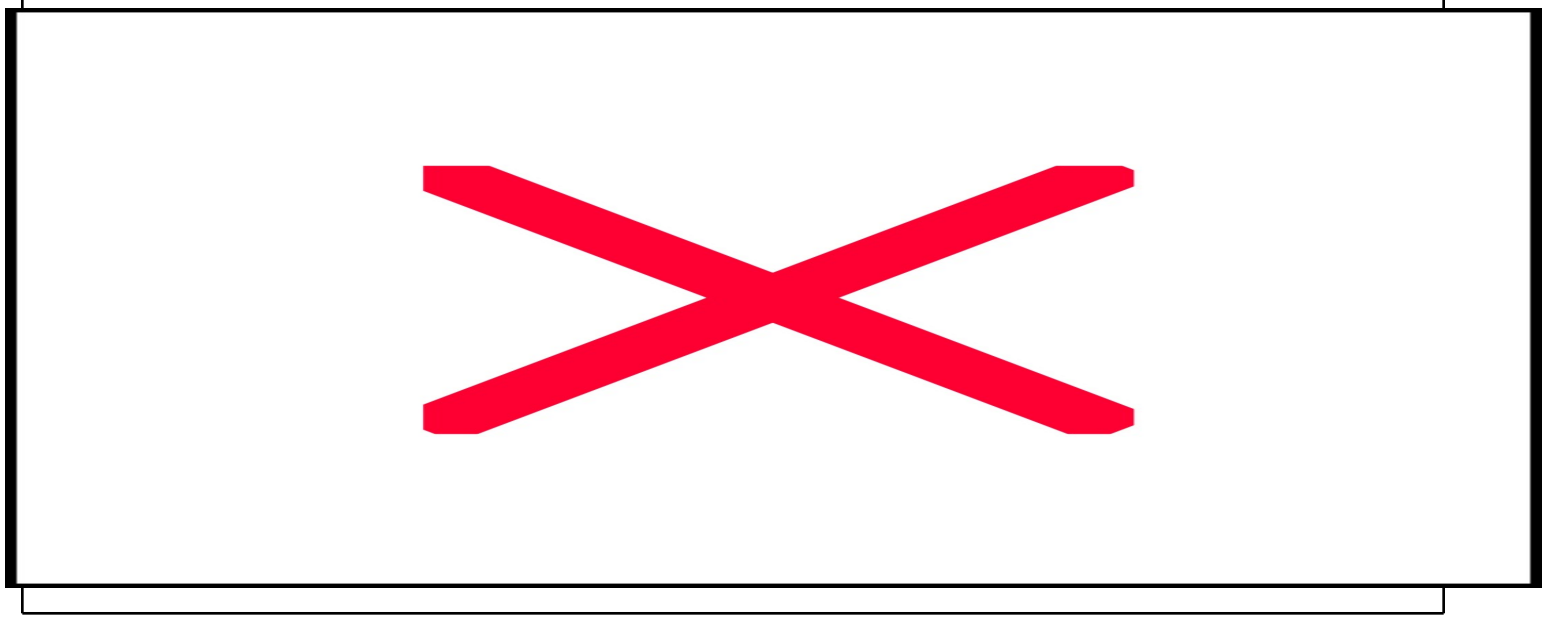
Rotational flow fields

- ~~Compute the rel. vorticity of the wind vector~~
- Compute the divergence
- Draw the flow field (wind vectors)



Taylor series expansion

- It is sometimes convenient to estimate the value of a continuous function f(x) about a point $x = x_0$ with a power series of the form:



- In the last approximation, we neglected the higher order terms