AOSS 321, Winter 2009
Earth System Dynamics

Lecture 6 & 7
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What are the fundamental forces in the Earth's system?

- Surface forces:
  - Pressure gradient force
  - Viscous force
- Body force: Gravitational force
- Apparent forces: Centrifugal and Coriolis
- Electromagnetic force (important for charged particles, e.g. in the ionosphere, not important for the neutral atmosphere)

- Total Force is the sum of all of these forces.

Viscous force(1): Friction

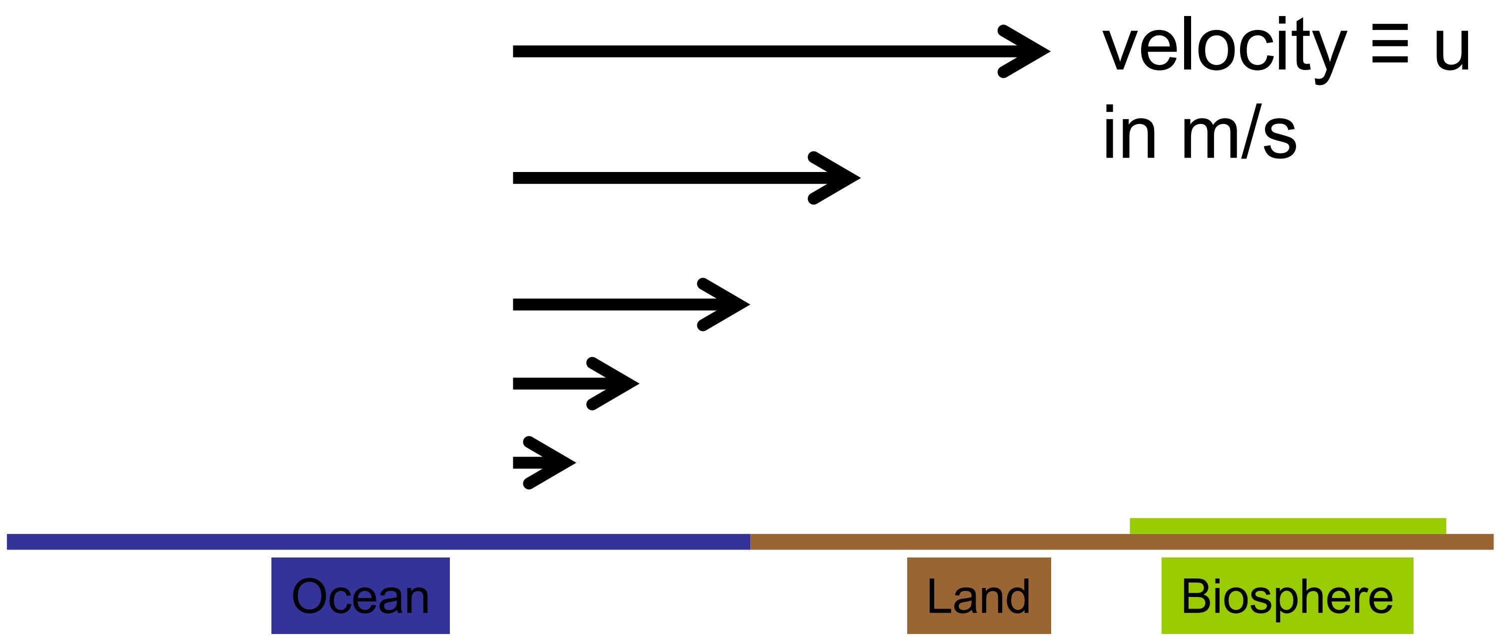
- In a fluid there is friction which resists the flow.
- It is dissipative, and if the fluid is not otherwise forced, it will slow the fluid and bring it to rest.
- Away from boundaries in the atmosphere this frictional force is often small, and it is often ignored.
- Close to the boundaries, however, we have to consider friction.

Viscous force (2)

velocity ≡ u
in m/s

velocity must be 0 at surface

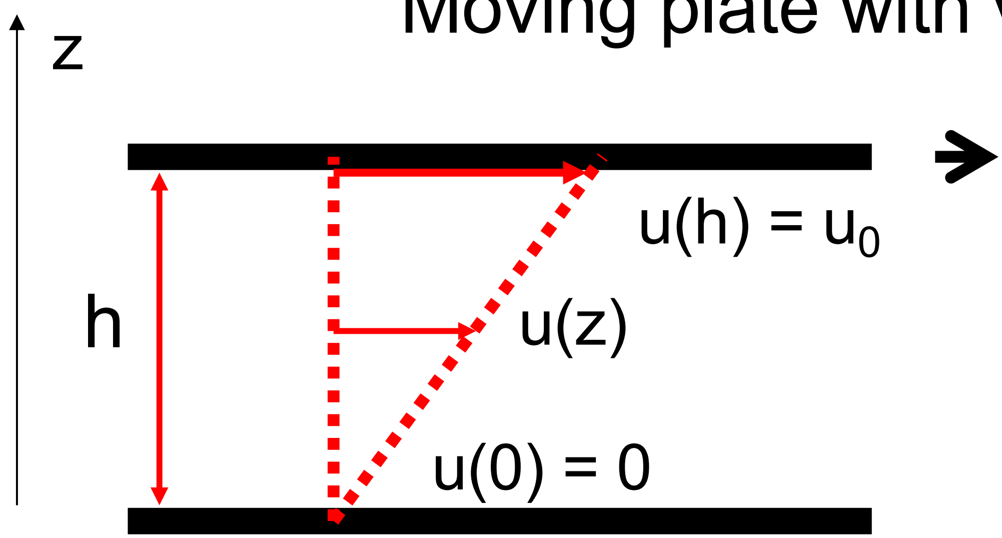
Viscous force (3)

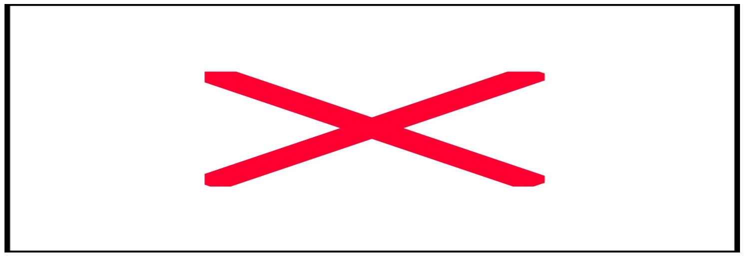


Velocity is zero at the surface; hence, there is some velocity profile.

Viscous force (4)
(How do we think about this?)

Moving plate with velocity u_0

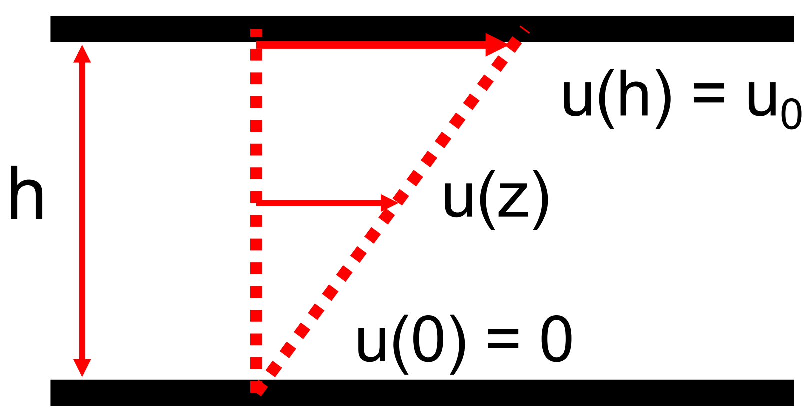




Linear velocity profile!

Viscous force (5)
(How do we think about this?)

The drag on the moving plate is the same of as the force required to keep the plate moving. It is proportional to the area (A), proportional to the velocity u_0 of the plate, and inversely proportional to the distance h between the plates; hence,



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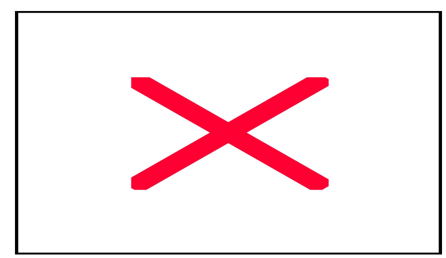
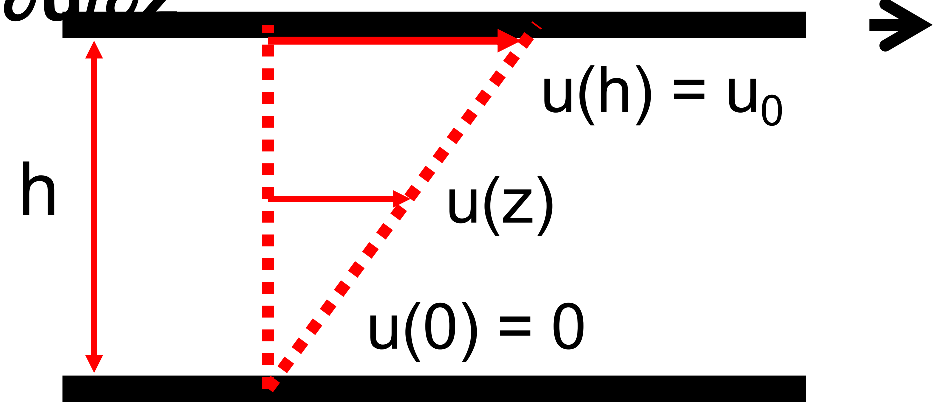
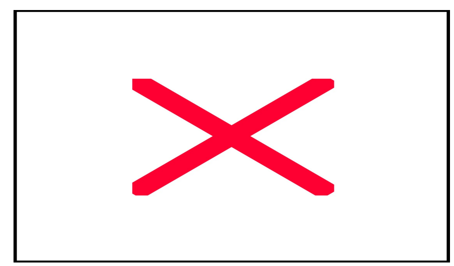


Proportional usually means we assume linear relationship. This is a model based on observation, and it is an approximation. The constant of proportionality assumes some physical units. What are they?

Viscous force (6)
(How do we think about this?)

Recognize the $\mathbf{u_0/h}$ can be represented by $\partial \mathbf{u}/\partial z$

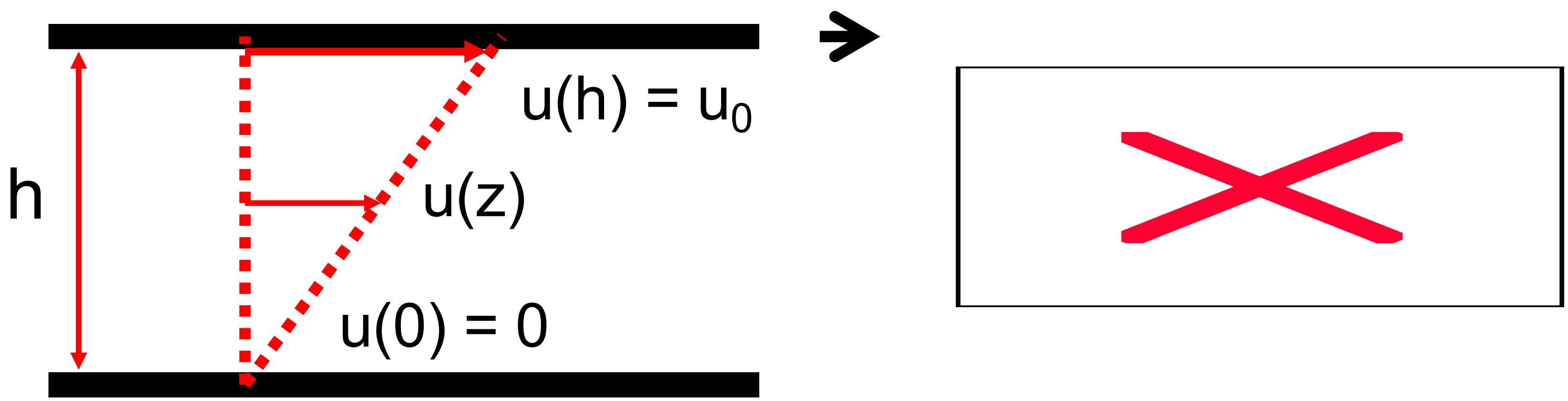




Force per unit area is F/A is defined as shearing stress (τ). Like pressure the shearing stress is proportional to area.

Viscous force (7) & Shearing stress

$\tau_{zx} = \mu(\partial u/\partial z)$ is the component of the shearing stress in the x direction due to the vertical (z) shear of the x velocity component (velocity u)



F_{zx}/A: Force per unit area with units kg m^{-1} s^{-2}: hence, like pressure

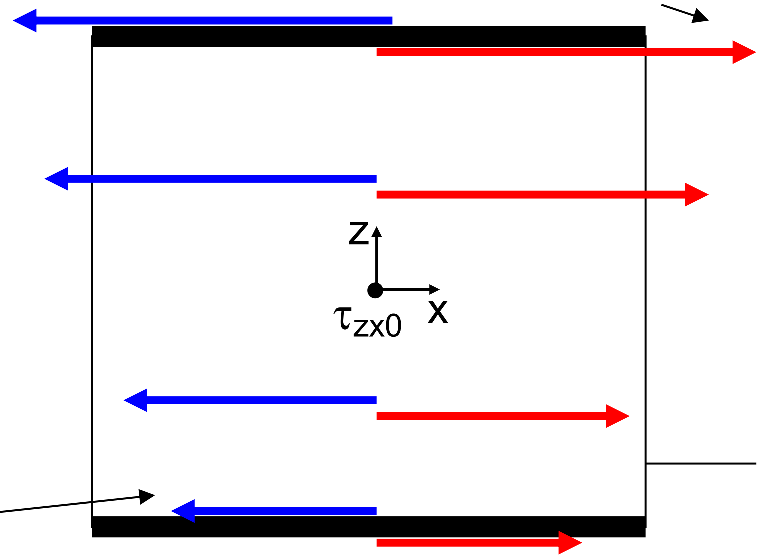
Viscous force (8) & Shearing stress

Remember Newton's third law:
"For every action, there is an equal and opposite reaction."

$\tau_{zx} = -(\tau_{zx0} + (\partial\tau_{zx}/\partial z)\Delta z/2)$

$\tau_{zx} = \tau_{zx0} + (\partial\tau_{zx}/\partial z)\Delta z/2$

Stress above a surface



Stress below a surface

Front view of 3D box

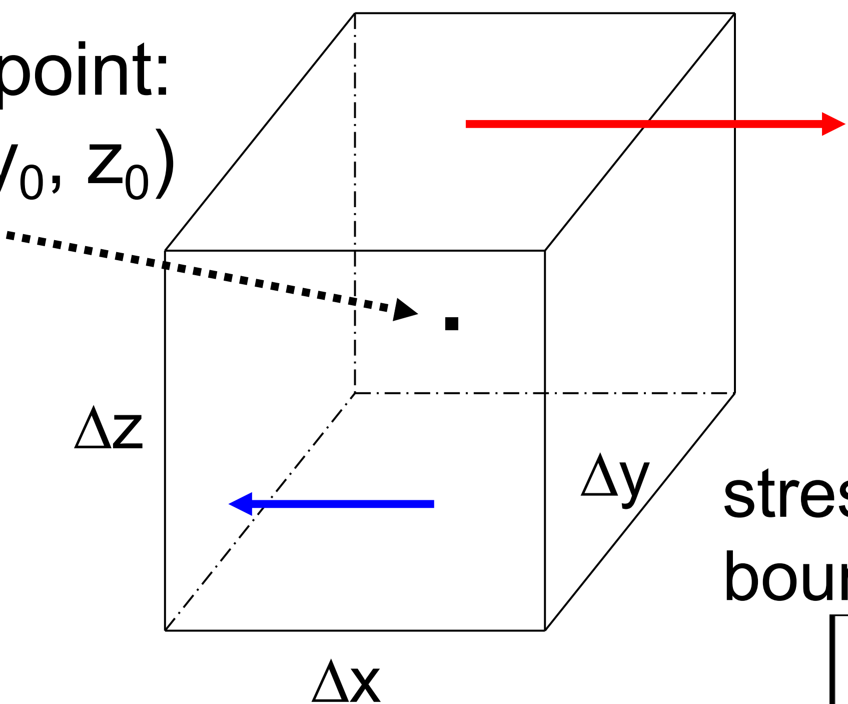
$\tau_{zx} = -(\tau_{zx0} - (\partial\tau_{zx}/\partial z)\Delta z/2)$

$\tau_{zx} = \tau_{zx0} - (\partial\tau_{zx}/\partial z)\Delta z/2$

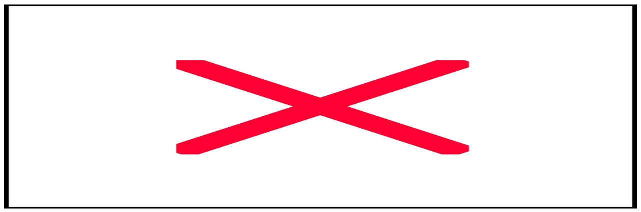
The opposing force introduces a '-' sign!
We are interested in the net force **inside** the volume.

Viscous force (9) & Shearing stress
(Do the same thing we did for pressure)

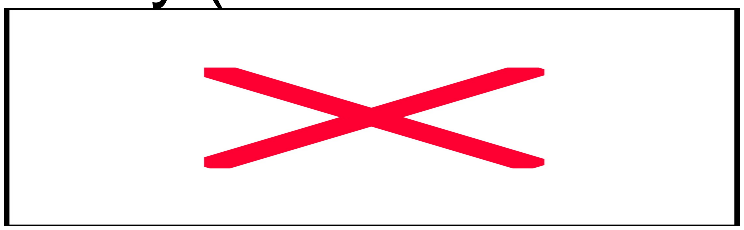
mid point:
(x_0, y_0, z_0)



stress below the upper boundary (inside the volume):



stress above the lower boundary (inside the volume):

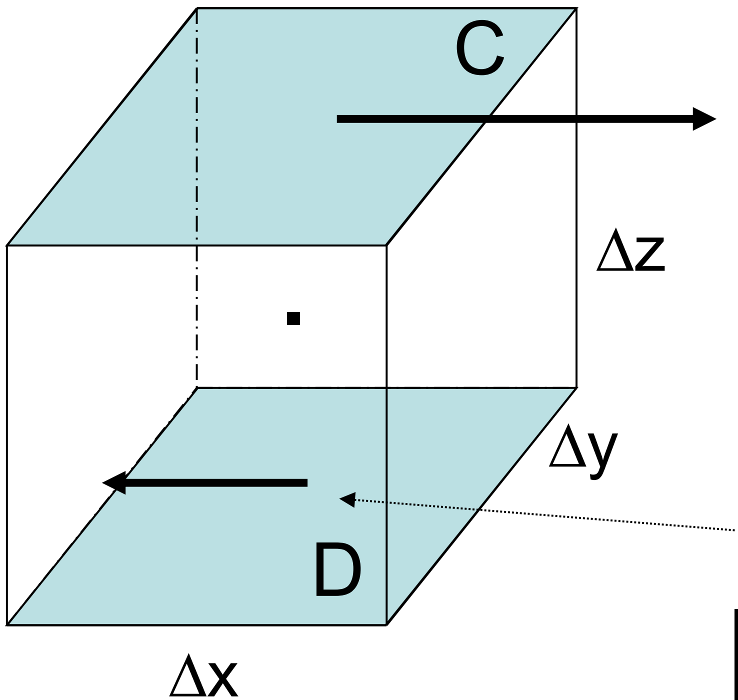


τ_{zx0} = stress at (x_0, y_0, z_0)

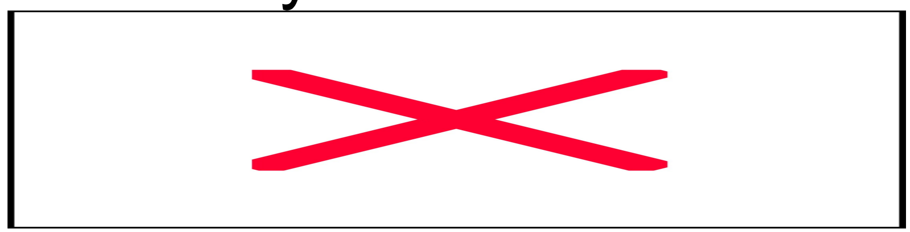
Viscous force (10) & Shearing stress

Viscous force = shearing stress * area

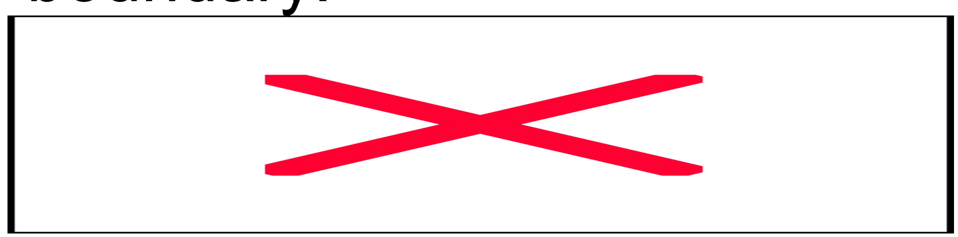
Area of side C: ΔyΔx



Viscous force below the upper boundary:



Viscous force above the lower boundary:

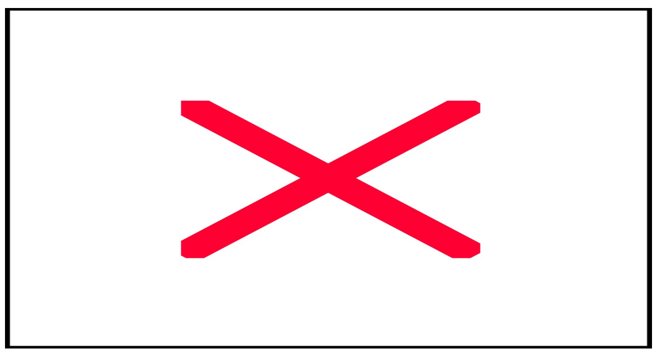


Area of side D: ΔyΔx

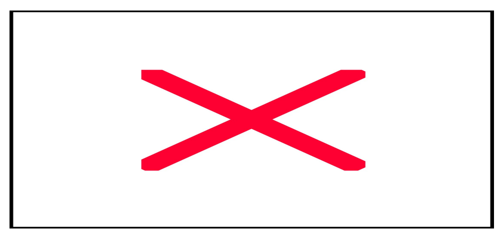
Viscous force (11)

Net viscous force (inside a volume) in the x direction due to the **vertical (z) shear** of the x velocity component (u):

Sum of the forces that act below the upper boundary C and above the lower boundary D on the fluid



We want the viscous force per unit mass:



Viscous force (12)
(using definition of τ)

Recall the definition of the shearing stress:

The picture can't be displayed.

It follows:

Viscous force (13)

μ is also called **dynamic viscosity** coefficient.
Assume that μ is constant, then:

The picture can't be displaye

$\nu \equiv \mu/\rho =$ **kinematic viscosity** coefficient

Viscous force (14)

Do the same for the shearing stress in x direction due to the **horizontal (x and y) shear** of the x velocity component (u): τ_{xx} and τ_{yx}

The total frictional (viscous) force F_r in the x direction is called F_{rx} .

It is the sum of all viscous forces acting in x:

Viscous force (15)

Do same for other directions of force

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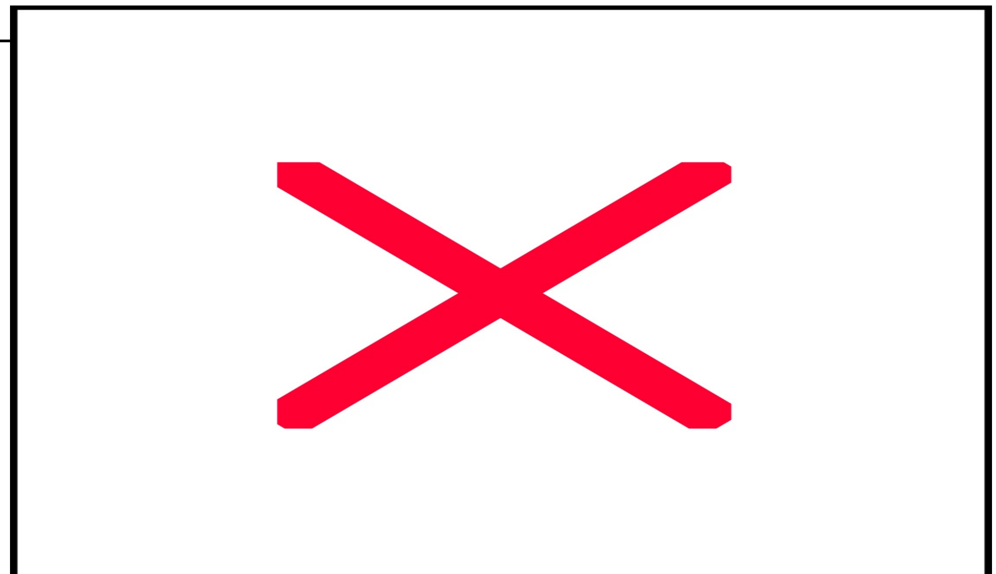
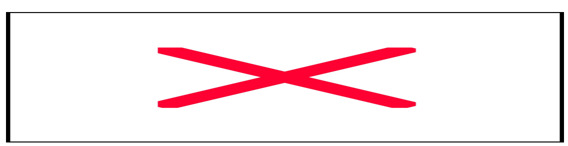
N-S (y) direction:



vertical direction:

velocity vector: 

Viscous force (16)

Viscous force per unit mass written in vector form:

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Kinematic viscosity coefficient

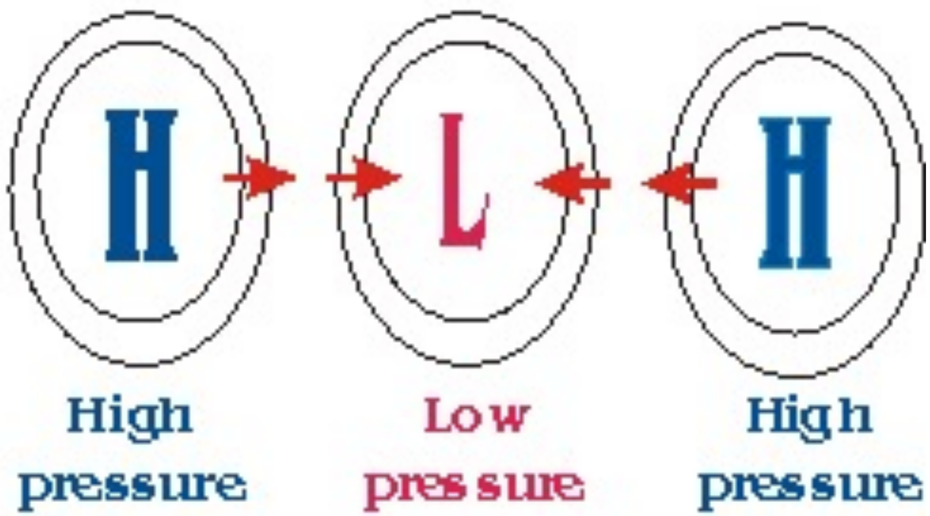
Where Laplacian is operating on the velocity vector:

Our surface forces

other forces

Here, we use the text's convention that the velocity is

Highs and Lows



Motion initiated by pressure gradient

Opposed by viscosity

← →

Summary: Surface forces

- Pressure gradient force and viscous force are examples of surface forces.
- They were proportional to the area of the surface of our particle of atmosphere.
- They are independent of the mass of the particle of atmosphere.
- They depend on characteristics of the particle of atmosphere; characteristics of the flow.

Body forces

- Body forces act on the center of mass of the parcel of fluid.
- Magnitude of the force is proportional to the mass of the parcel.
- The body force of interest to dynamic meteorology is gravity.

Newton's Law of Gravitation

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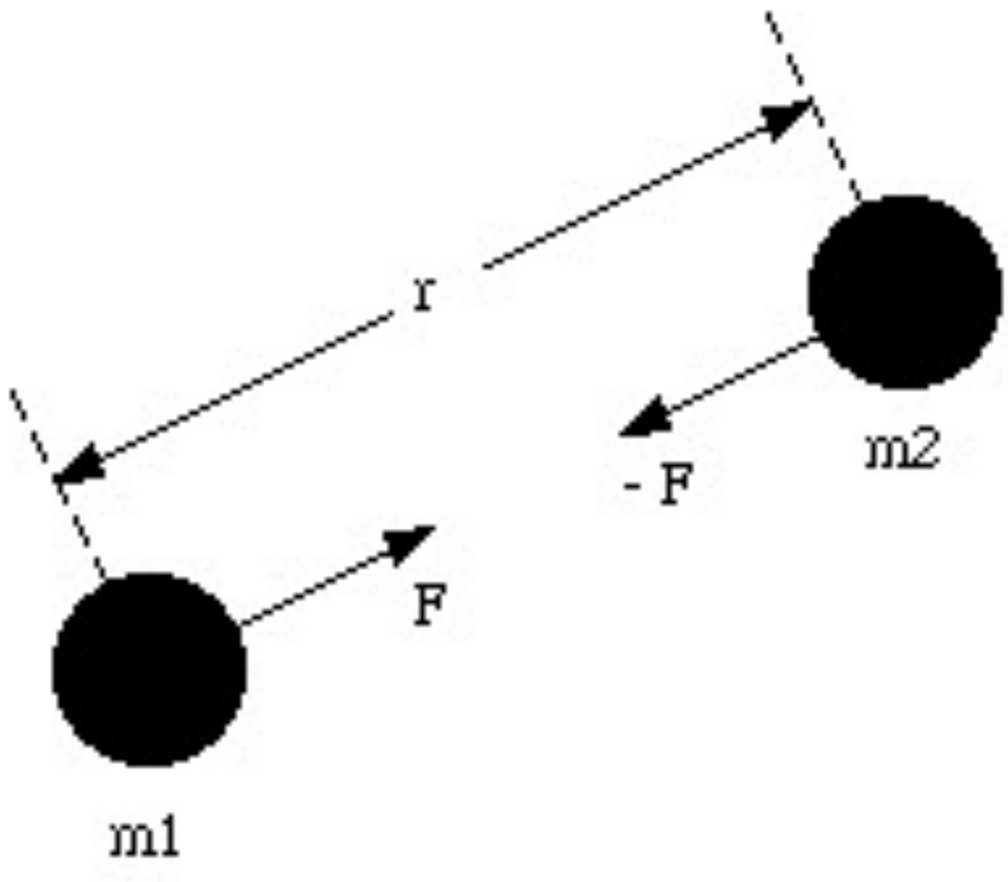
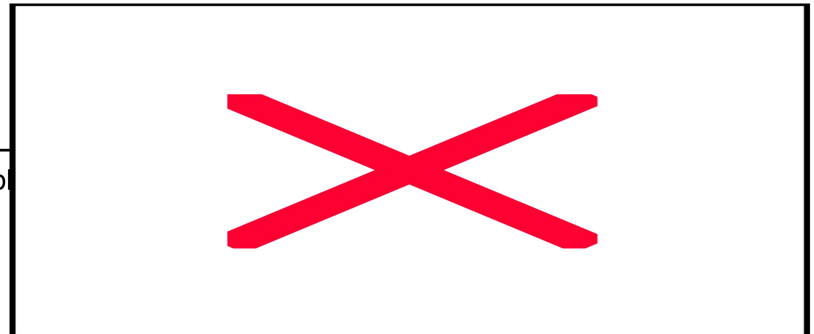
Newton's Law of Gravitation: Any two elements of mass in the universe attract each other with a force *proportional to their masses m_1 and m_2* and *inversely proportional to the square of the distance r* between the centers of the mass.

The force acts along the line joining the particles and has the magnitude proportional to G, the universal gravitational constant.

Gravitational Force



Gravitational force for dynamic meteorology



Newton's Law of Gravitation:

M = mass of Earth

m = mass of air parcel

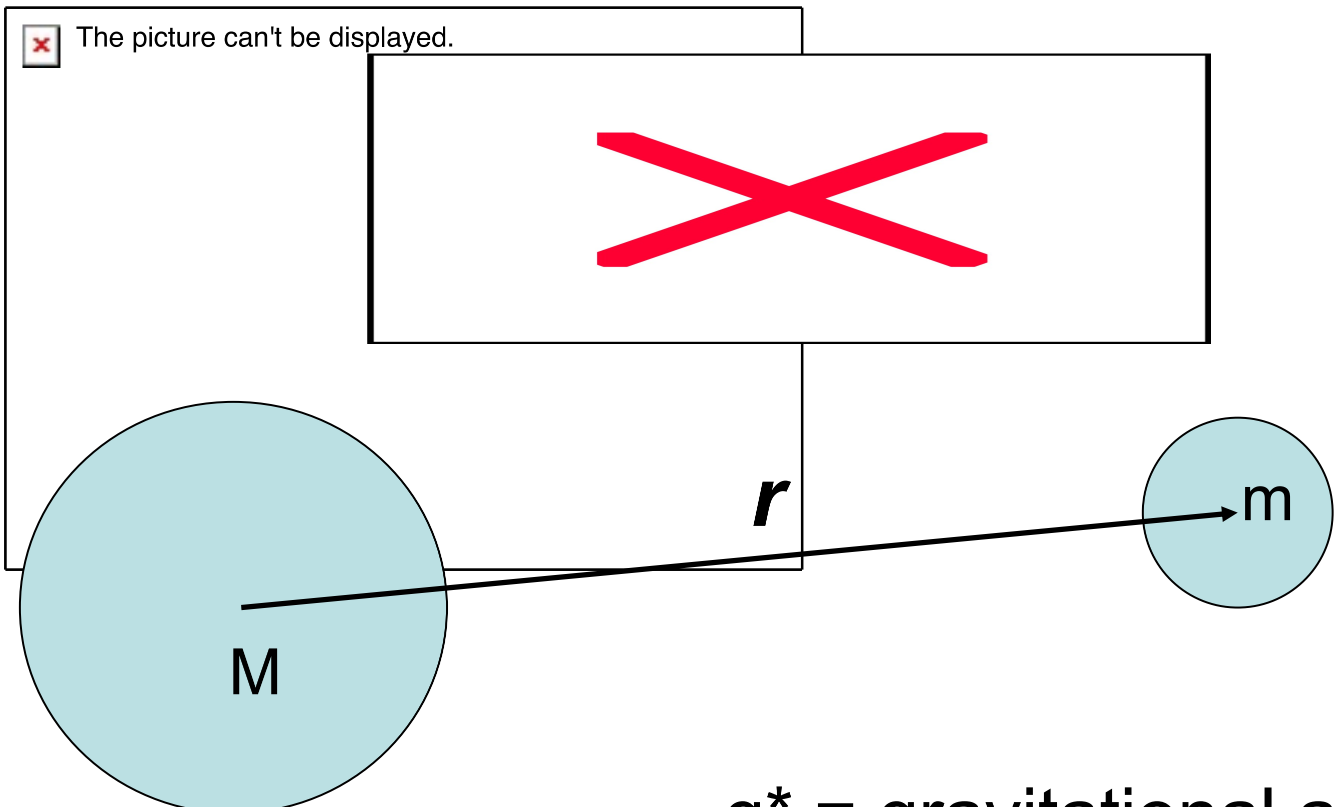
r = distance from center (of mass) of Earth to parcel

force directed down, towards Earth, hence '-' sign

G = gravitational constant (6.673 × 10^{-11} m^3 kg^{-1} s^{-2})

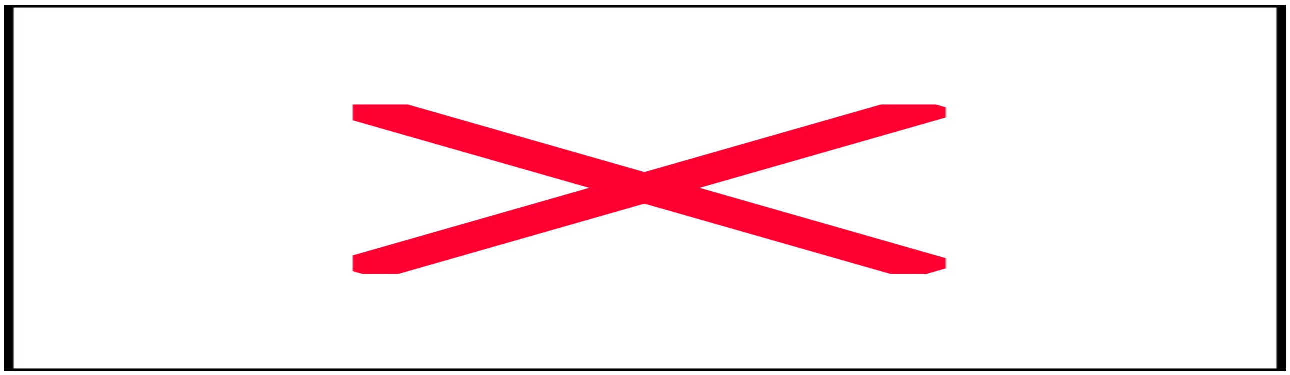
Gravitational force per unit mass

Force per unit mass exerted on the atmosphere by the gravitational attraction of the earth:



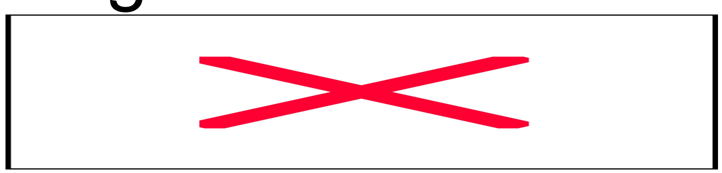
g* = gravitational acceleration

Adaptation to dynamical meteorology

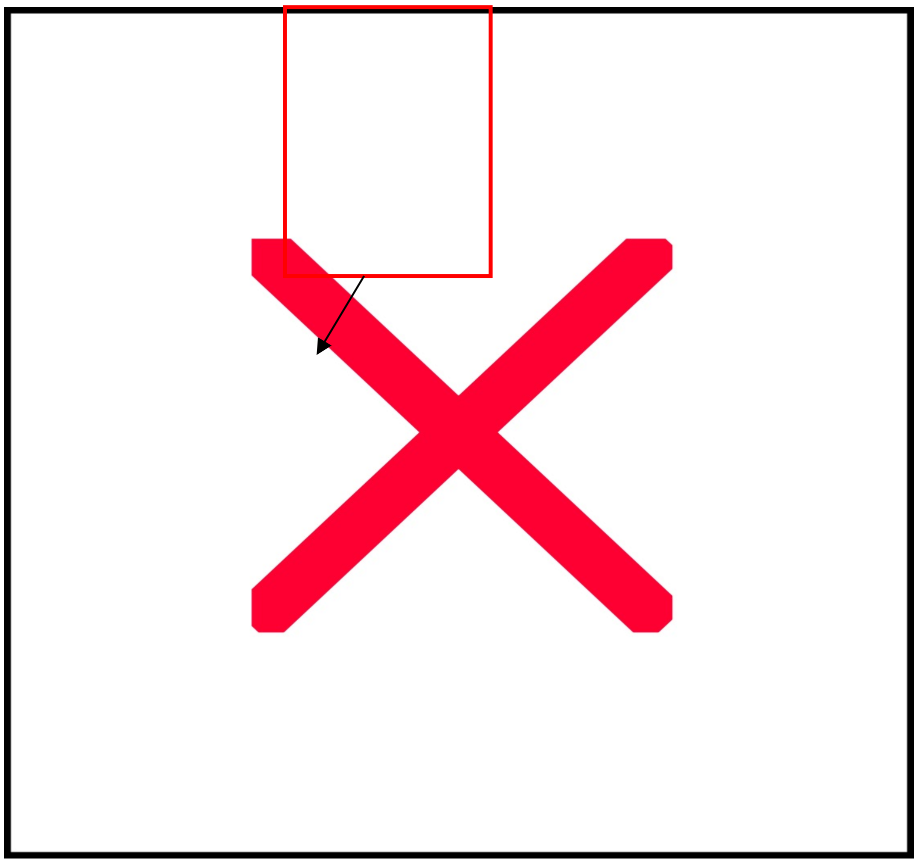


This is the gravitational acceleration at mean sea level.

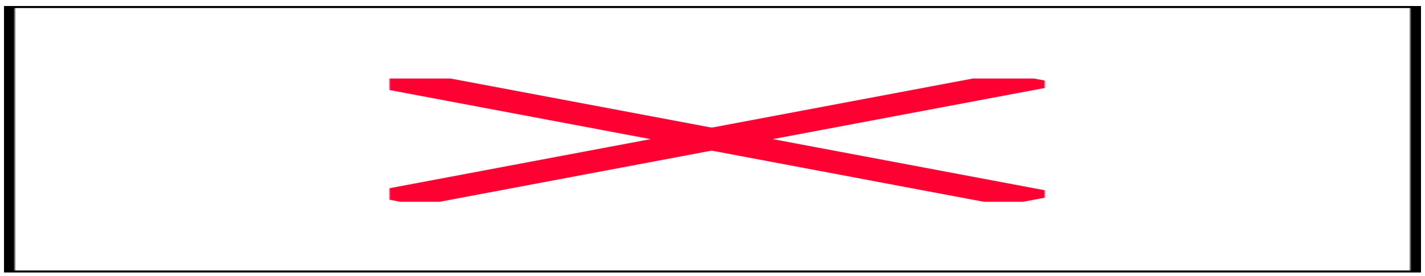
a: radius of the Earth
z: height above the mean sea level

For 

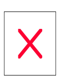
Gravitational force per unit mass



Our momentum equation so far



+ other forces

Material derivative of the velocity vector 

Pressure gradient force

Viscous force

Gravitational force

Apparent forces

- Due to the fact that we live on a rotating planet
- Centrifugal force
- Coriolis force

Back to Basics: Newton's Laws of Motion

- **Law 1:** Bodies in motion remain in motion with the same **velocity**, and bodies at rest remain at rest, unless acted upon by unbalanced forces.
- **Law 2:** The rate of change of momentum of a body with time is equal to the vector sum of all forces acting upon the body and is the same direction.
- **Law 3:** For every action (force) there is and equal and opposite reaction.

Back to basics: A couple of definitions

- Newton's laws assume we have an "inertial" coordinate system; that is, and absolute frame of reference – fixed, absolutely, in space.
- Any motion relative to the coordinate system fixed in space is known as inertial motion.
- Velocity is the change in position of a particle (or parcel). It is a vector and can vary either by a change in magnitude (speed) or direction.

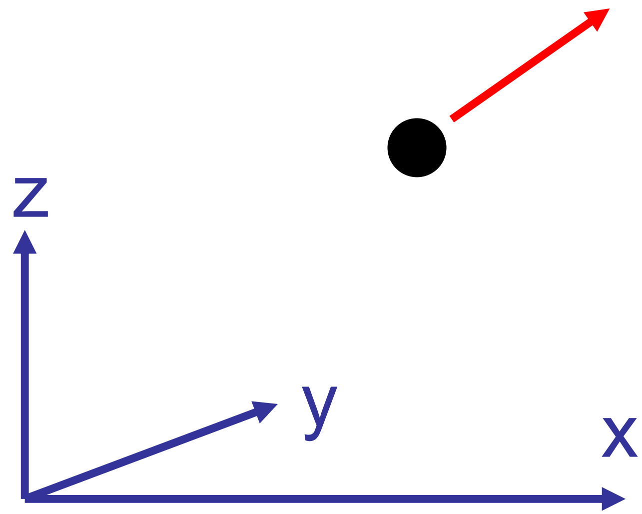
Non-inertial coordinate systems

Unit 6,frames 16-21:

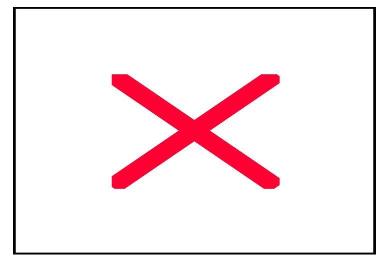
http://www.atmos.washington.edu/2005Q1/101/CD/MAIN3.swf

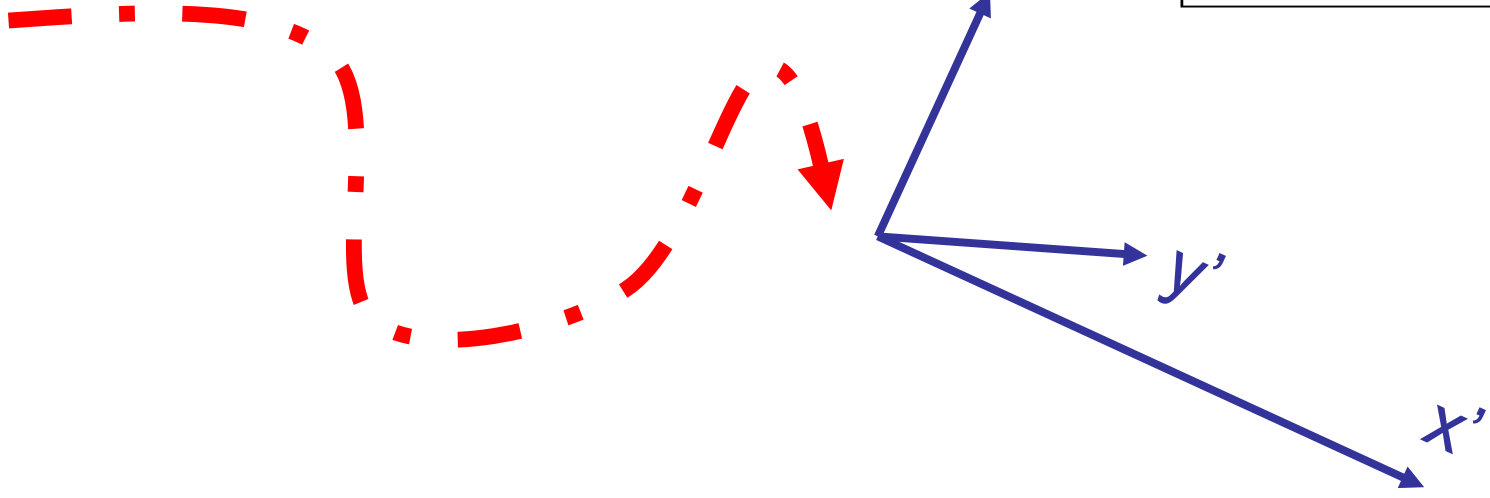
- The Earth rotates, it is an accelerating frame of reference, hence:
- Our coordinate system ‘north, south, east, west’ is a **non-inertial**, non-absolute coordinate system.
- The ‘north’ direction in Ann Arbor is different from the ‘north’ direction in San Francisco or Sydney, Australia.

Two coordinate systems

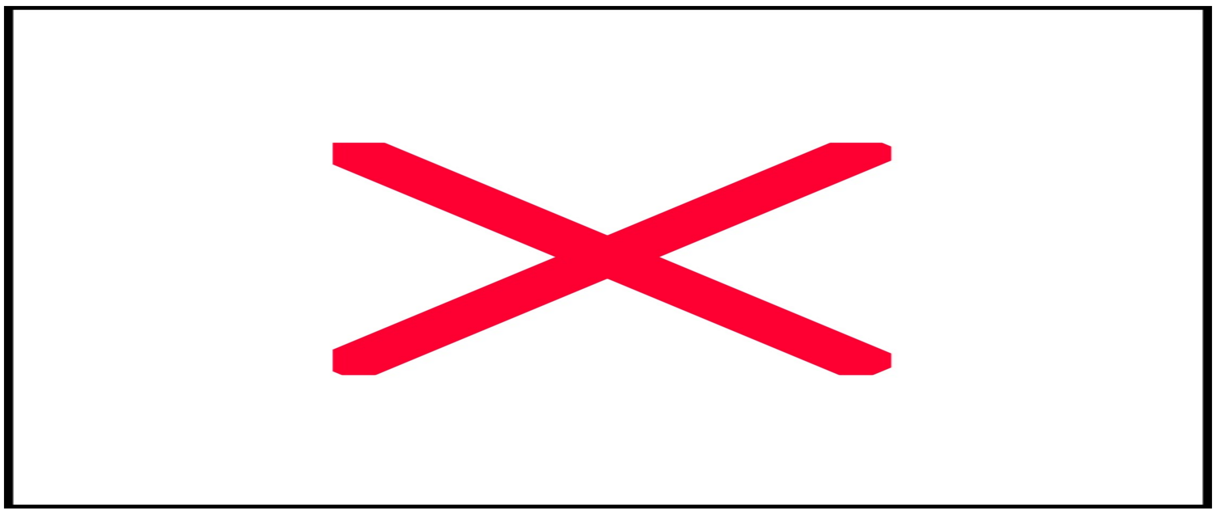


Can describe the velocity and forces (acceleration) in either coordinate system.

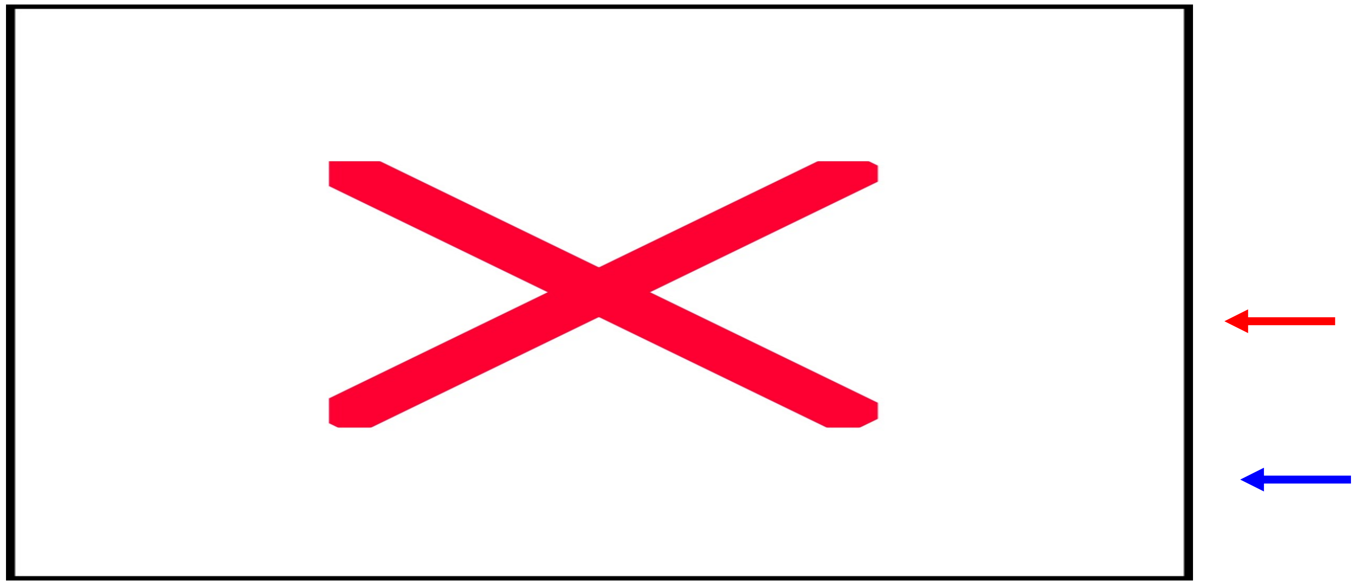




One coordinate system related to another by:



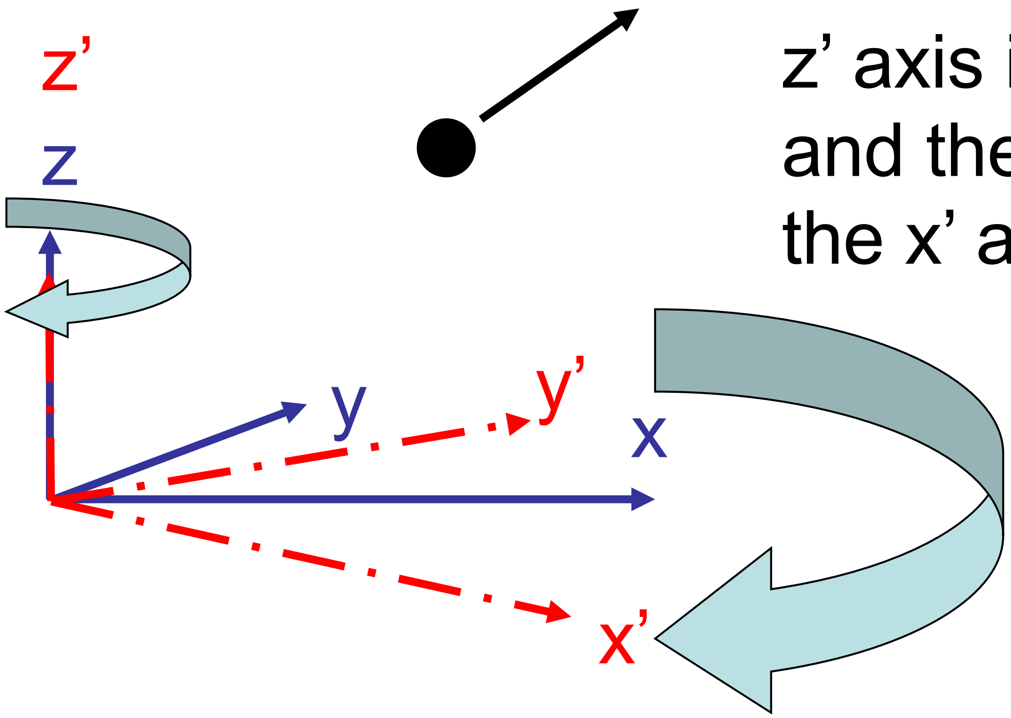
Example: Velocity (x' direction)



So we have the **velocity relative to the coordinate system** and the **velocity of one coordinate system relative to the other**.

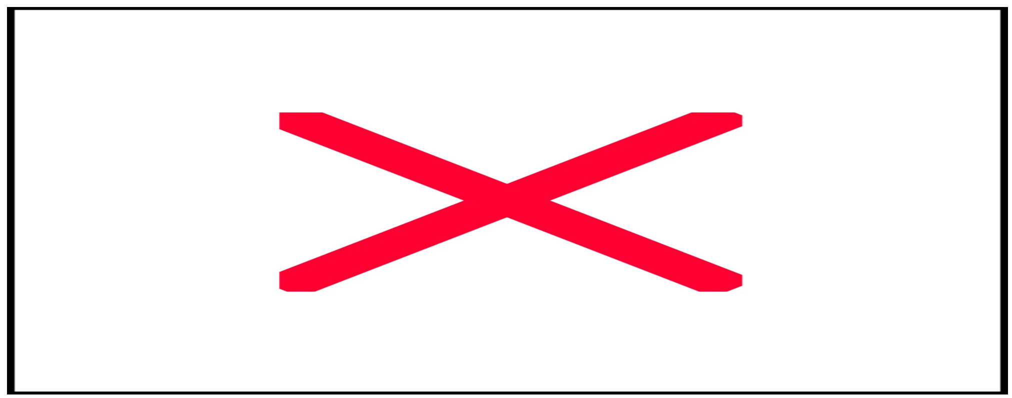
This velocity of one coordinate system relative to the other leads to **apparent forces**. They are real, observable forces to the observer in the moving coordinate system.

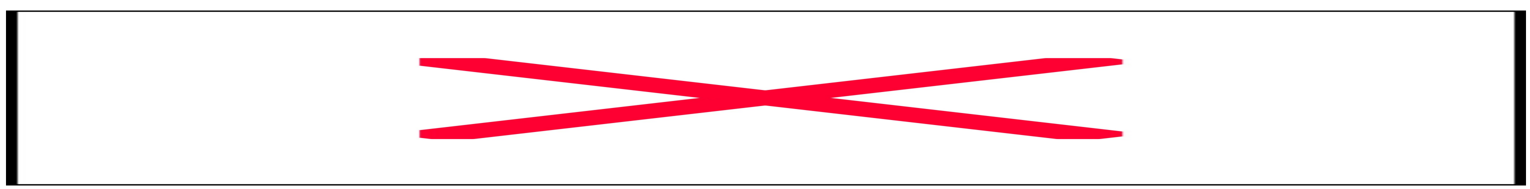
Two coordinate systems



z' axis is the same as z, and there is rotation of the x' and y' axis

Example:
One coordinate system related to another by

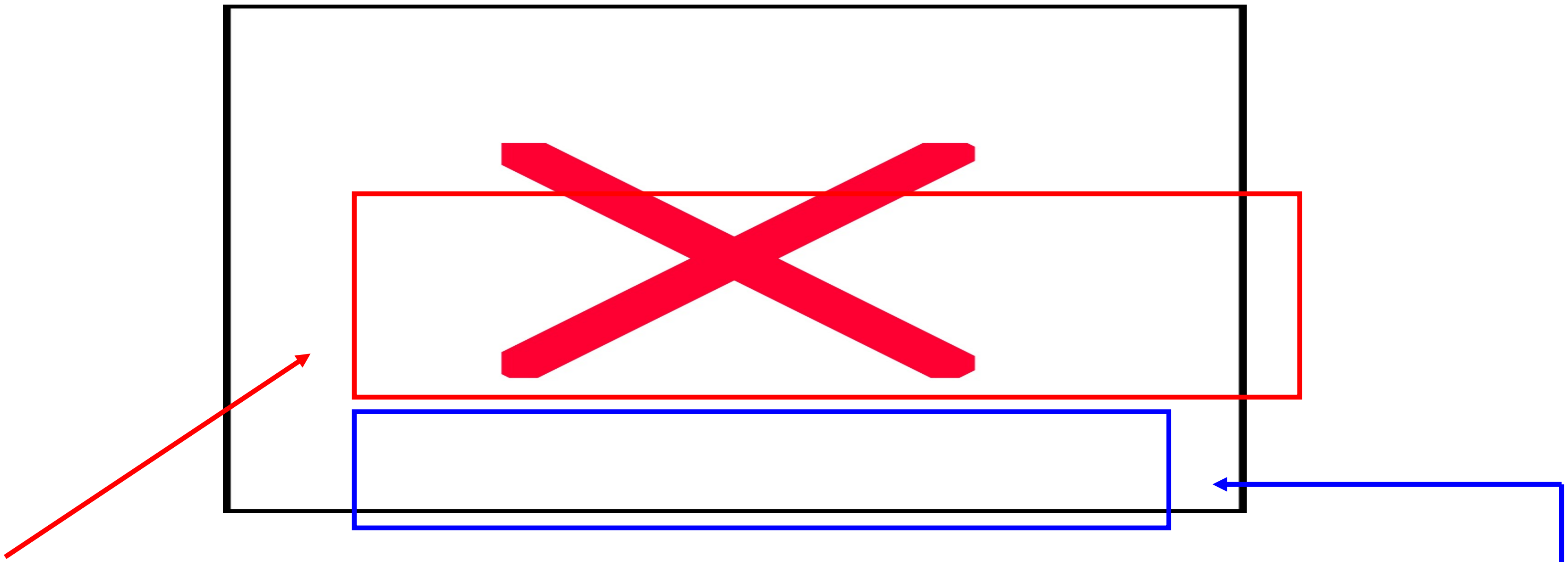




We also use the symbol Ω for ω.
Ω is the angular speed of rotation of the earth.

T is time needed to complete rotation.
On Earth it takes a sidereal day = 86164 s.

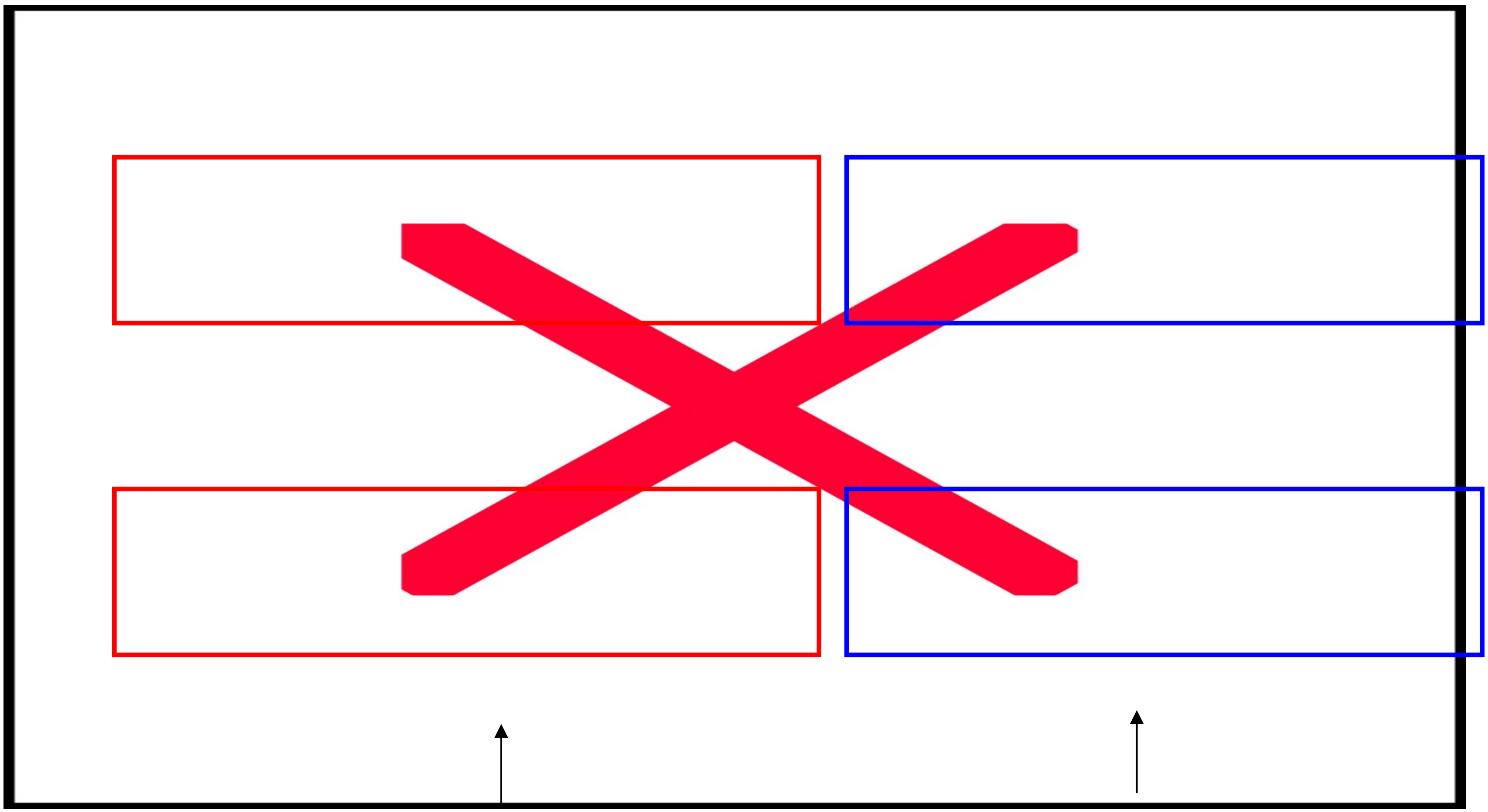
Acceleration (force) *in rotating coordinate system*



The apparent forces that are **proportional to rotation and the velocities** in the inertial system (x,y,z) are called the **Coriolis** forces.

The apparent forces that are **proportional to the square of the rotation and position** are called **centrifugal forces**.

Acceleration (force) *in rotating coordinate system*



Coriolis forces **Centrifugal forces**