# *AOSS 321, Winter 2009*
# *Earth System Dynamics*

## *Lecture 8*
## *2/3/2009*


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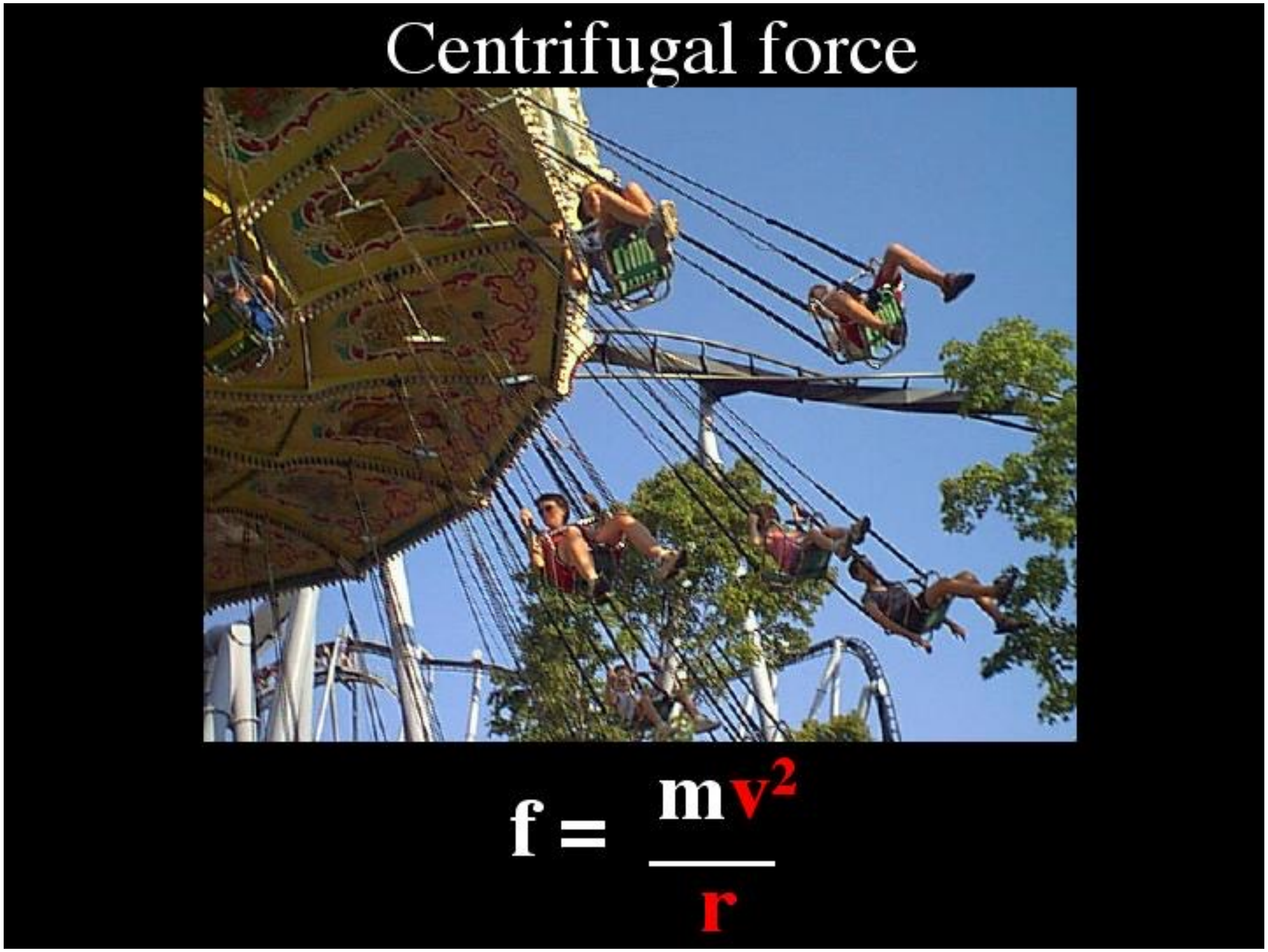
# *What are the fundamental forces in the Earth's system?*

- Pressure gradient force
- Viscous force
- Gravitational force
- Apparent forces: Centrifugal and Coriolis
- Can you think of other classical forces and would they be important in the Earth's system? *Electromagnetic force*

- Total Force is the sum of all of these forces.

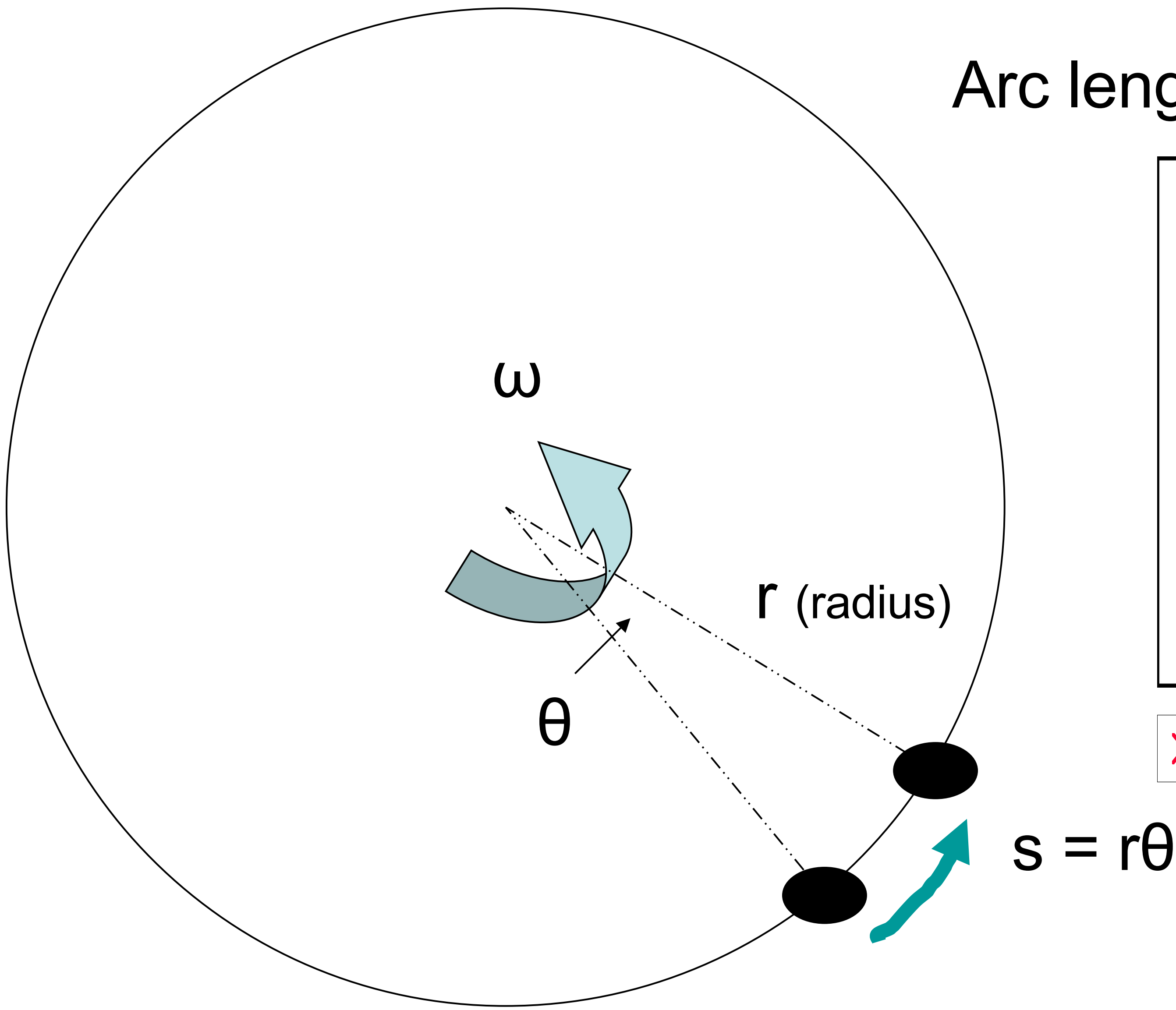
# *Apparent forces: A physical approach*

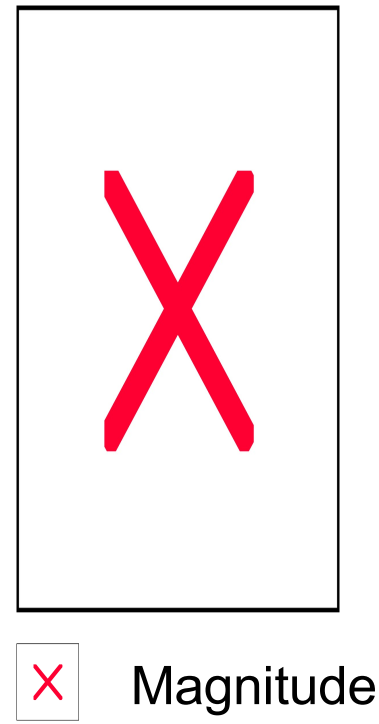



Check out Unit 6 (winds), frames 20 & 21:
http://www.atmos.washington.edu/2005Q1/101/CD/MAIN3.swf

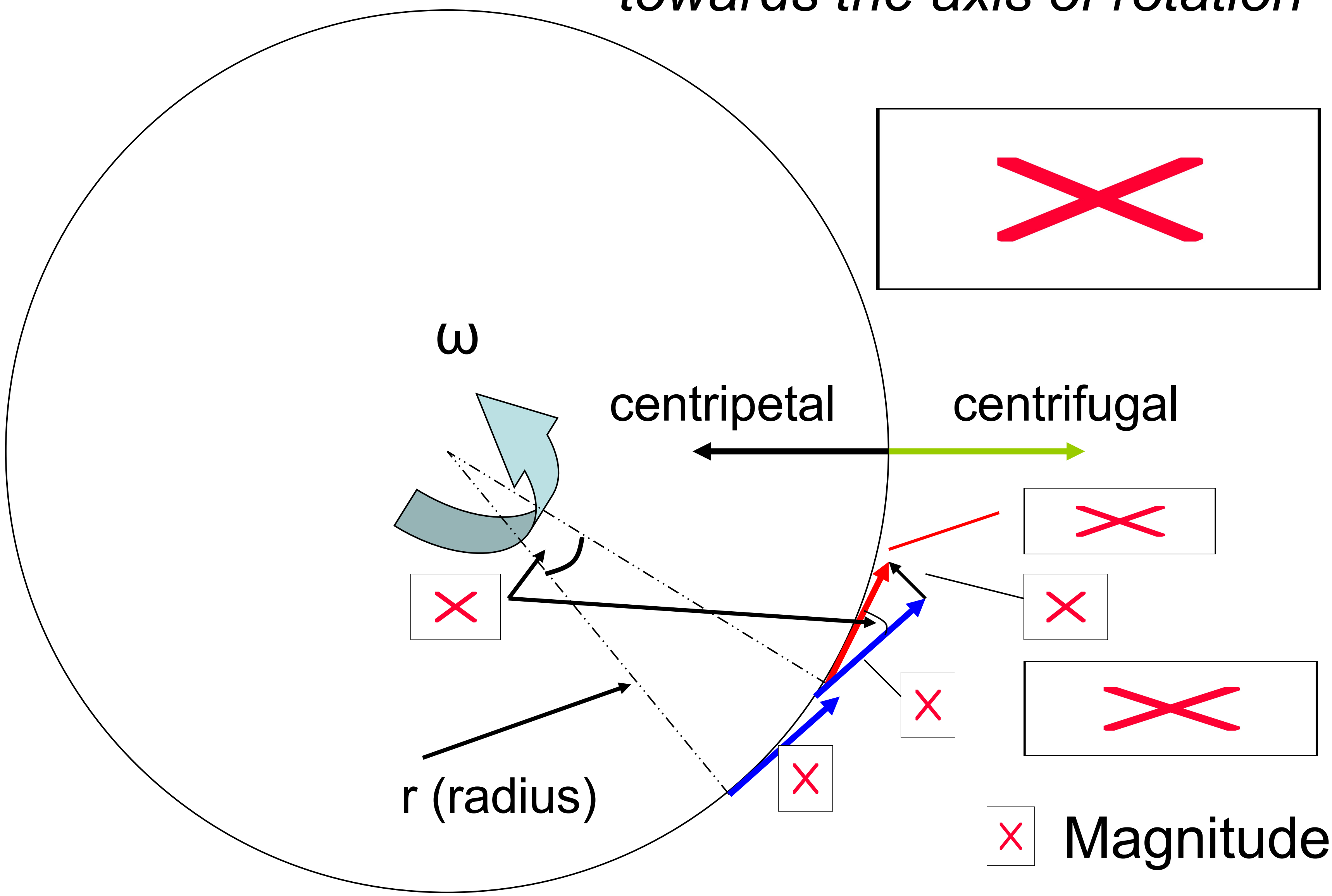
## *Circle Basics*

Arc length $\equiv s = r\theta$



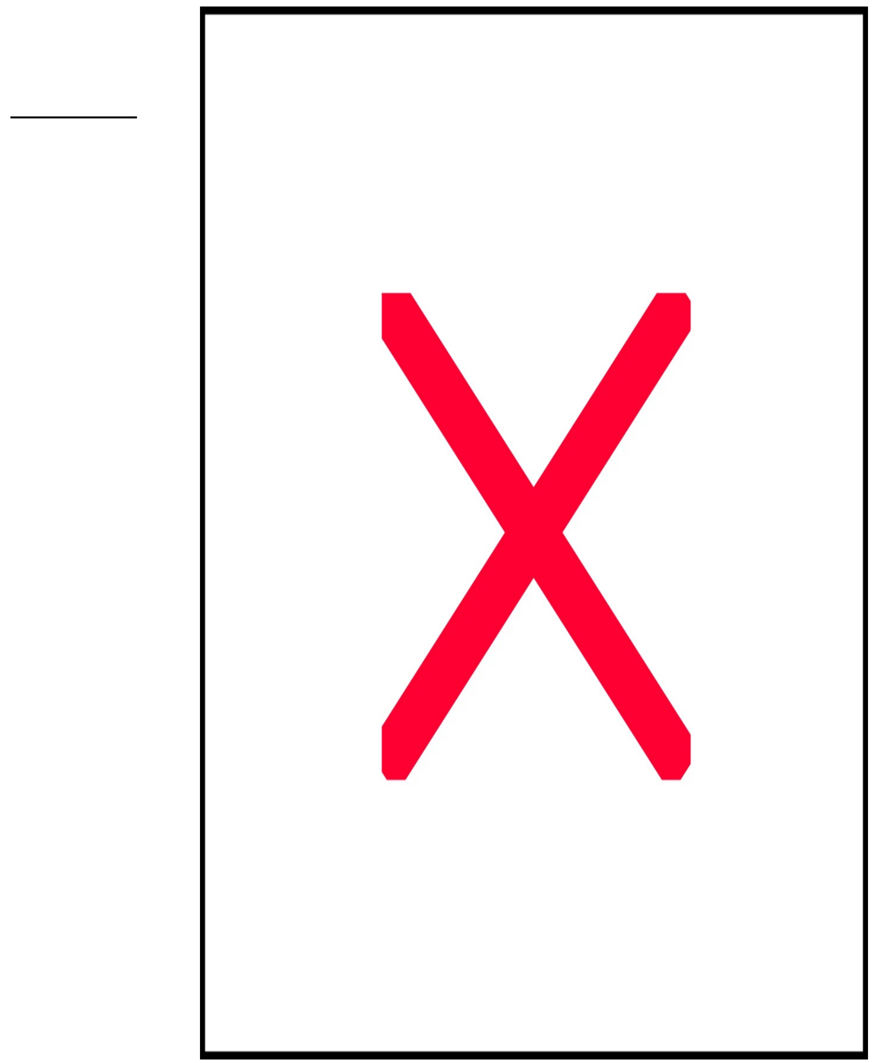



*Centripetal acceleration: towards the axis of rotation*



# *Centripetal force: for our purposes*

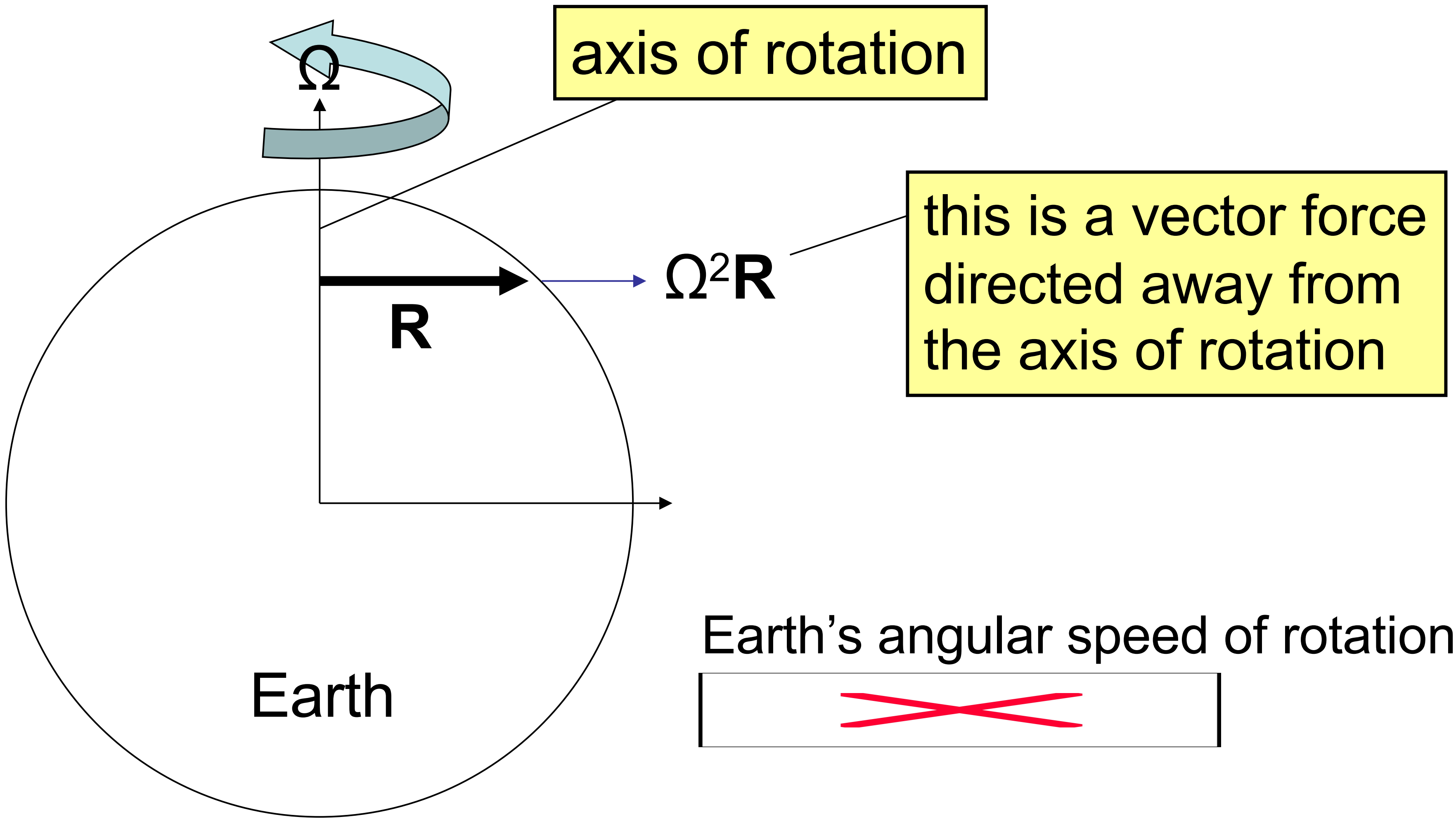
Directed toward the axis of rotation



# *Now we are going to think about the Earth*

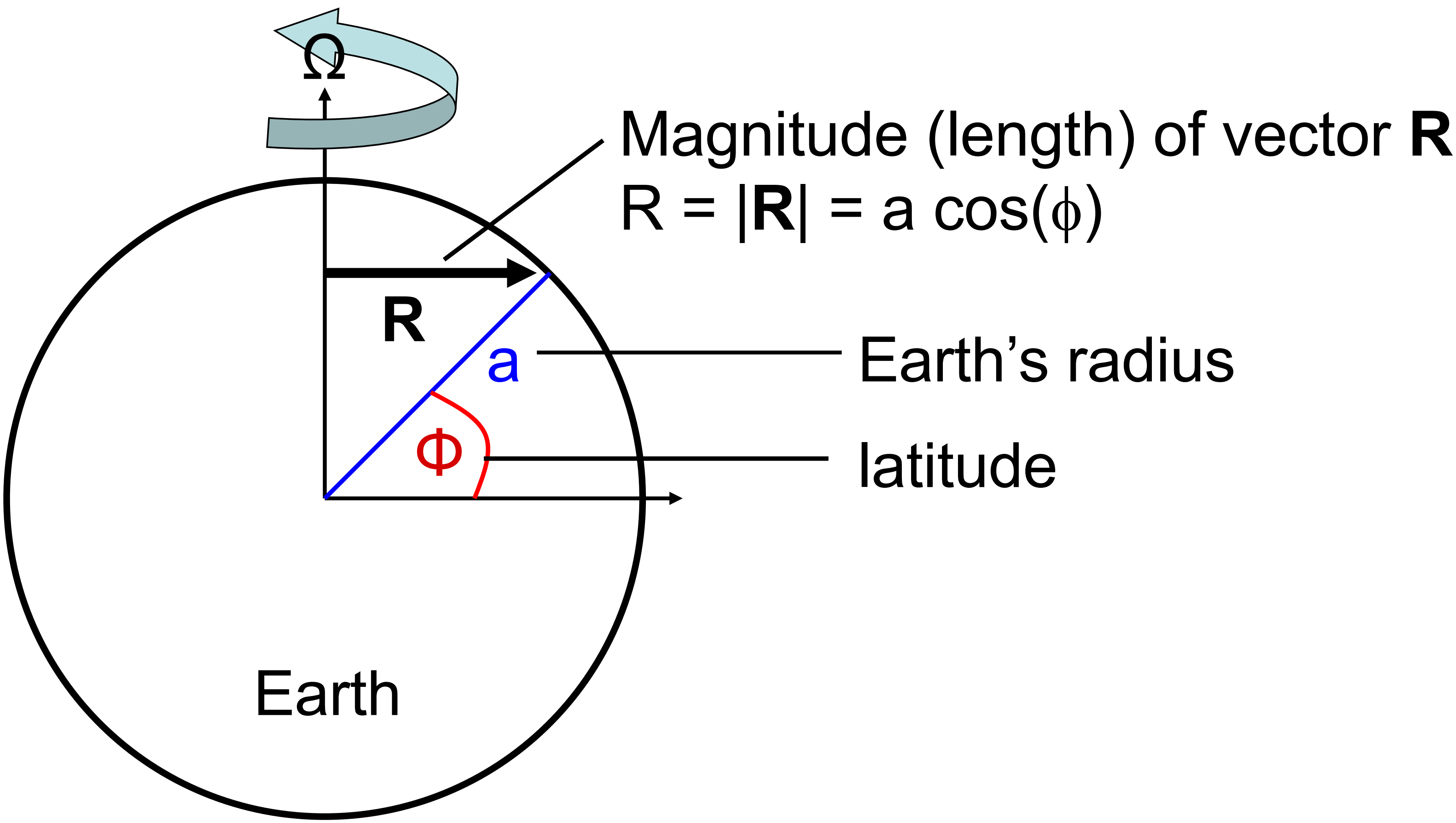
- The preceding was a schematic to think about the centripetal acceleration problem.
  Remember Newton's third law:
  "For every action, there is an equal and opposite reaction."
- View from fixed system: uniform centripetal acceleration towards the axis of rotation
- View from rotating system: centrifugal acceleration (directed outward) equal and opposite to the centripetal acceleration (directed inward)

# *What direction does the Earth's centrifugal force point?*

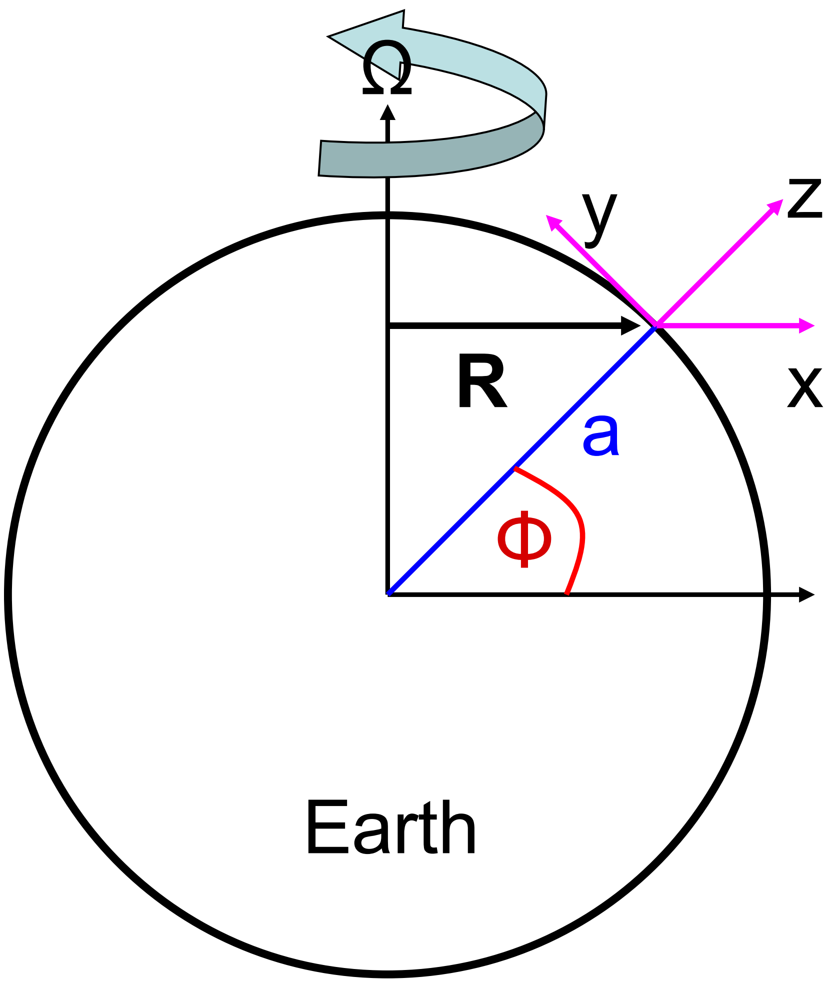



Earth's angular speed of rotation:

# *Magnitude of **R***



# *Tangential coordinate system*



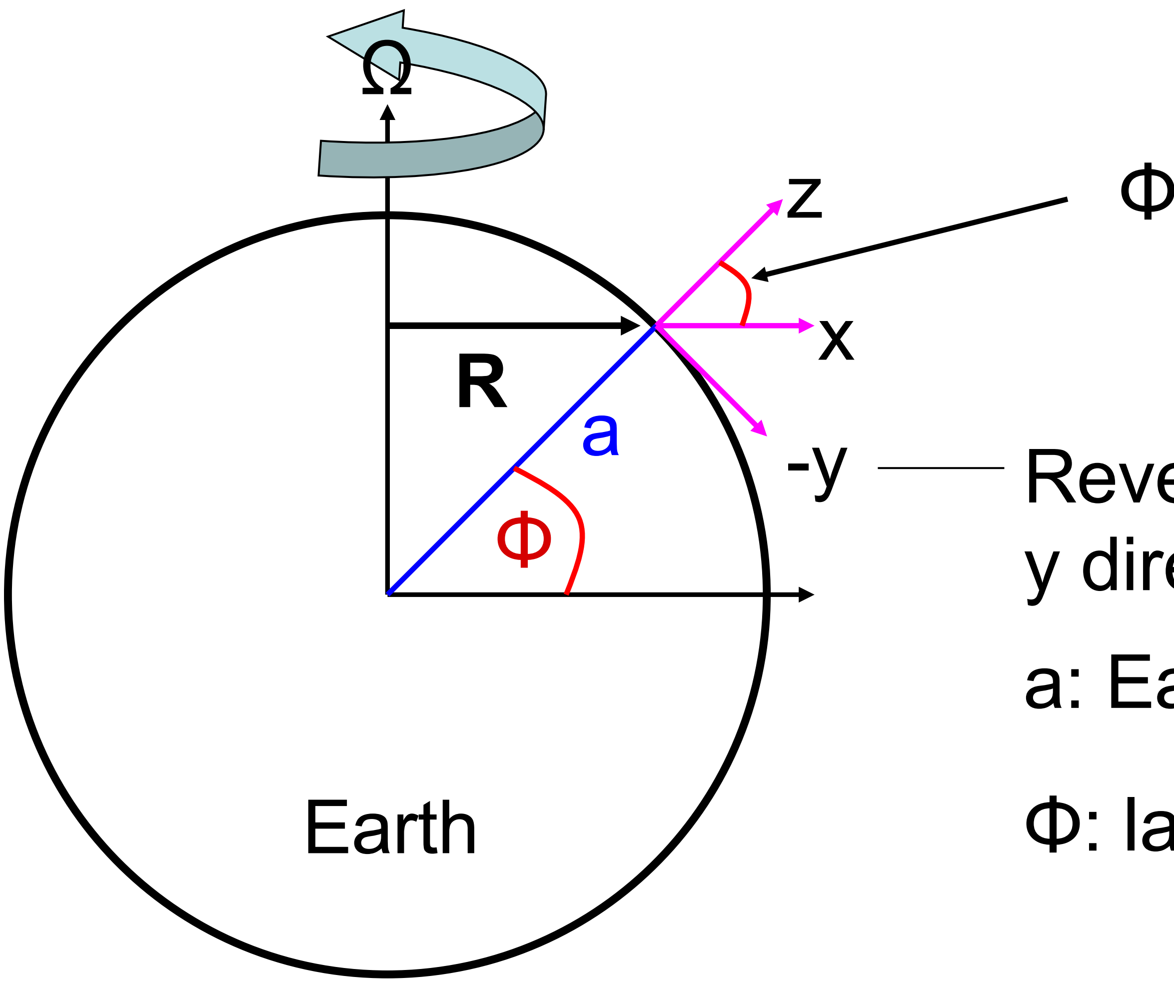

Place a coordinate system on the surface.

x = west-east (longitude)
y = south-north (latitude)
z = local vertical

a: Earth's radius

Φ: latitude

# *Angle between **R** and axes*

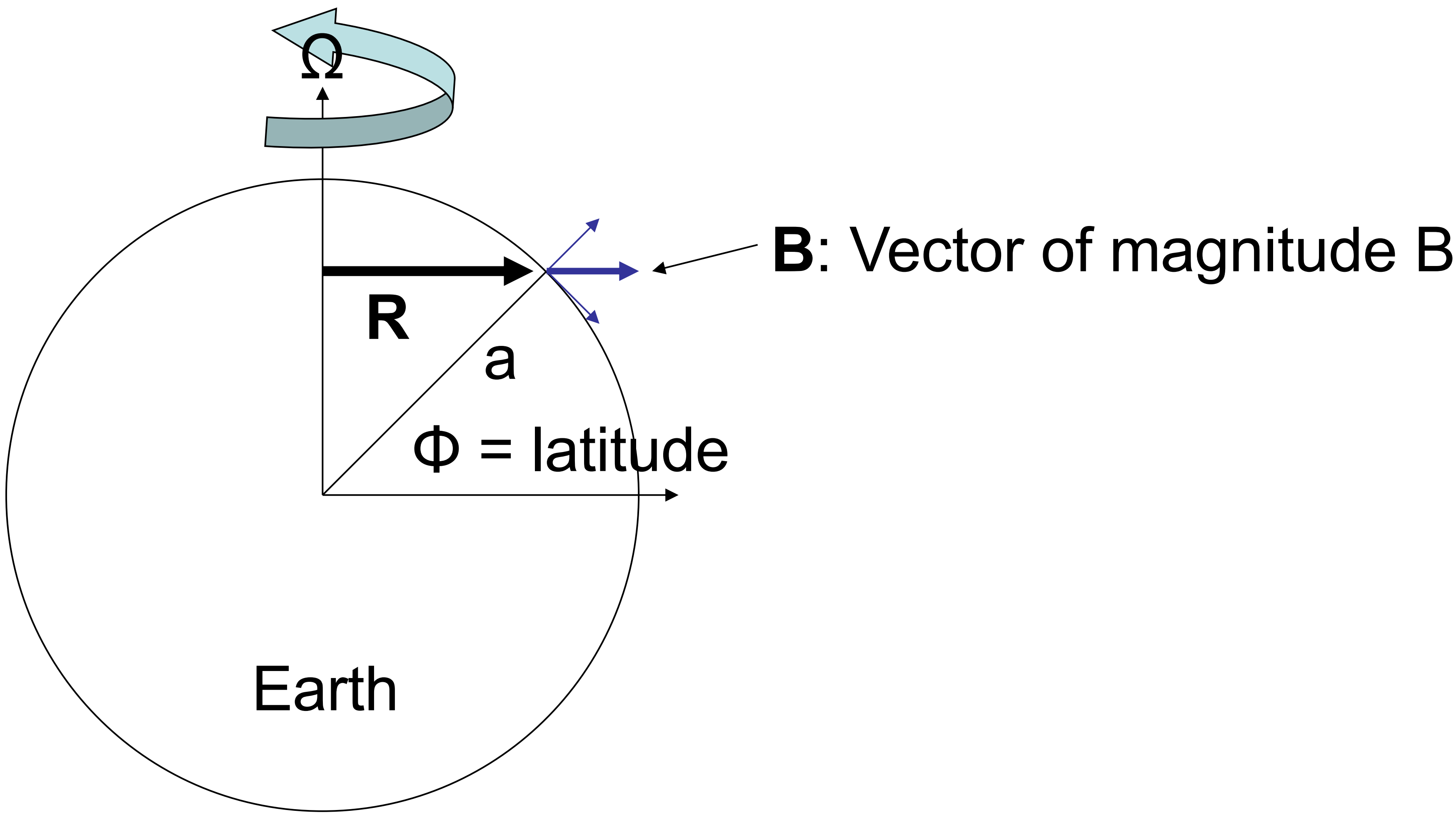



Reversed (negative) y direction

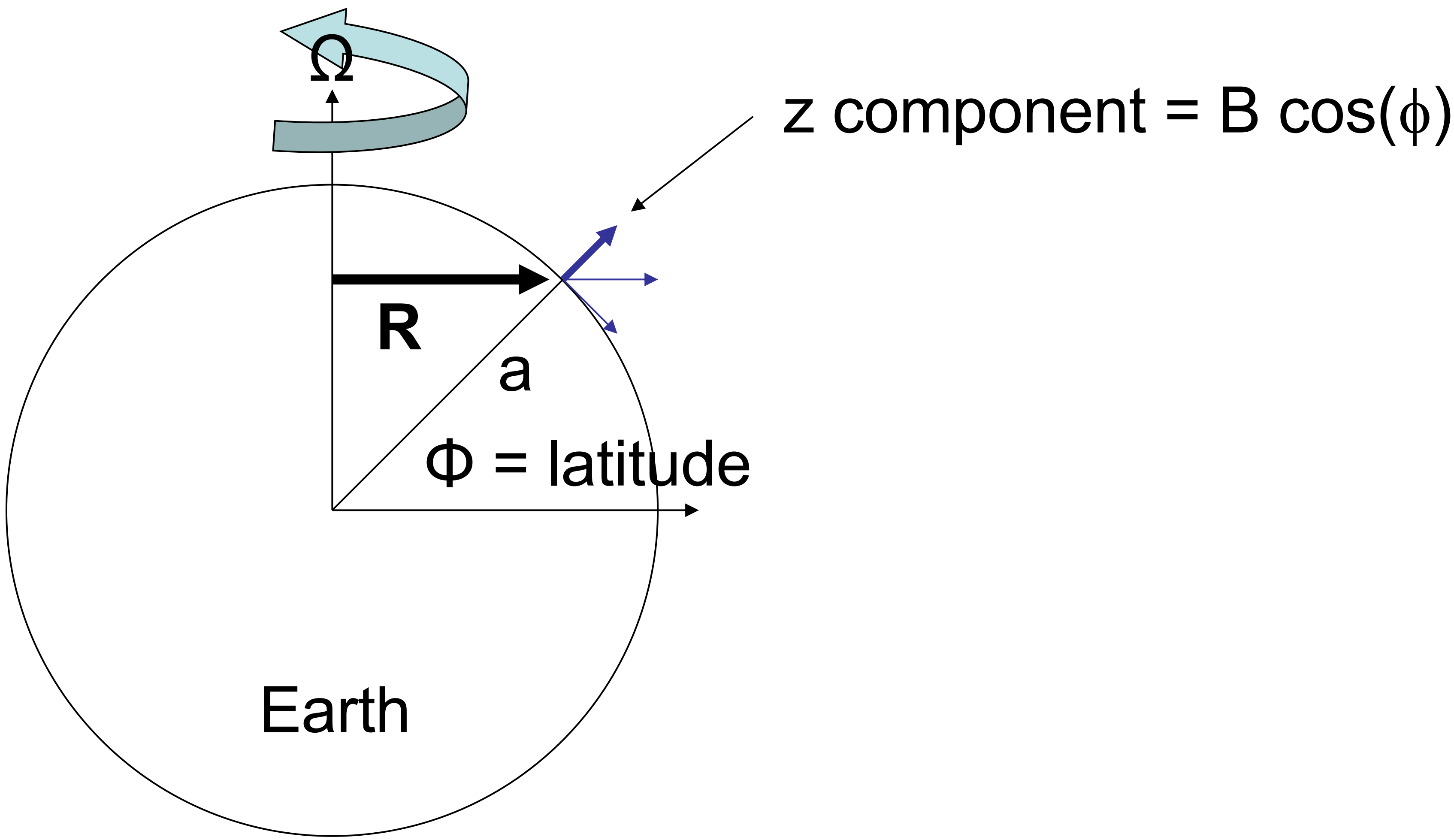
a: Earth's radius

Φ: latitude

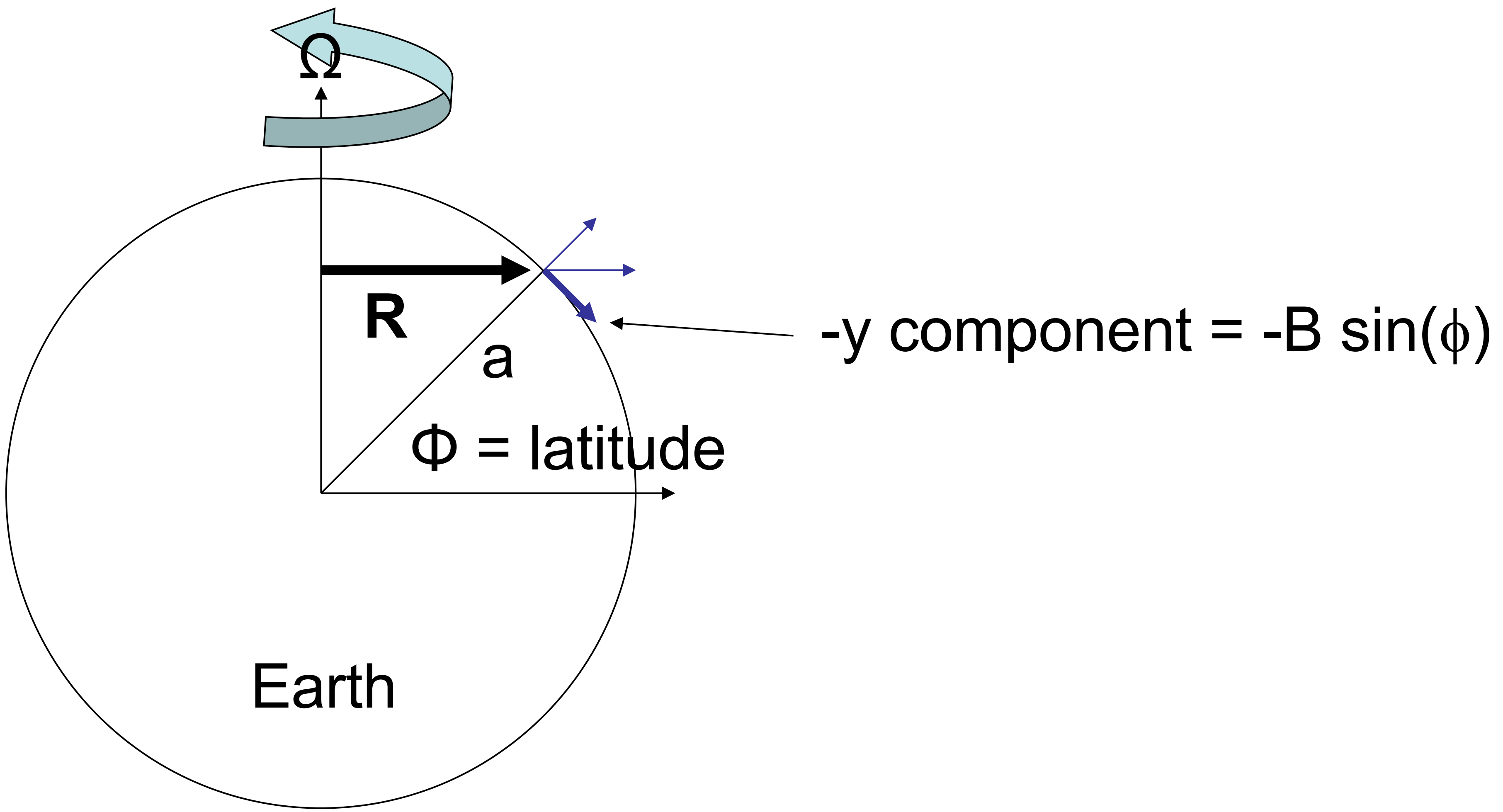
# *Assume magnitude of vector in direction **R***



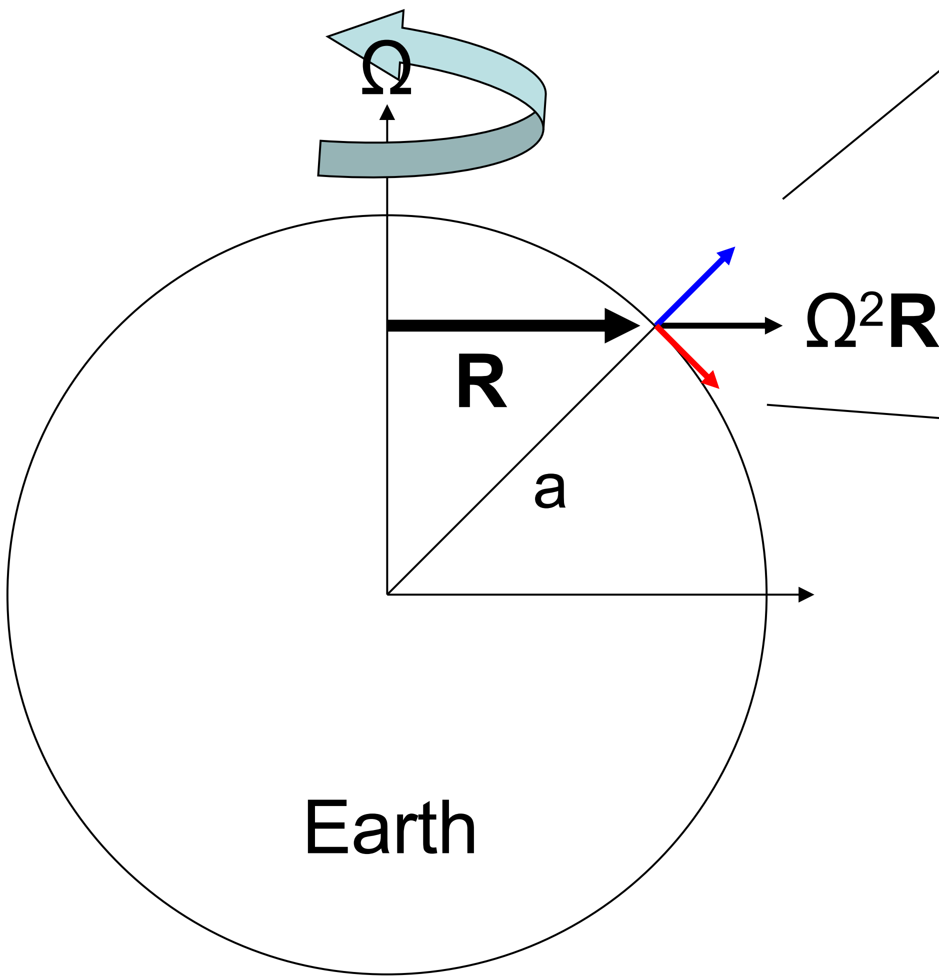
# *Vertical component*



# *Meridional component*



# *What direction does the Earth's centrifugal force point?*

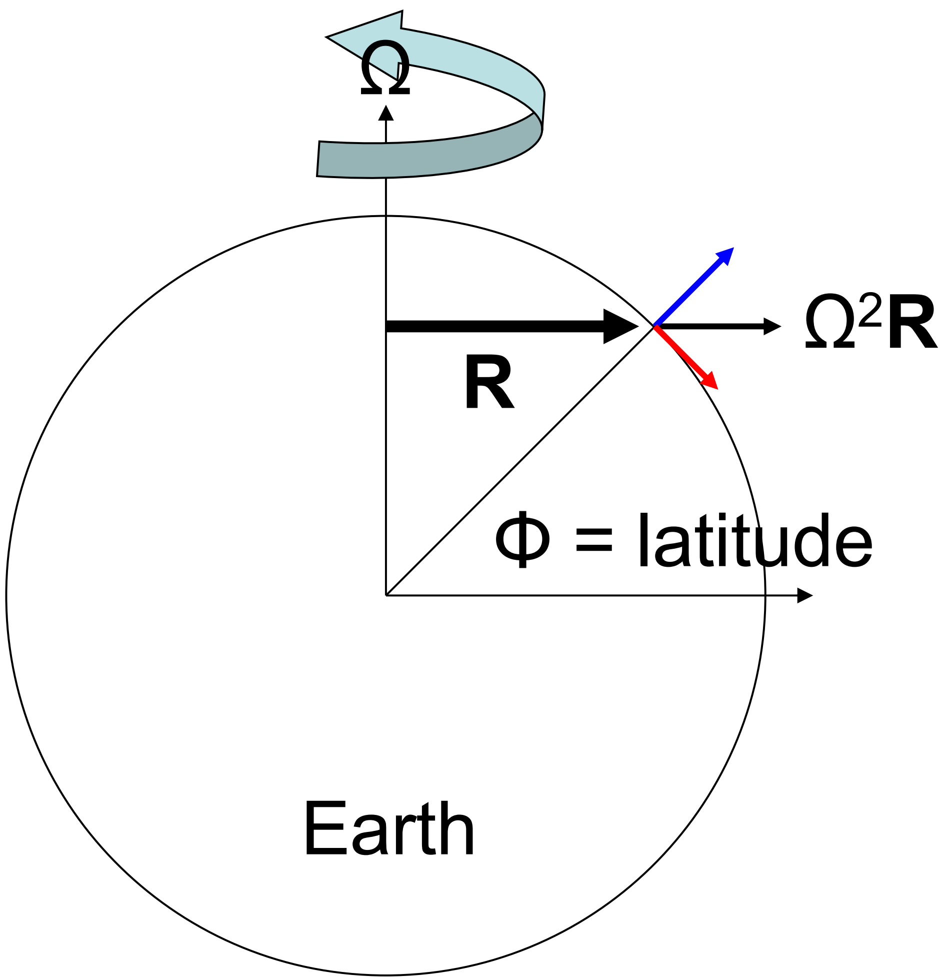



So there is a component that is in the same coordinate direction as gravity (and local vertical).

And there is a component pointing towards the equator

We are now explicitly considering a coordinate system tangent to the Earth's surface.

# *What direction does the Earth's centrifugal force point?*



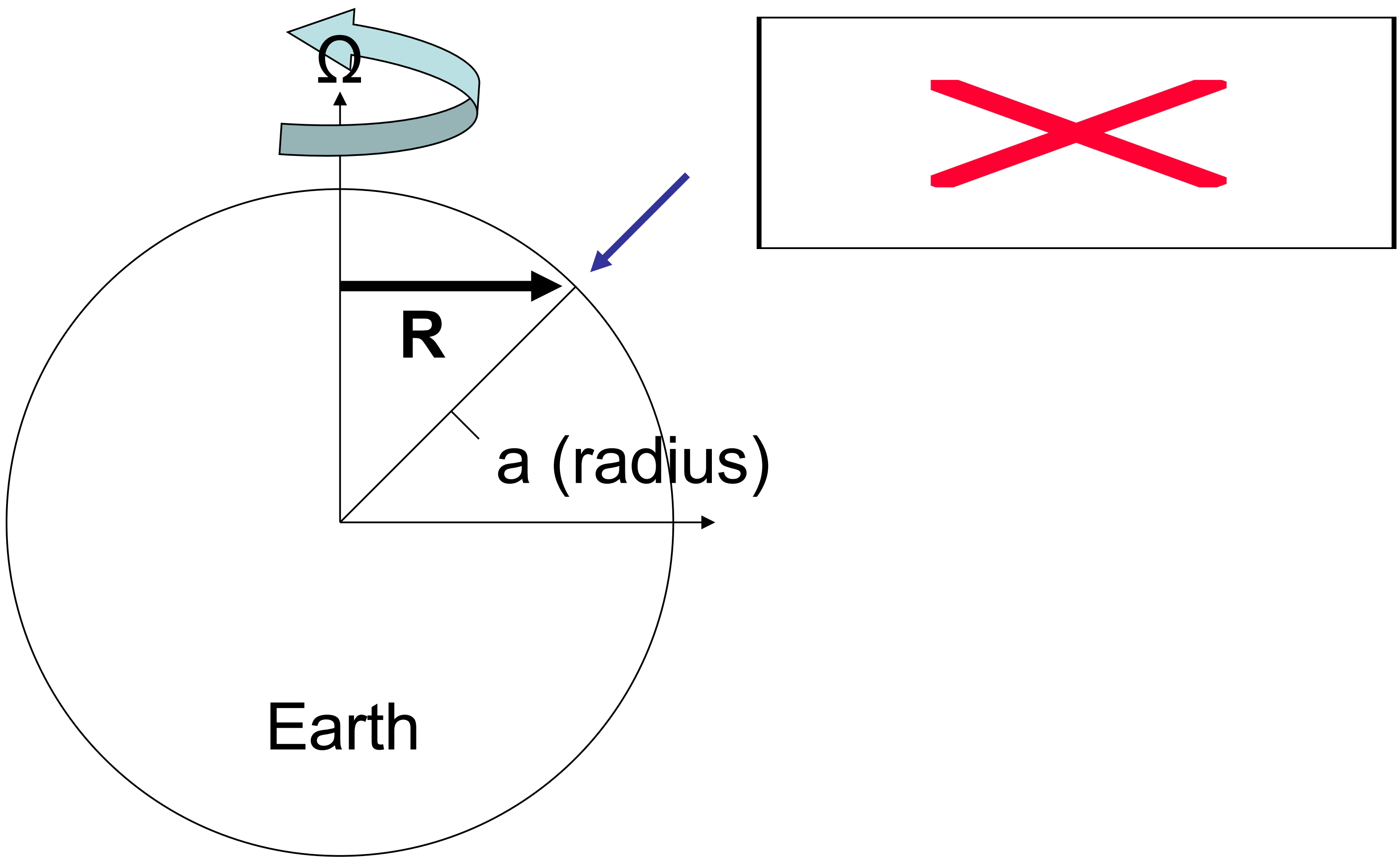

So there is a component that is in the same coordinate direction as gravitational acceleration:

$\sim a\Omega^2\cos^2(\phi)$

And there is a component pointing towards the equator

$\sim -\ a\Omega^2\cos(\phi)\sin(\phi)$

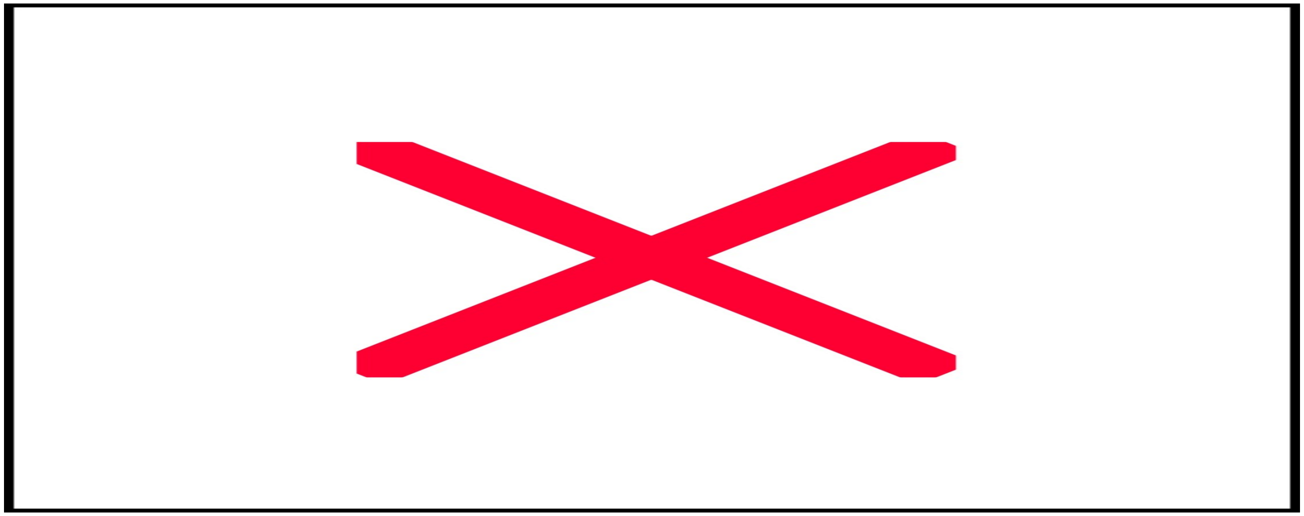
# *What direction does the gravitational acceleration point?*



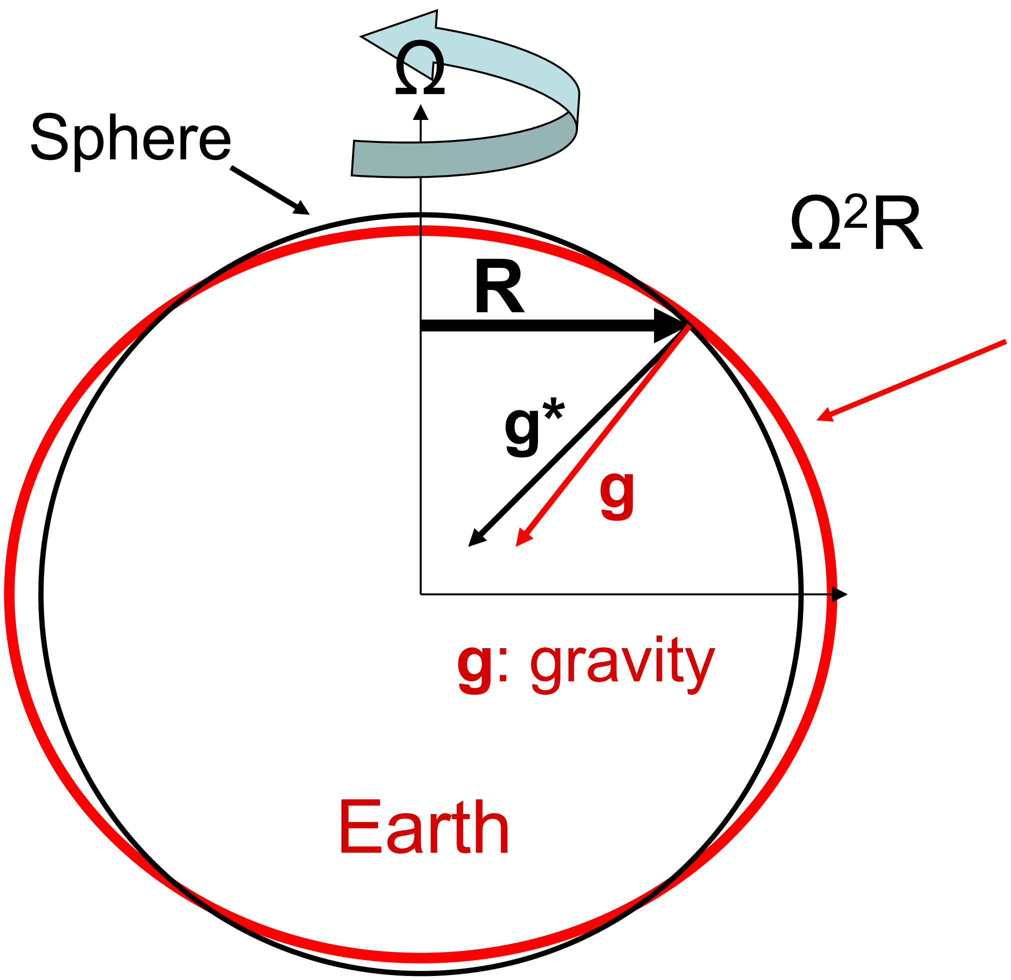
*So we re-define*

***gravitational acceleration g****

*as* ***gravity g***



# *What direction does the Earth's centrifugal force point?*




And there is a component pointing towards the equator.

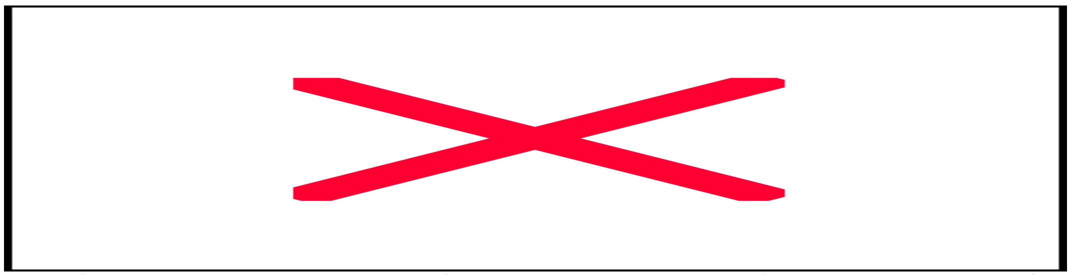
The Earth has bulged to compensate for the equatorward component (how much?)

Hence we don't have to consider the horizontal component explicitly.

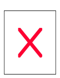
# *Centrifugal force of Earth*

- Vertical component incorporated into re-definition of gravity.
- Horizontal component does not need to be considered when we consider a coordinate system tangent to the Earth's surface, because the Earth has bulged to compensate for this force.
- Hence, centrifugal force does not appear EXPLICITLY in the equations.

# *Our momentum equation so far*



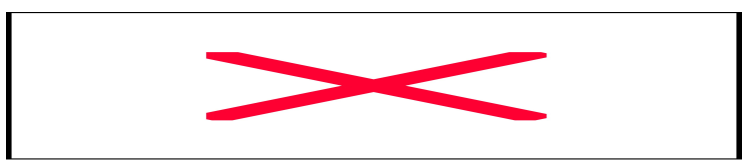
+ other forces

Material derivative of the velocity vector 

Pressure gradient force

Viscous force

Gravity force = gravitational force + centrifugal force



with $g = 9.81\ m\ s^{-2}$

# *Where is the low pressure center?*


How and why does the system rotate?

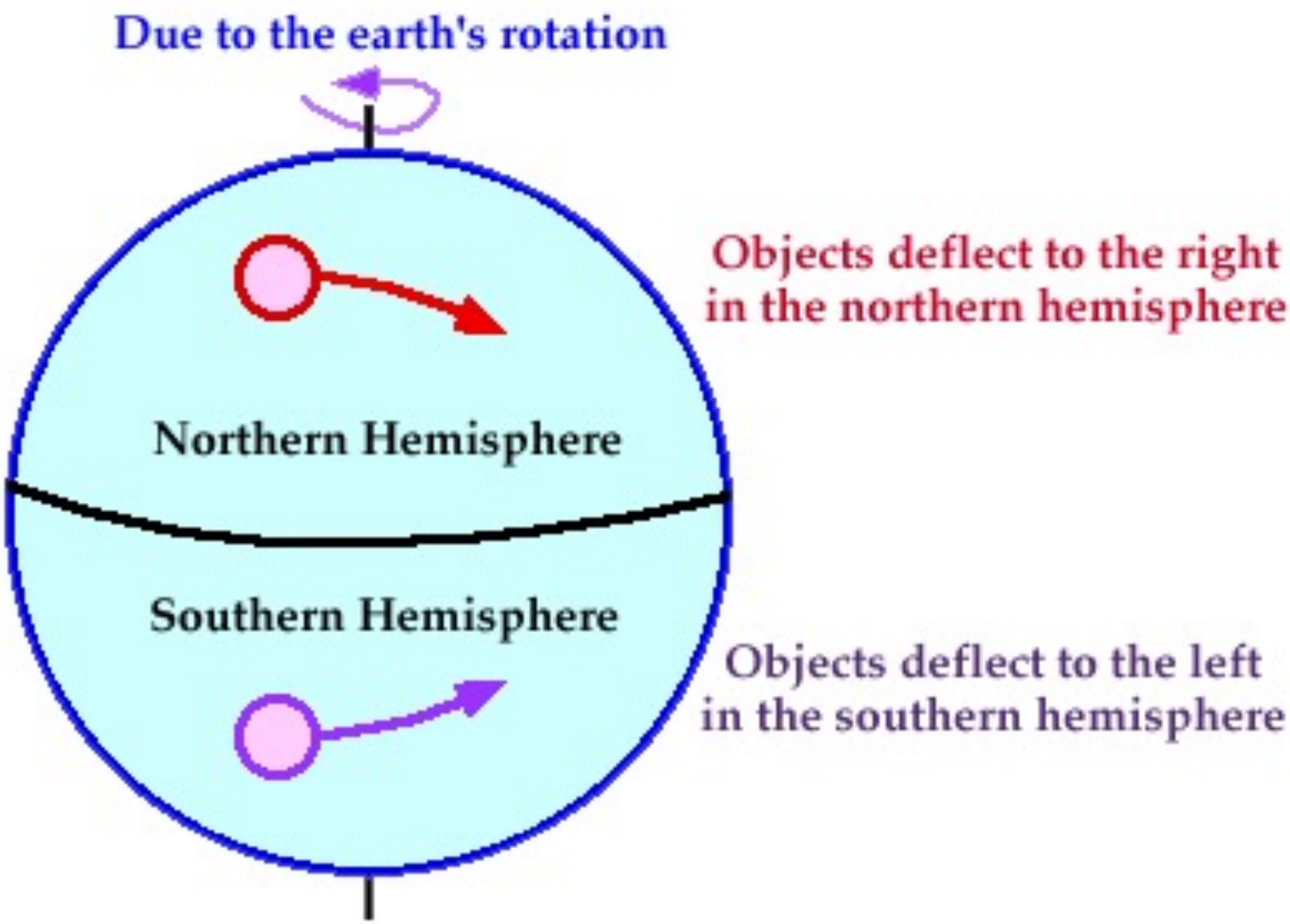
# *Apparent forces: A physical approach*

- Consider a dynamics field experiment in which one student takes a position on a merry-go-round and another student takes a position above the ground in an adjacent tree.
- Merry-go-round is spinning, a ball is pushed
- On the Merry-go-round: the ball is **deflected** from its path. This is due to the **Coriolis force**.
- http://ww2010.atmos.uiuc.edu/(Gh)/guides/mtr/fw/crls.rxml

# *Apparent forces: Coriolis force*

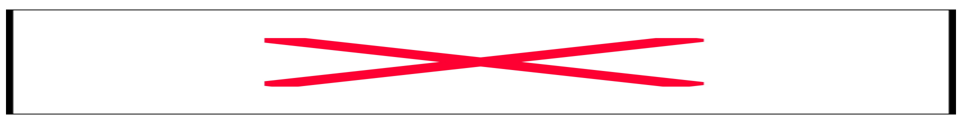
- Observe the flying aircrafts
- http://www.classzone.com/books/earth_science/terc/content/visualizations/es1904/es1904page01.cfm?chapter_no=visualization
- What happens?
- http://www.physics.orst.edu/~mcintyre/coriolis/

# *Effects of the Coriolis force on motions on Earth*



# *Angular momentum*

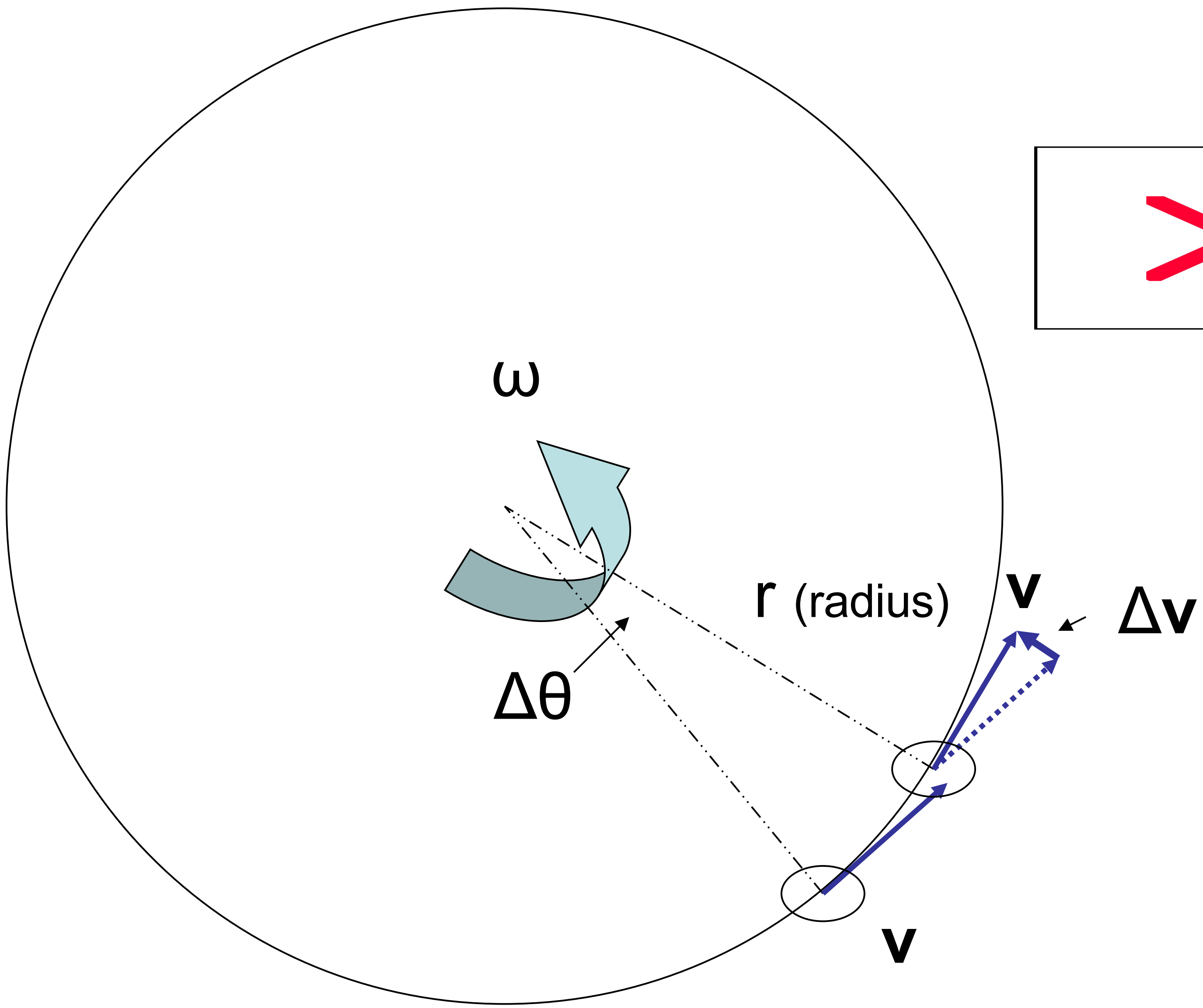
- Like momentum, **angular momentum is conserved** in the absence of torques (forces) which change the angular momentum.
- The absolute angular momentum per unit mass of atmosphere is



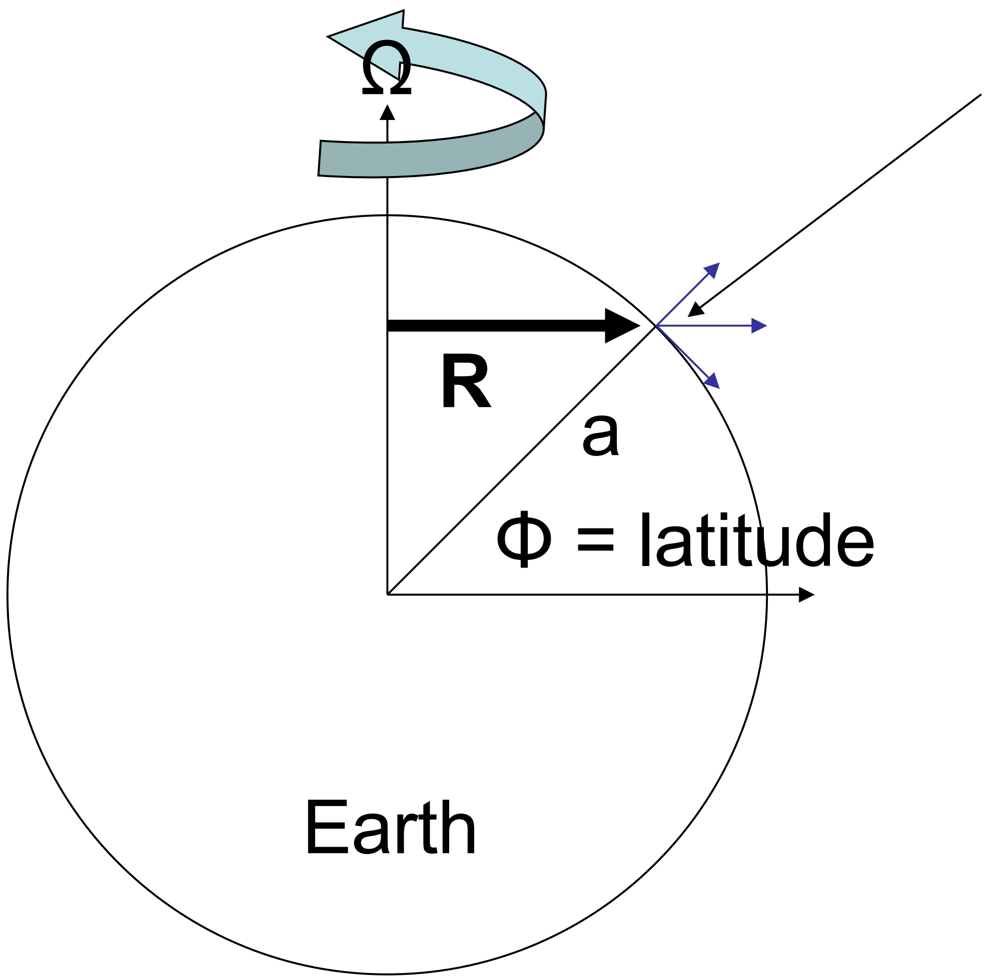
- This comes from considering the conservation of momentum of a body in constant body rotation in the polar coordinate system.
- Coriolis force & angular momentum: Check out Unit 6, frames 25-32

http://www.atmos.washington.edu/2005Q1/101/CD/MAIN3.swf

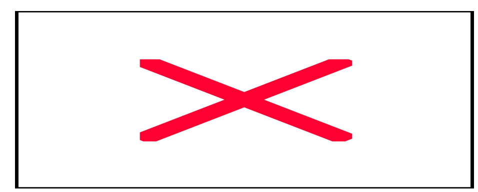
# *Angular speed (circle)*



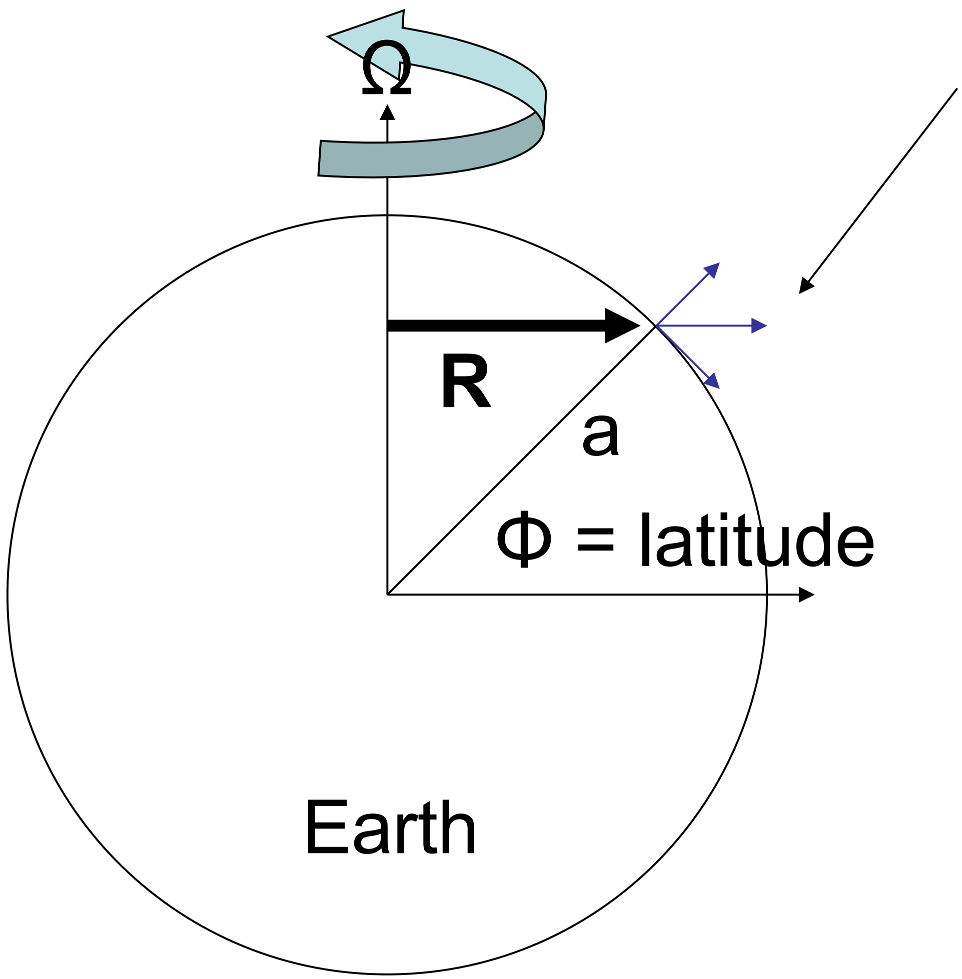
# *Earth's angular momentum (1)*



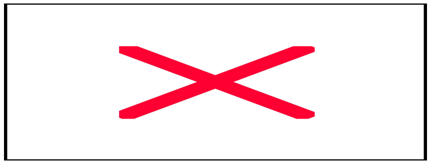

What is the speed of this point due **<u>only</u>** to the rotation of the Earth?



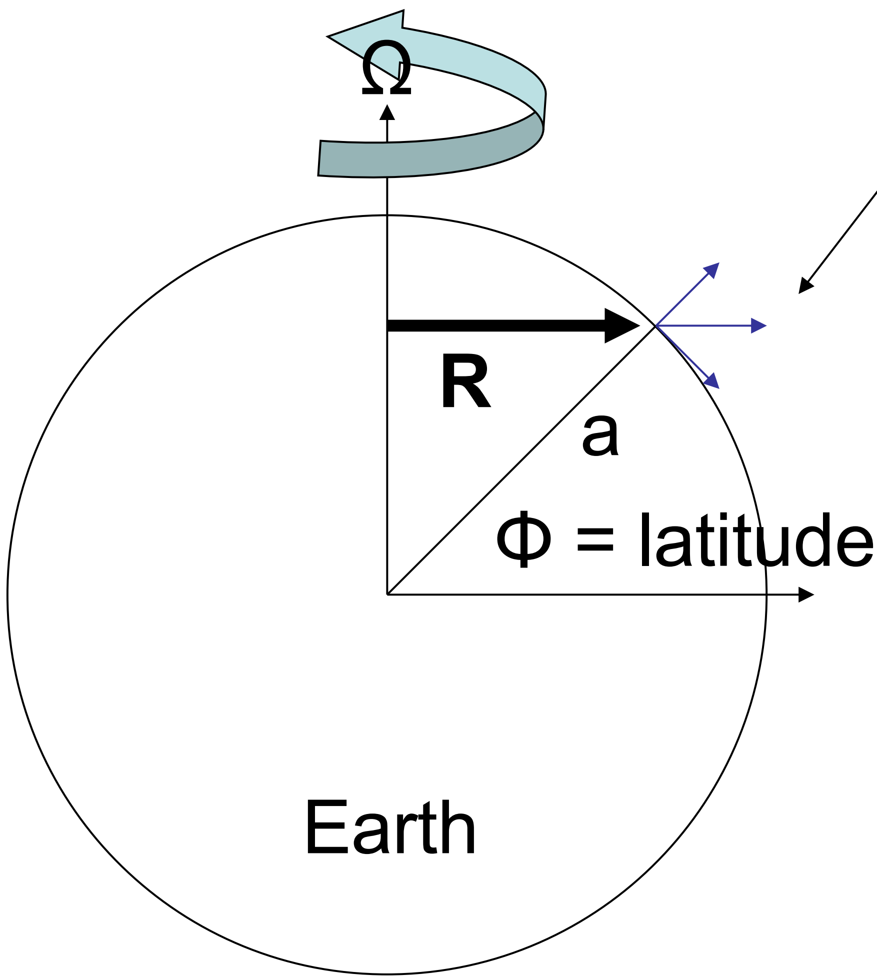
# *Earth's angular momentum (2)*



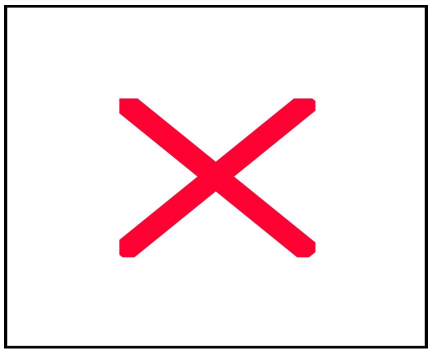

Angular momentum is



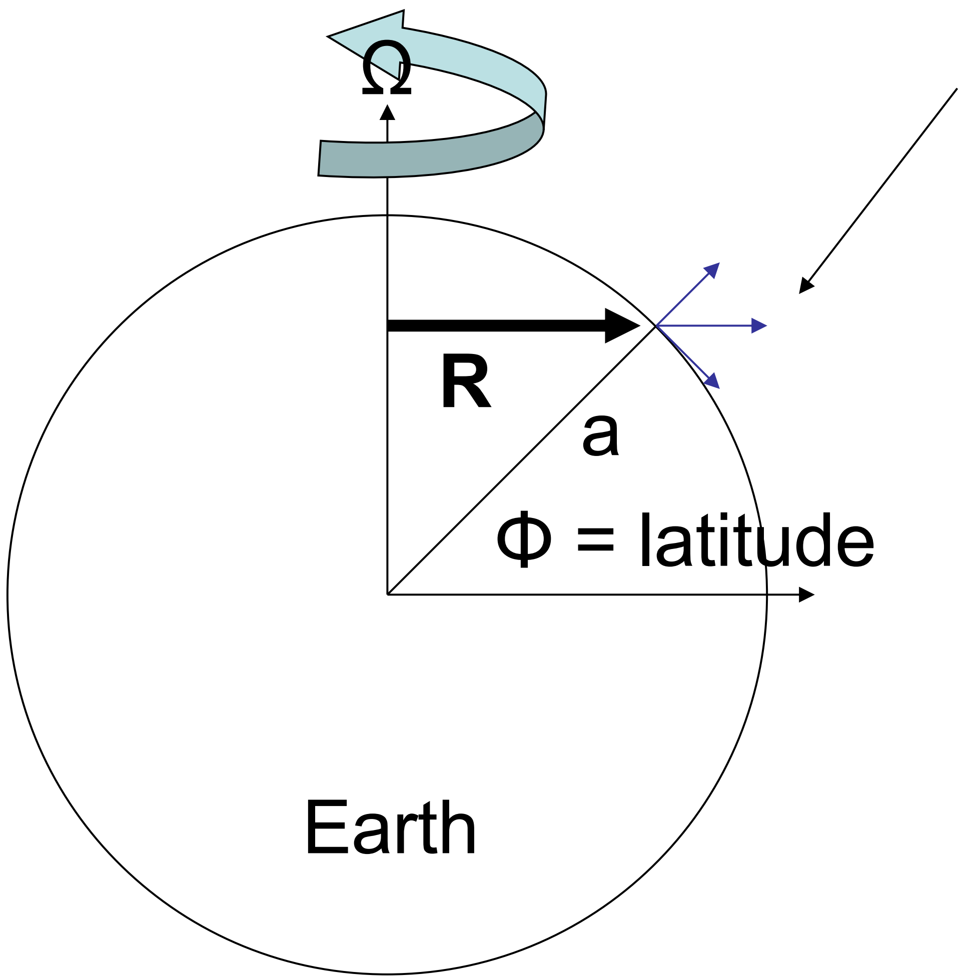
# *Earth's angular momentum (3)*



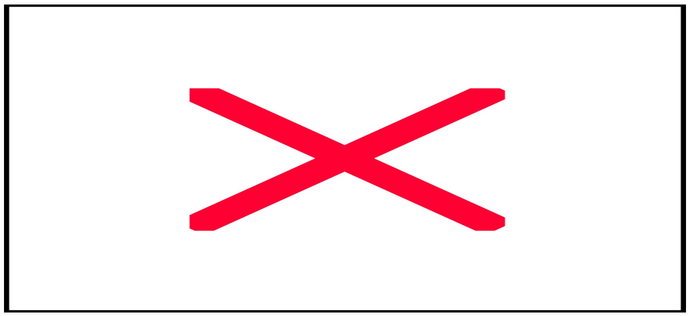

Angular momentum due **only** to rotation of Earth is



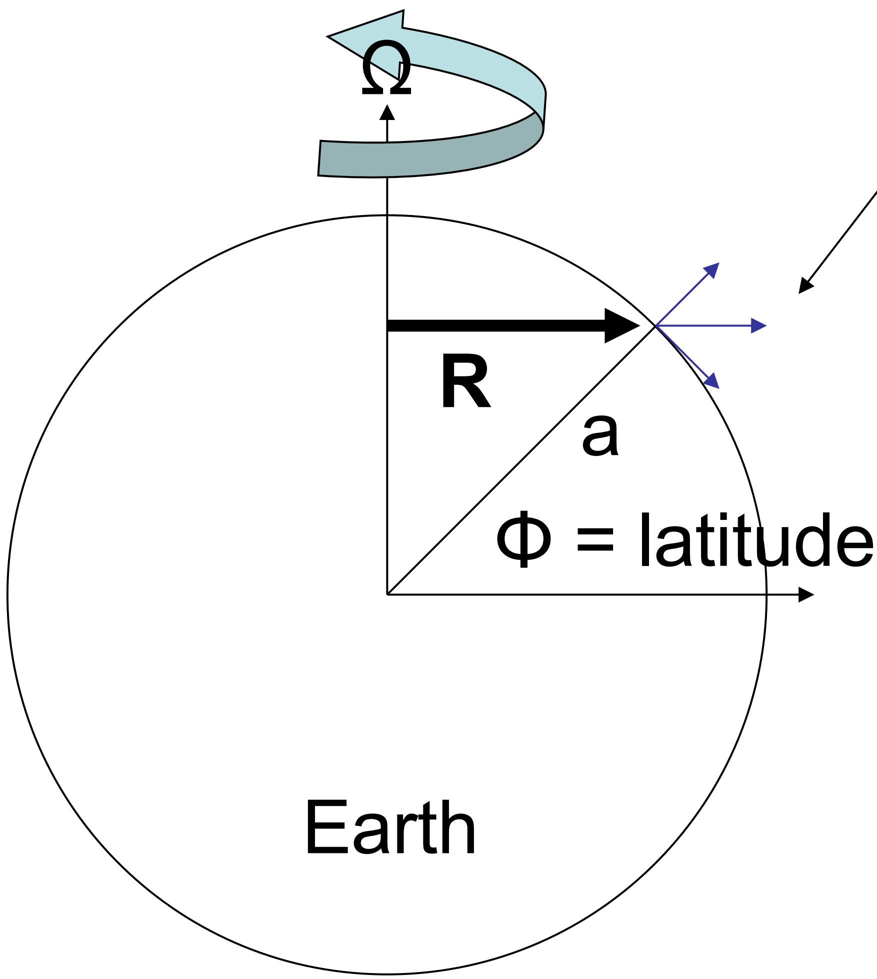
# *Earth's angular momentum (4)*



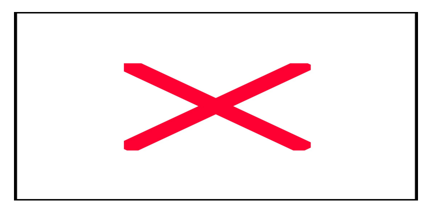

Angular momentum due **<u>only</u>** to rotation of Earth is



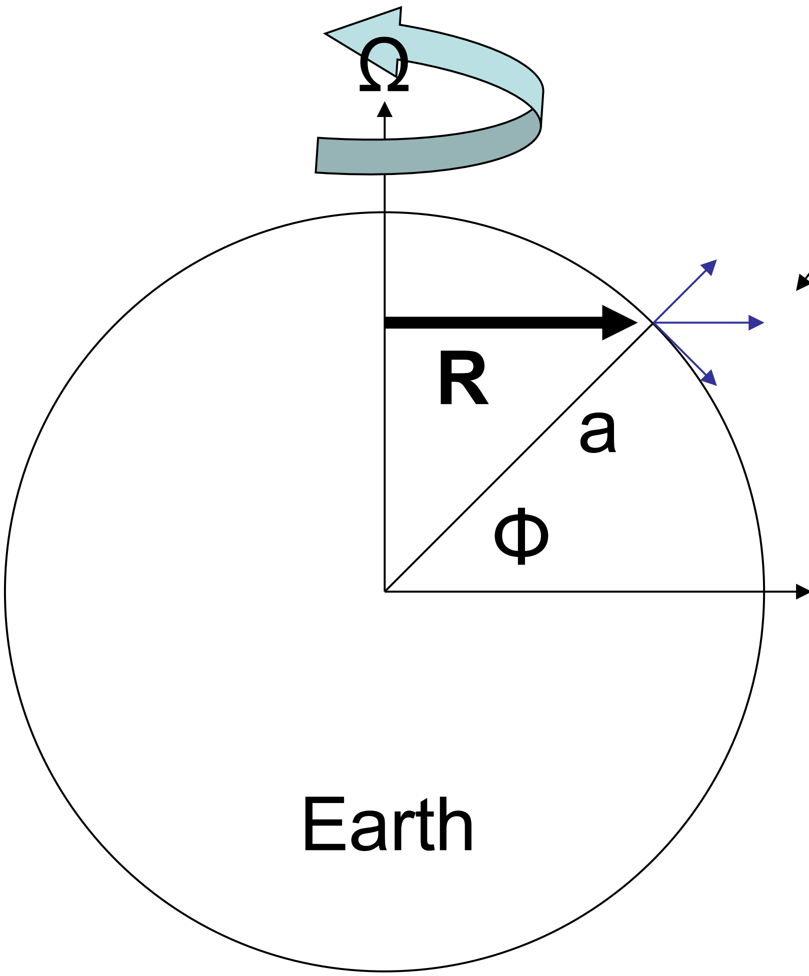
# *Angular momentum of parcel (1)*



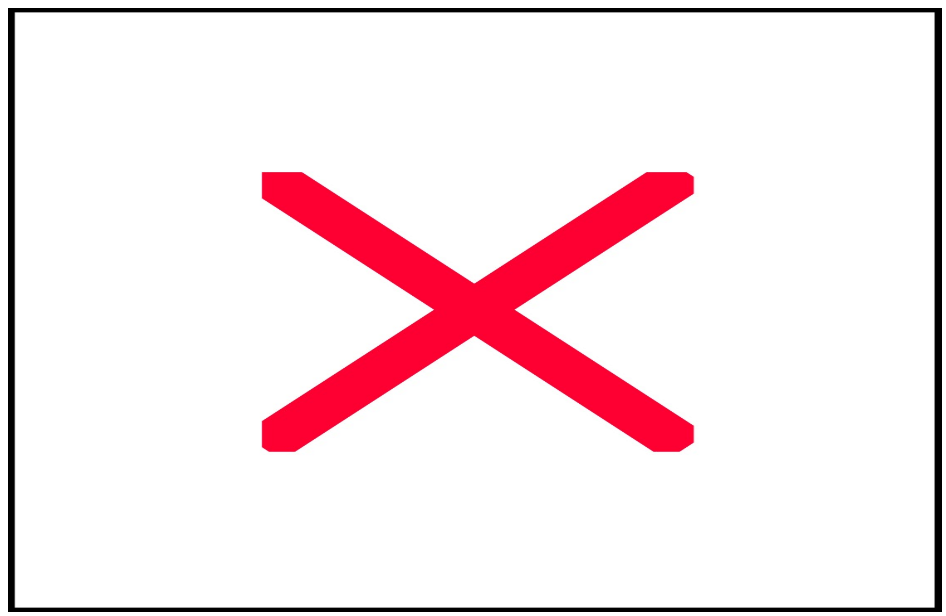

Assume there is some x velocity, u. Angular momentum associated with this velocity is



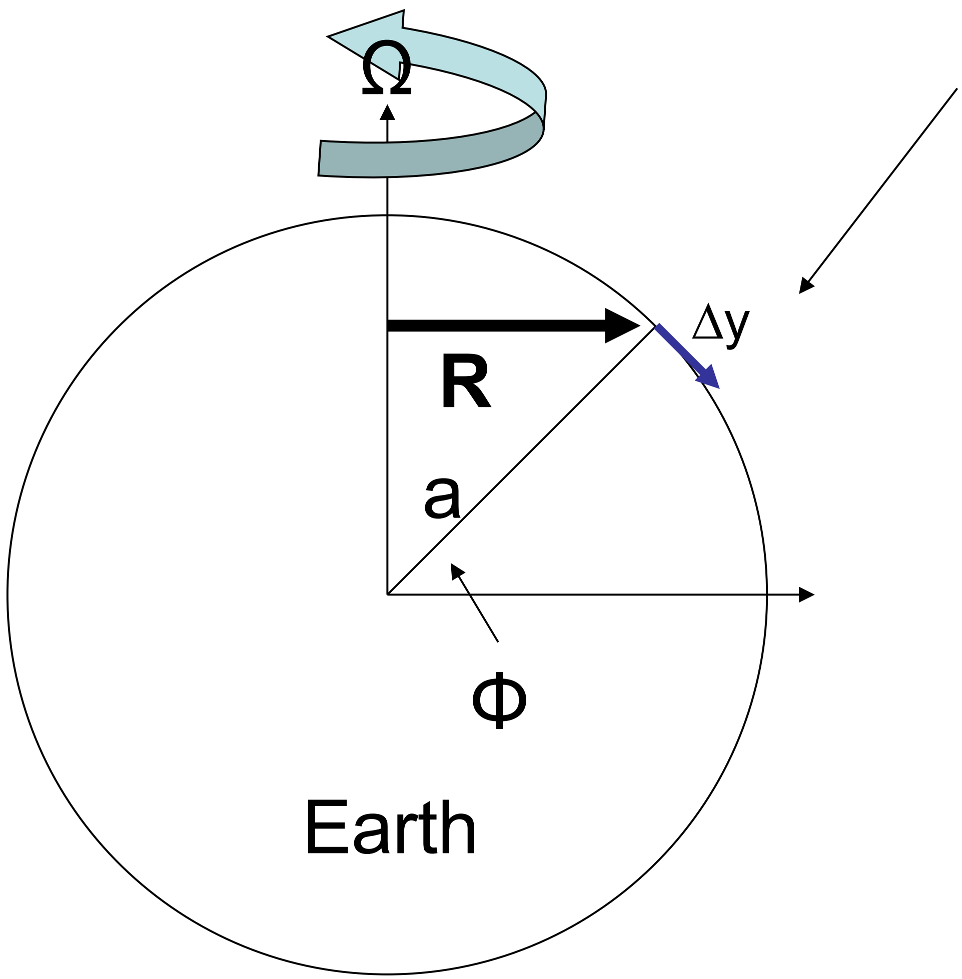
# *Total angular momentum*




Angular momentum due **both** to rotation of Earth and relative velocity u is

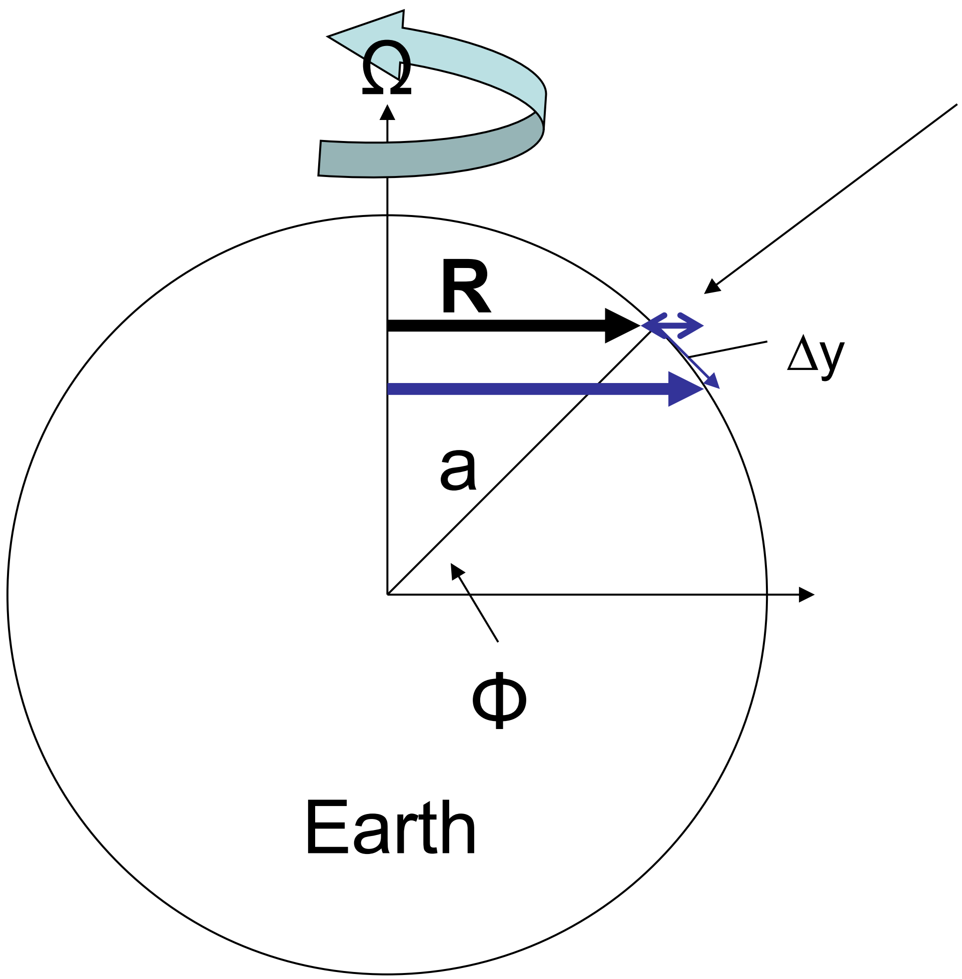


# *Displace parcel south (1)*
# *(Conservation of angular momentum)*



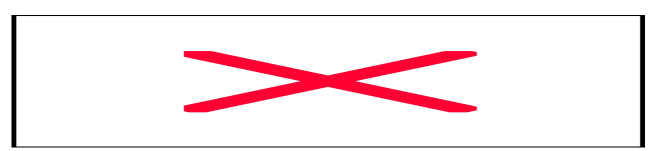

Let's imagine we move our parcel of air south (or north). What happens? Δy

# *Displace parcel south (2)*
# *(Conservation of angular momentum)*

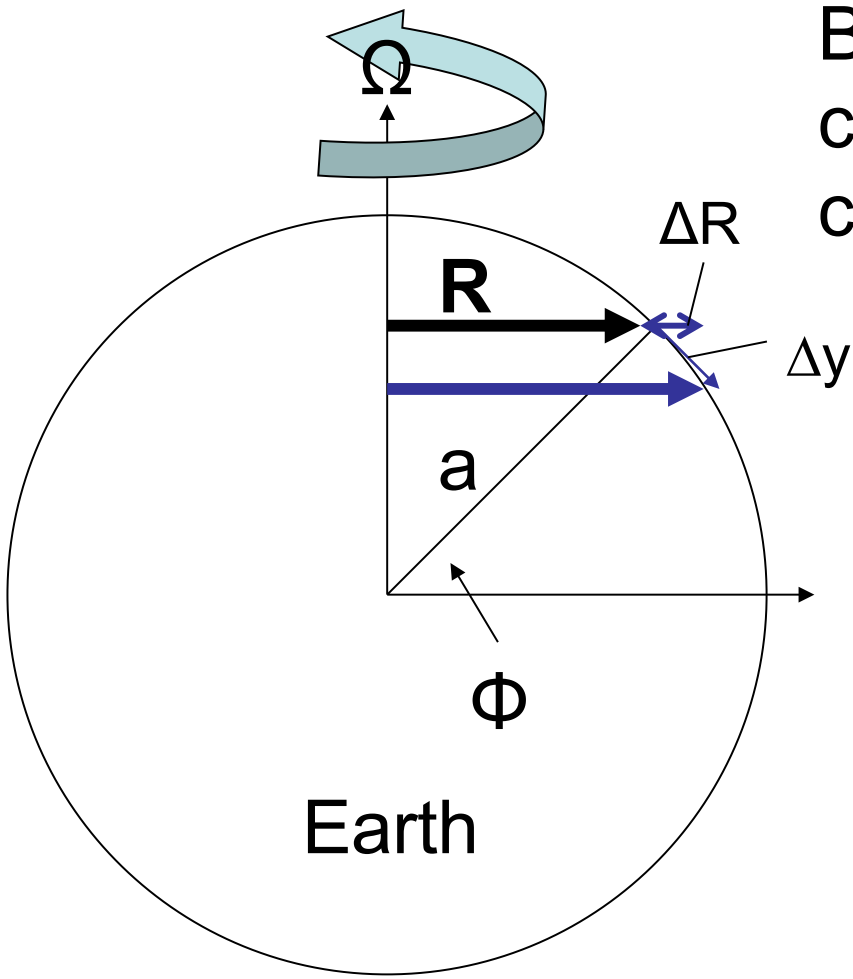



We get some change ΔR (R gets longer)

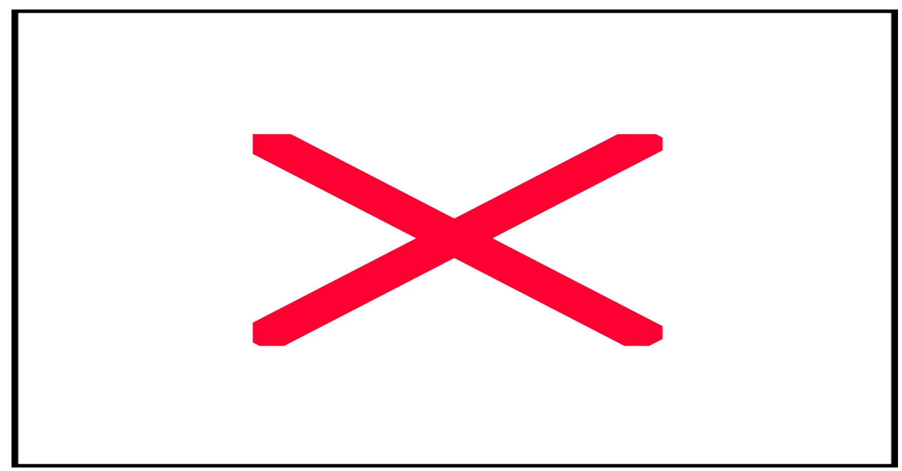
For the southward displacement we get



# *Displace parcel south (3) (Conservation of angular momentum)*

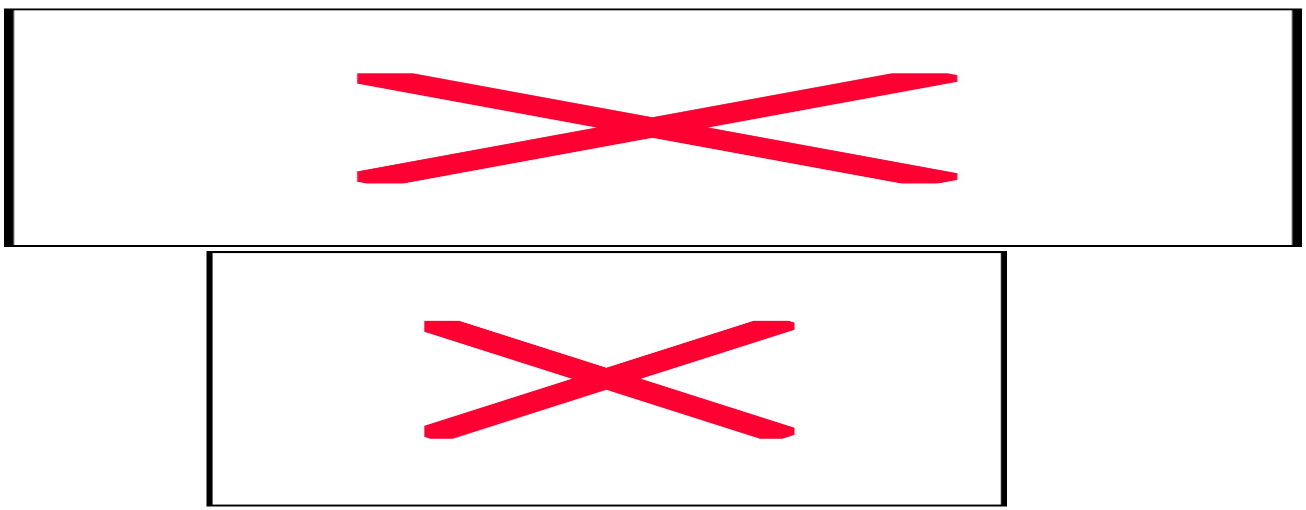



But if angular momentum is conserved, then u must change.

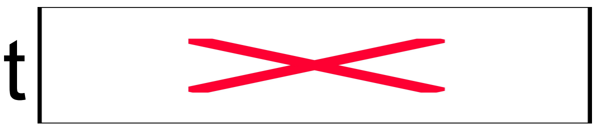


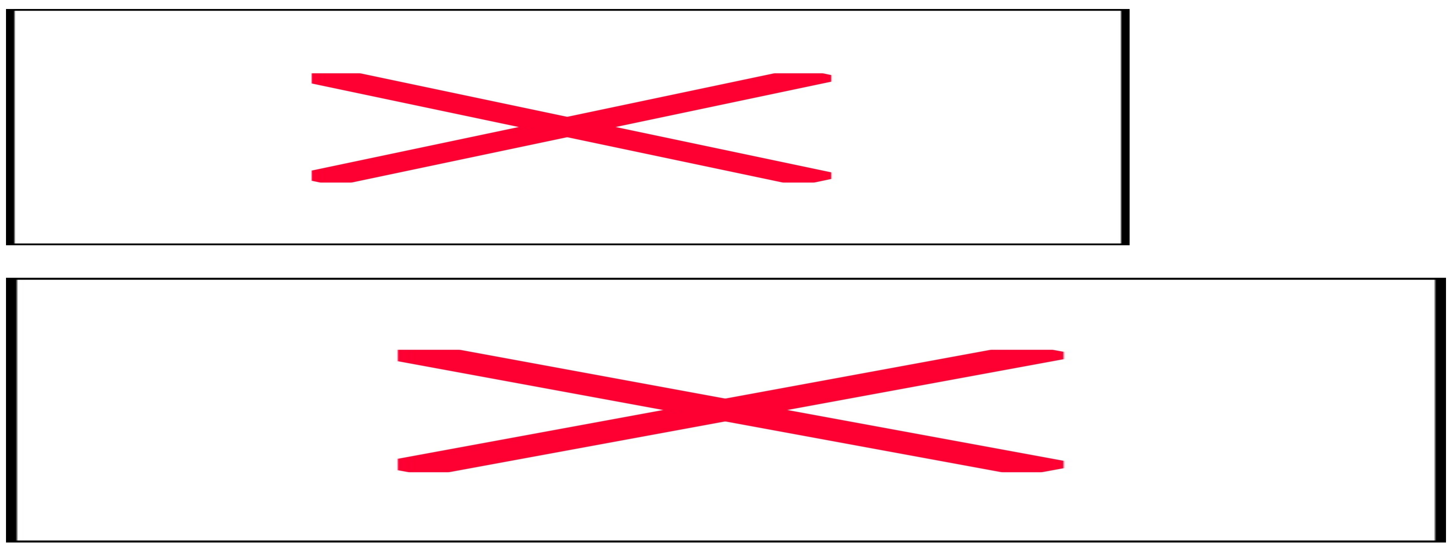
# *Displace parcel south (4)*
# *(Conservation of angular momentum)*

Expand right hand side, ignore second-order difference terms, solve for Δu (change in eastward velocity after southward displacement):



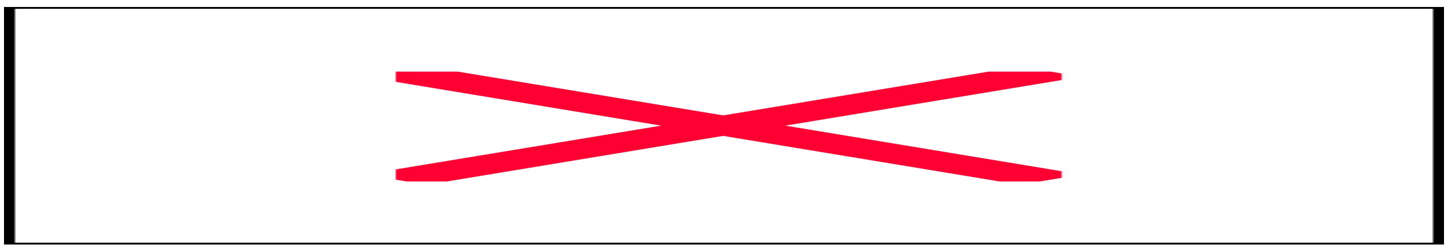
*Displace parcel south (5)*
*(Conservation of angular momentum)*

For our southward displacement 



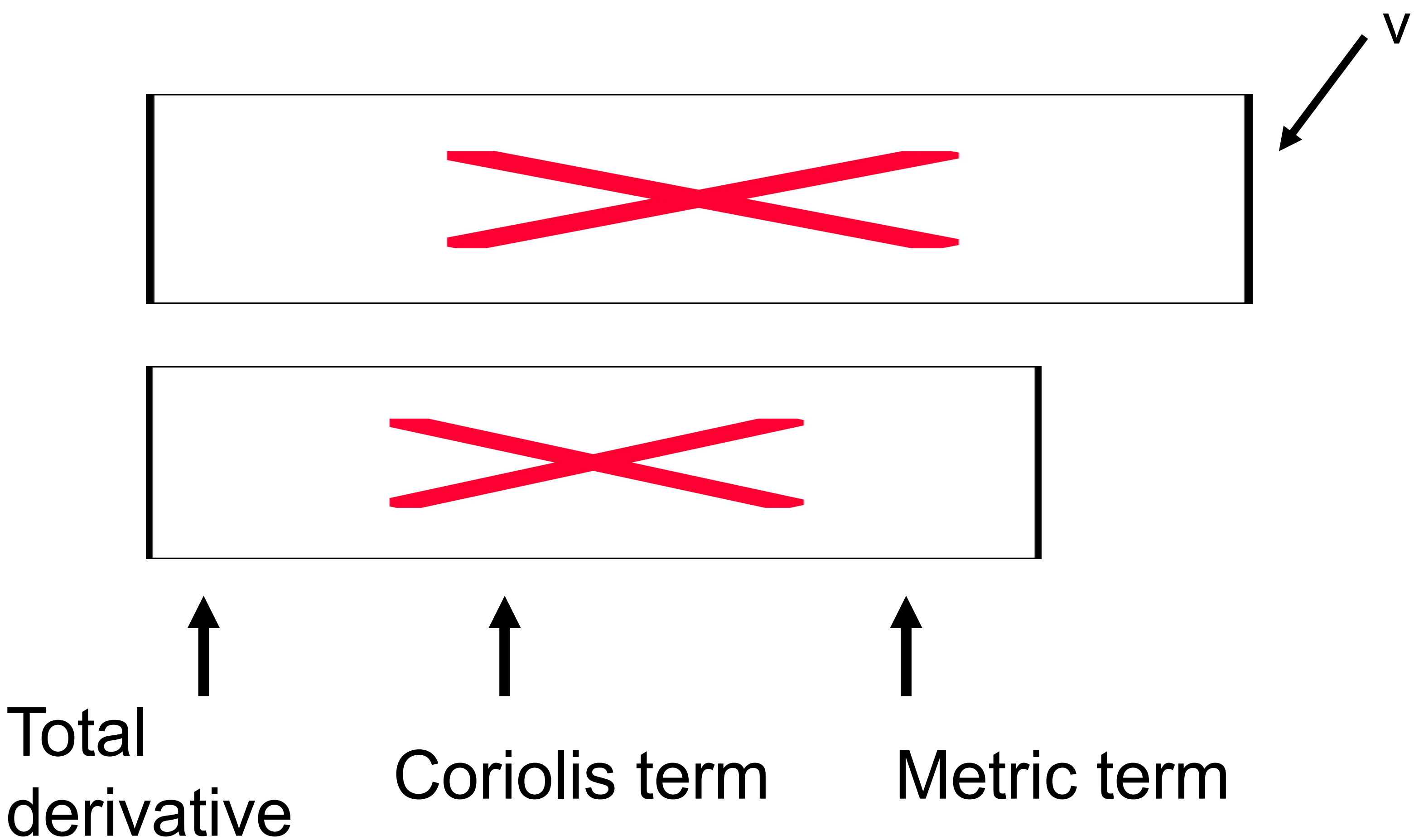
# *Displace parcel south (6)*
# *(Conservation of angular momentum)*

Divide by Δt:

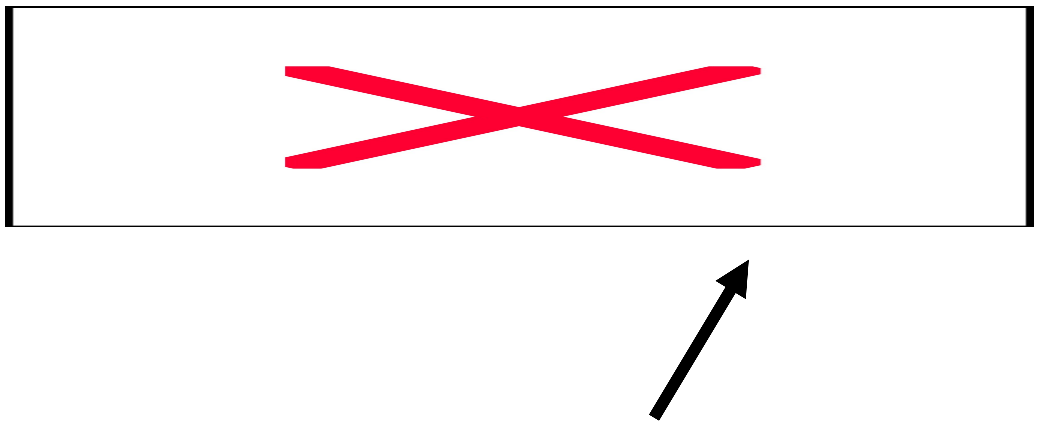


# *Displace parcel south (7)*
# *(Conservation of angular momentum)*

Take the limit $\Delta t \Rightarrow 0$:



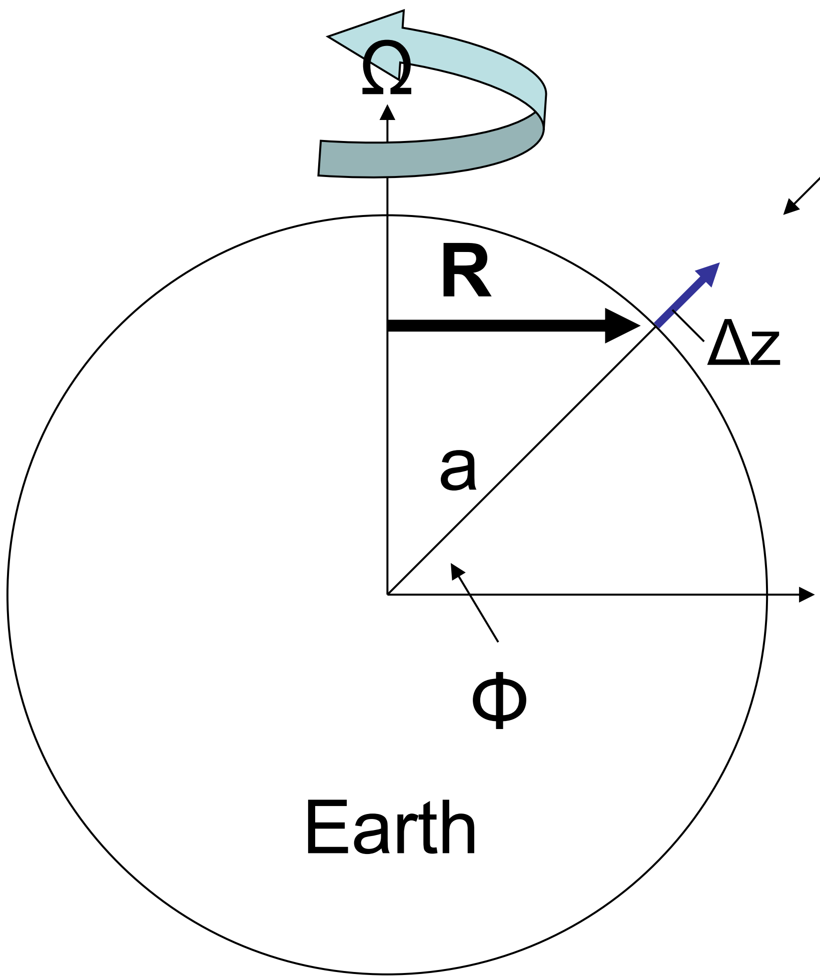
*Displace parcel south (8)*
*(Conservation of angular momentum)*



What's this? "Curvature or metric term." It takes into account that y curves, it is defined on the surface of the Earth. More later.

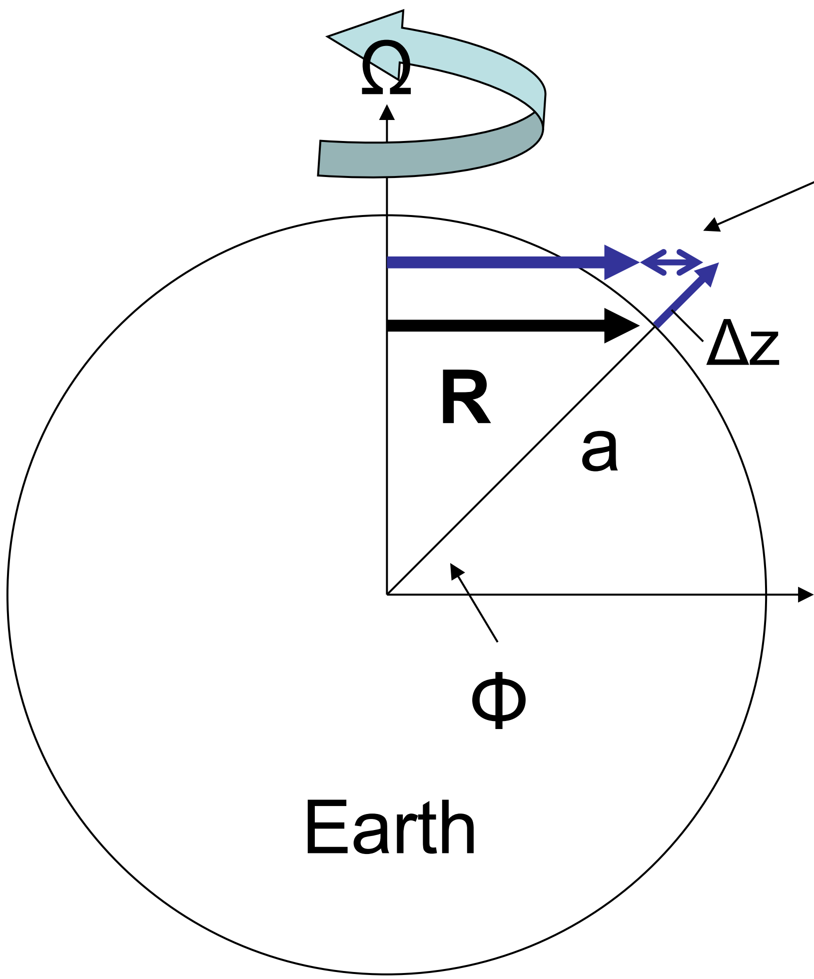
Remember this is ONLY FOR a NORTH-SOUTH displacement.

# *Displace parcel up (1)*
# *(Conservation of angular momentum)*



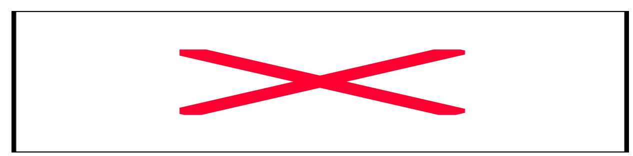

Let's imagine we move our parcel of air up (or down). What happens? Δz

# *Displace parcel up (2)*
# *(Conservation of angular momentum)*




We get some change ΔR (R gets longer)

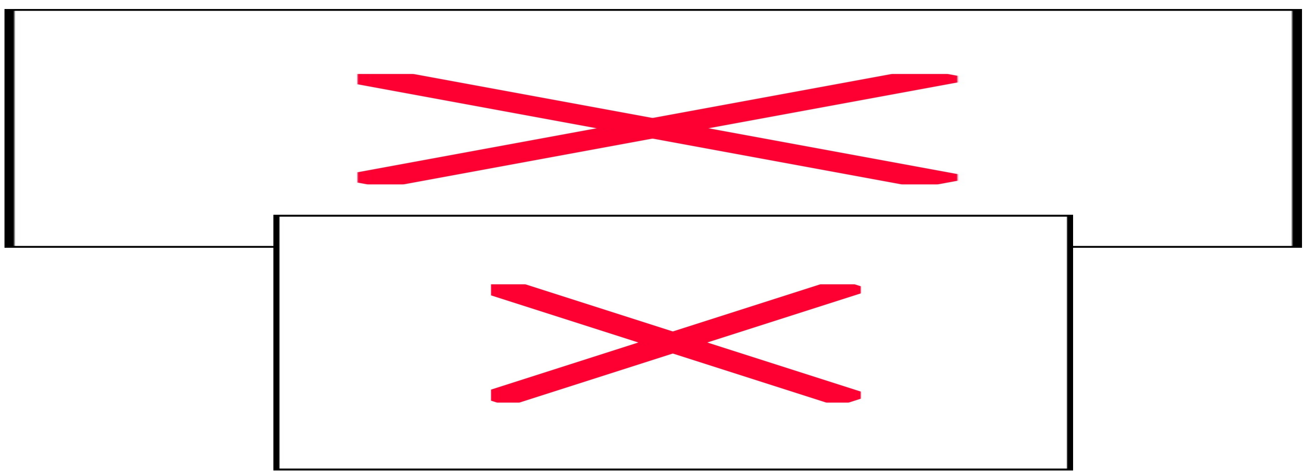
For our upward displacement



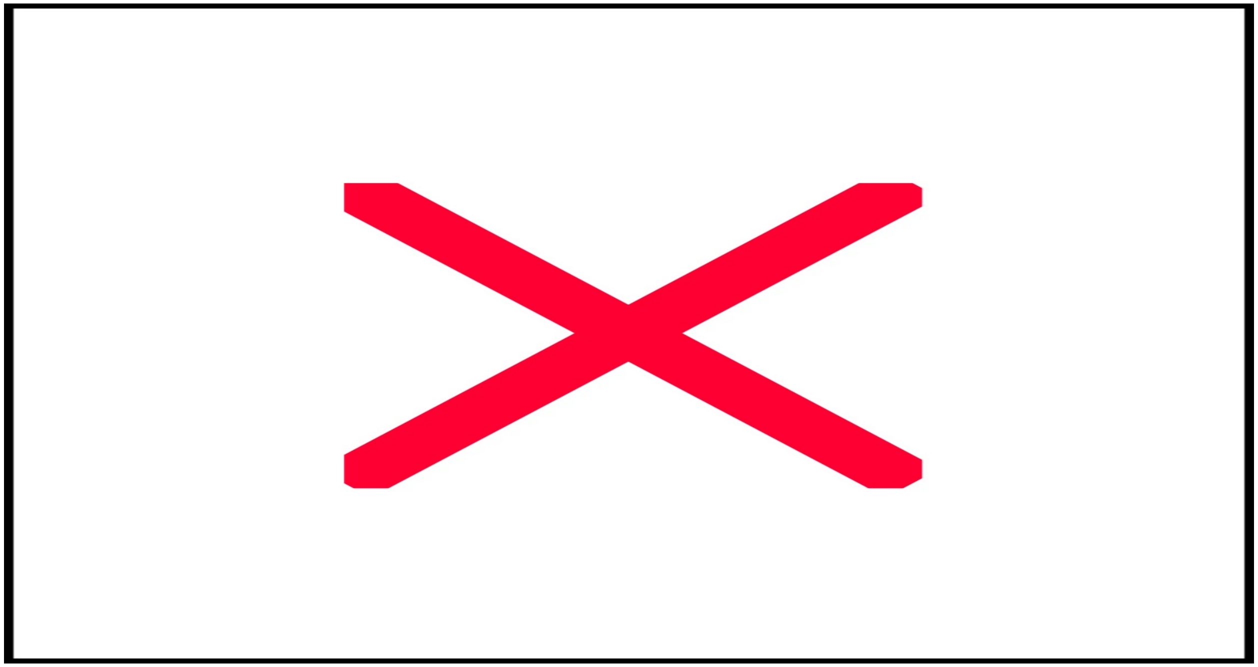
## *Displace parcel up (3)*
## *(Conservation of angular momentum)*

Do the same form of derivation
(as we did for the southward displacement)

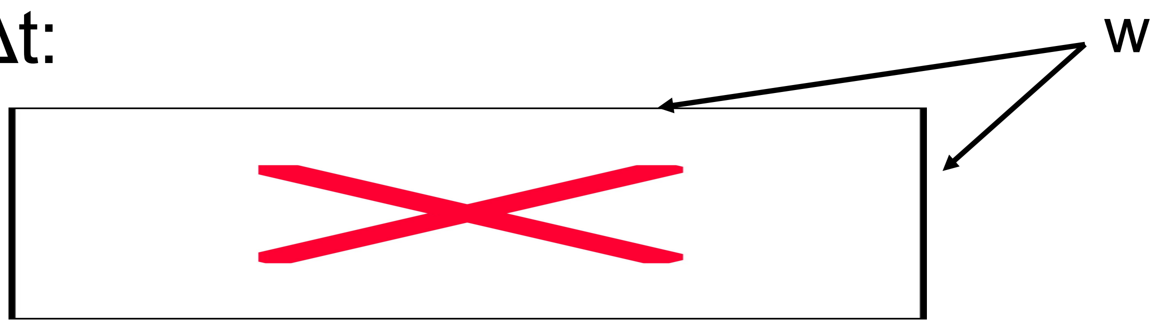
Expand right hand side, ignore second-order difference terms, solve for $\Delta u$ (change in eastward velocity after vertical displacement):



*Displace parcel up (4)*
*(Conservation of angular momentum)*

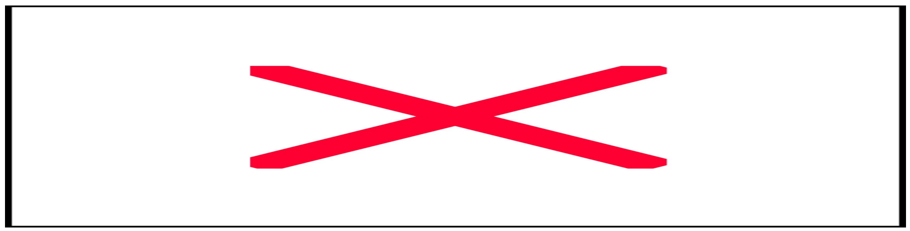


Divide by Δt:



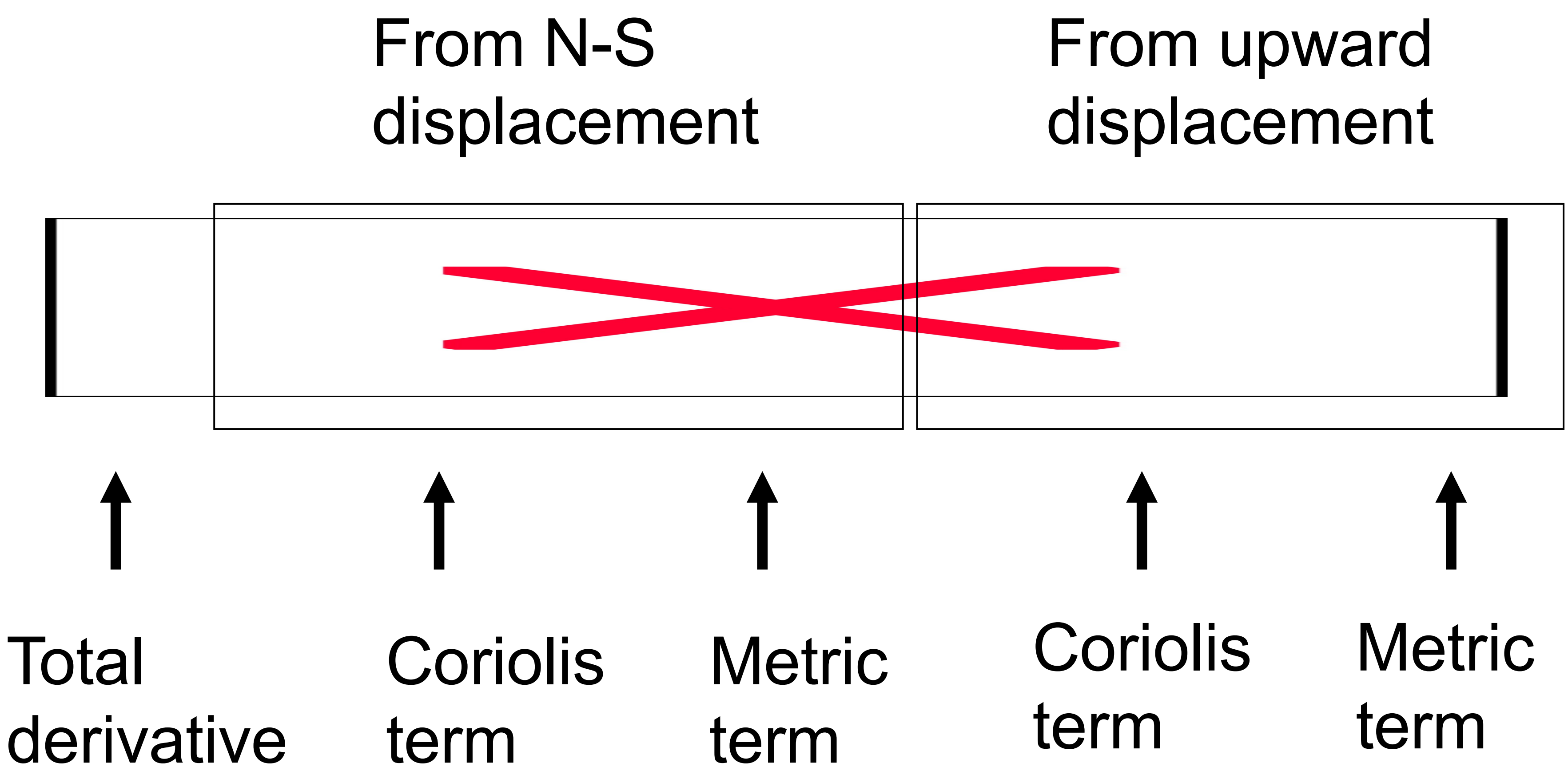
# *Displace parcel up (5)*
# *(Conservation of angular momentum)*

Take the limit $\Delta t \Rightarrow 0$:

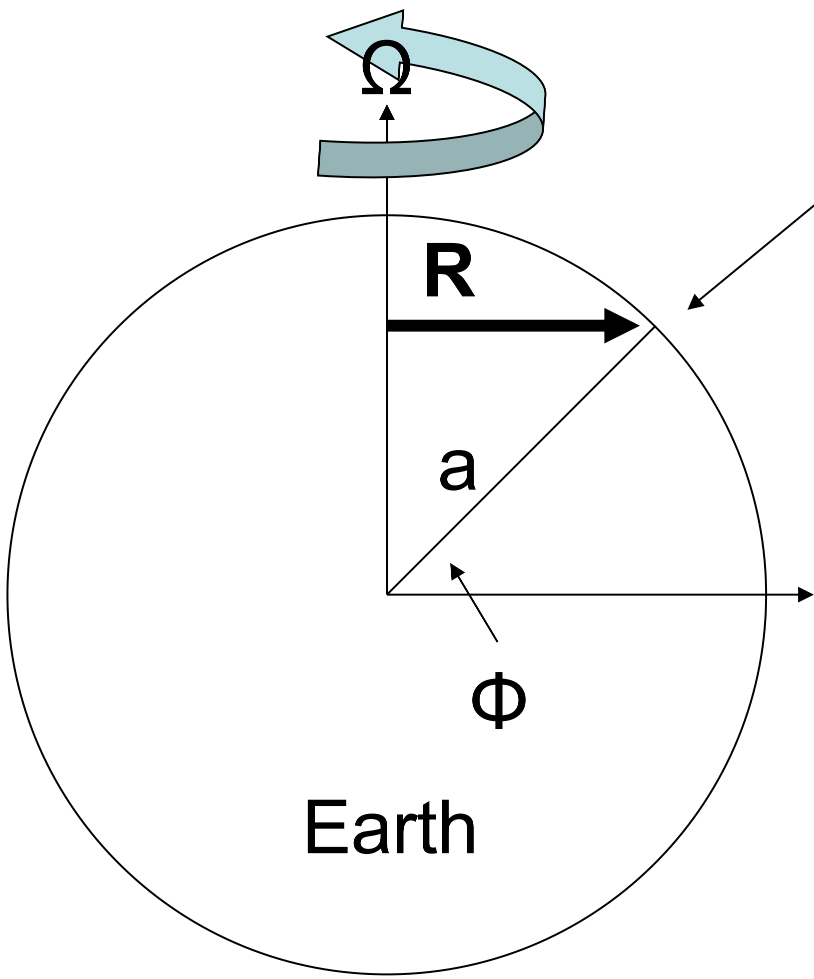


Remember this is ONLY FOR a VERTICAL displacement.

## *So far we got (Conservation of angular momentum)*

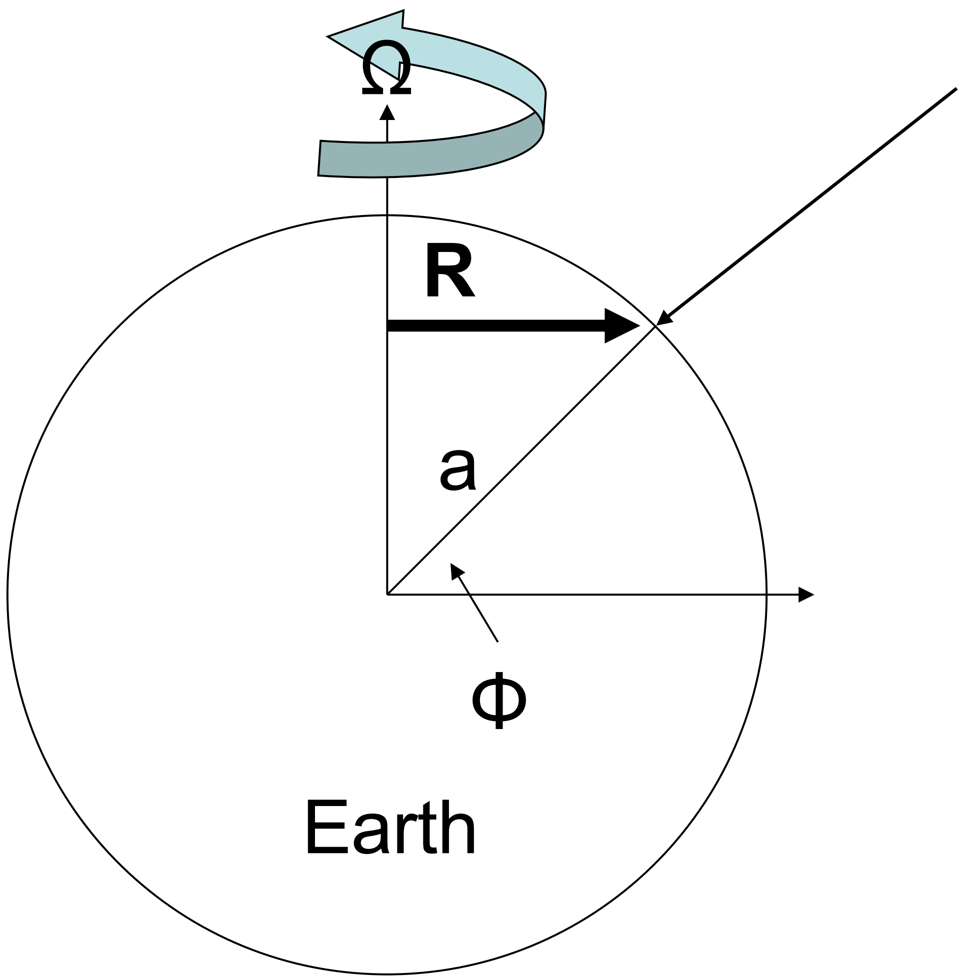


# *Displace parcel east (1)*
# *(Conservation of angular momentum)*




Let's imagine we move our parcel of air east (or west). What happens? Δx

# *Displace parcel east (2) (Conservation of angular momentum)*



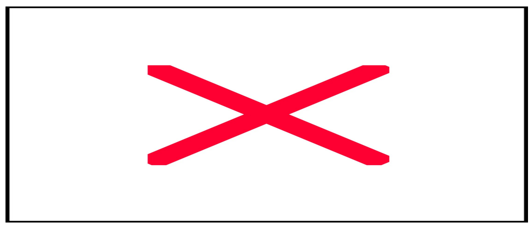

Well, there is no change of ΔR.

But the parcel is now rotating faster than the earth:

**Centrifugal force on the object will be increased**

# *Displace parcel east (3)*
# *(Conservation of angular momentum)*

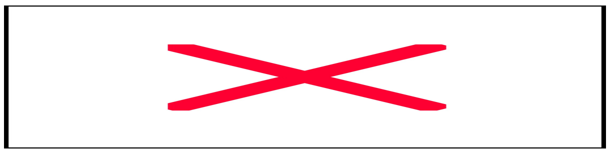
- Remember: Angular momentum



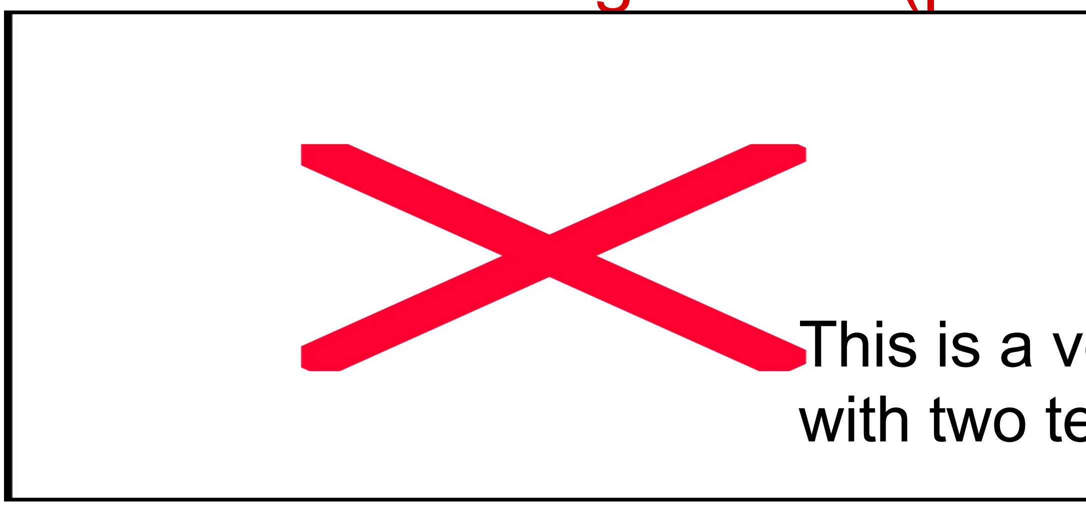
- The east displacement changed u (=dx/dt). Hence again we have a question of conservation of angular momentum.
- We will think about this as an **excess (or deficit) of centrifugal force** per unit mass relative to that from the Earth alone.

# *Displace parcel east (4)*
# *(Conservation of angular momentum)*

Remember: centrifugal force per unit mass



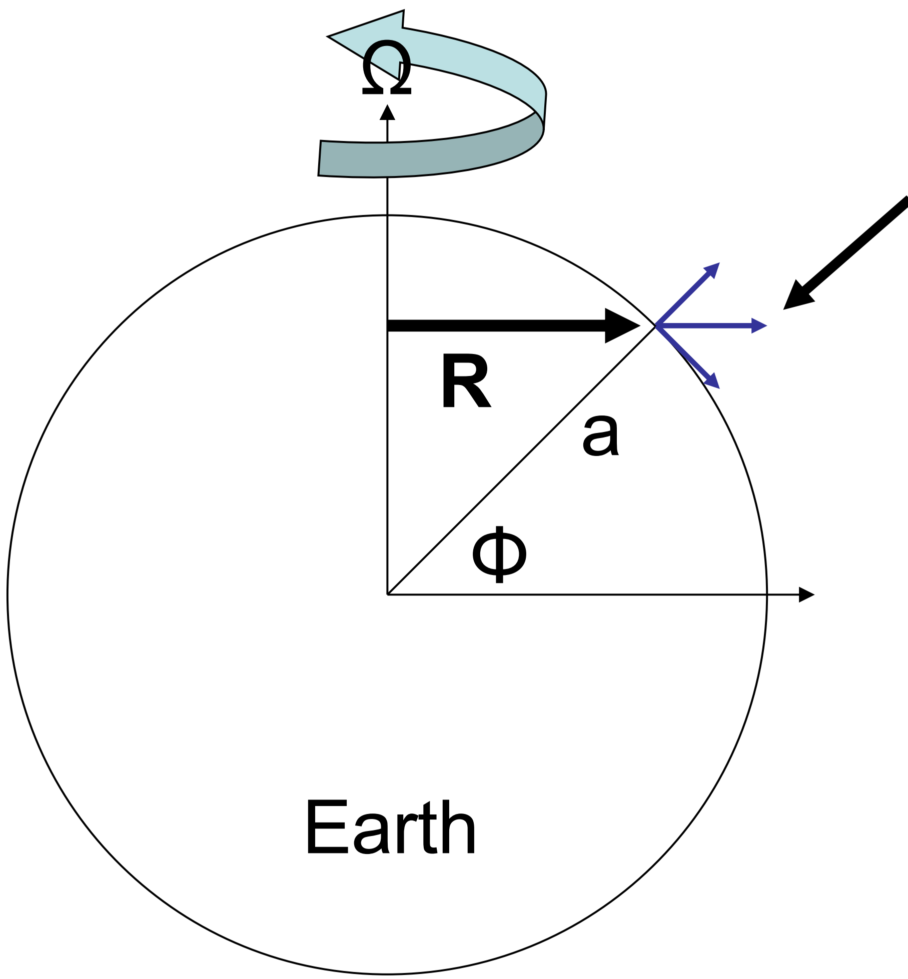
Therefore: excess centrifugal force (per unit mass):

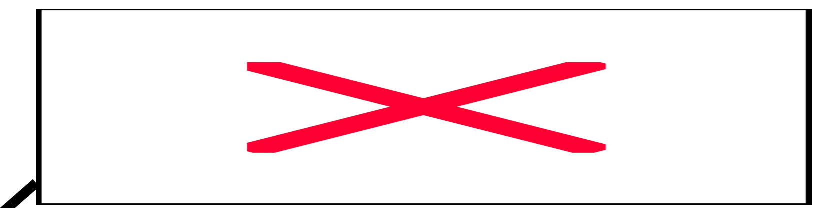


This is a vector force with two terms!

Coriolis term Metric term

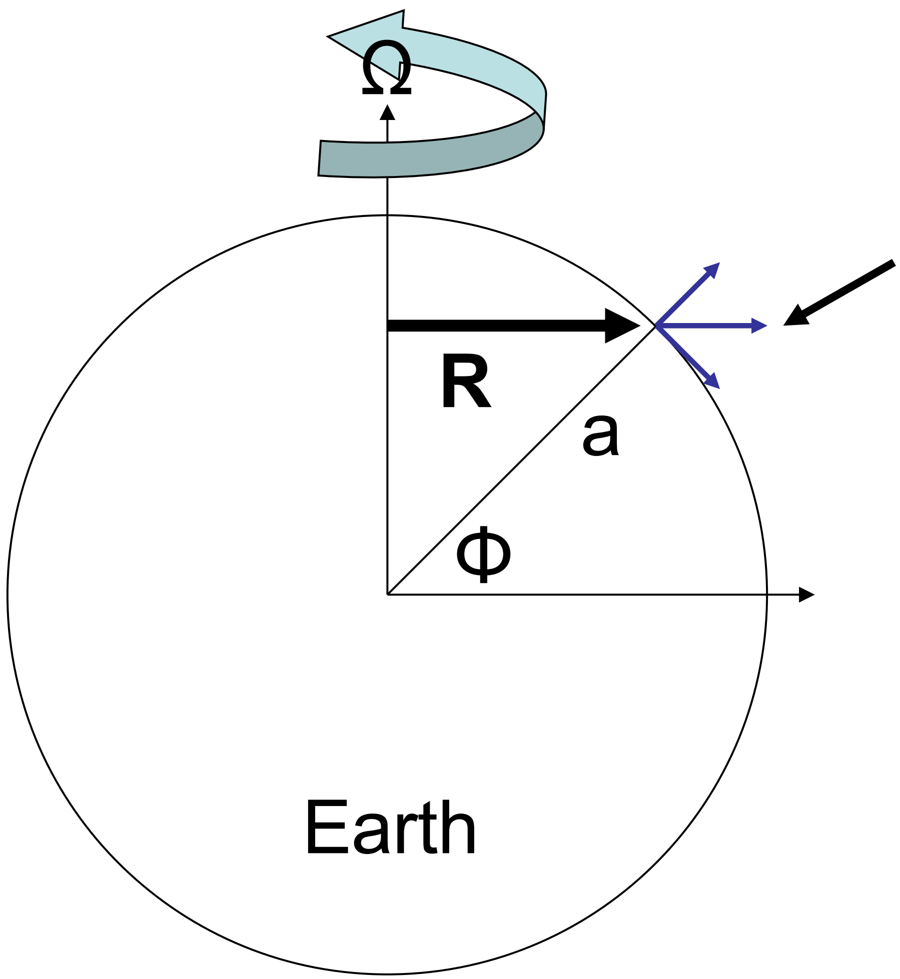
# *Displace parcel east (5)*
# *(Conservation of angular momentum)*



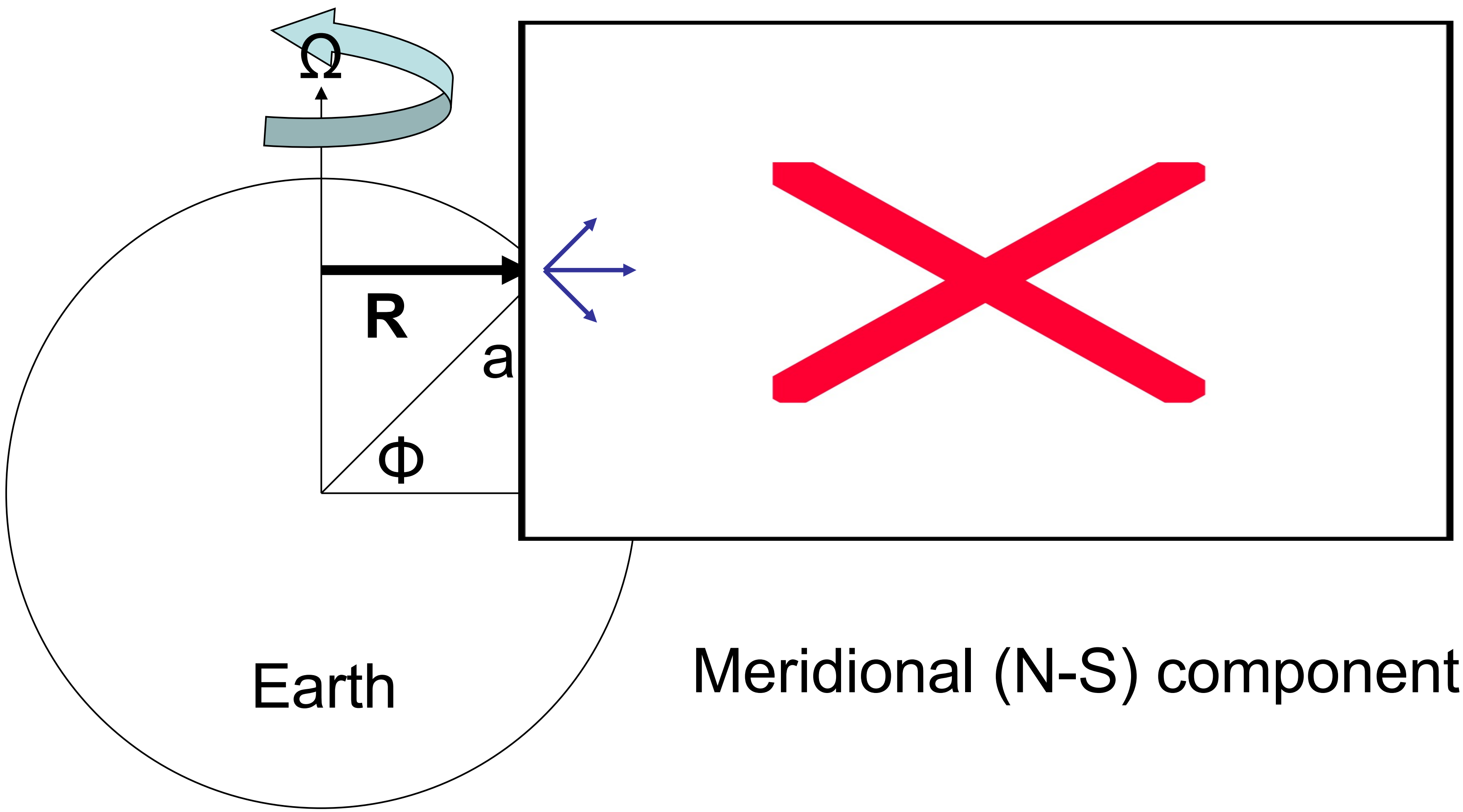
Vector with component in north-south and vertical direction:
Split the two directions.

# *Displace parcel east (6)*
# *(Conservation of angular momentum)*



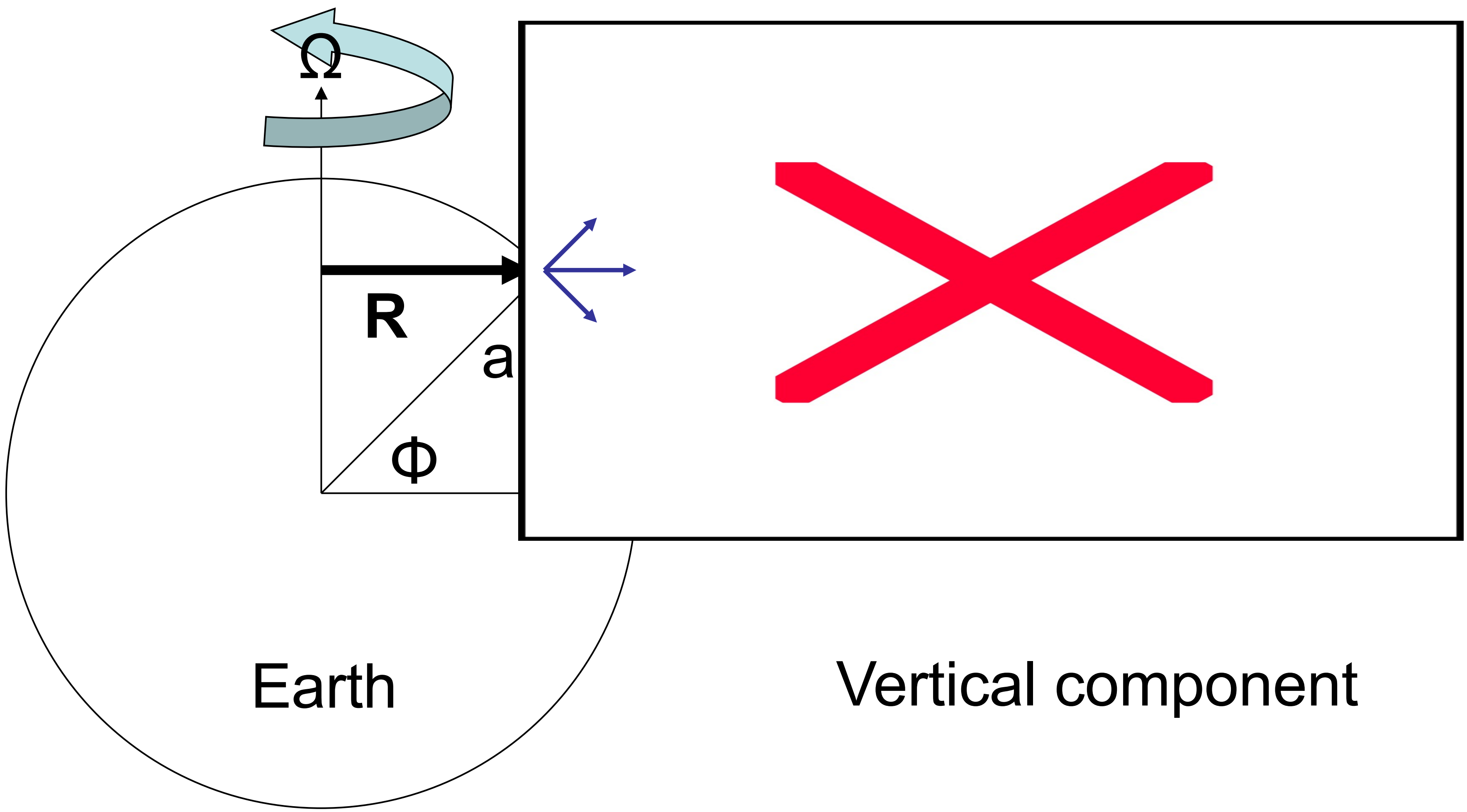

For the Coriolis component magnitude is $2\Omega u$.
For the curvature (or metric) term the magnitude is $u^2/(a\cos(\phi))$

## *Displace parcel east (7)*
## *(Conservation of angular momentum)*



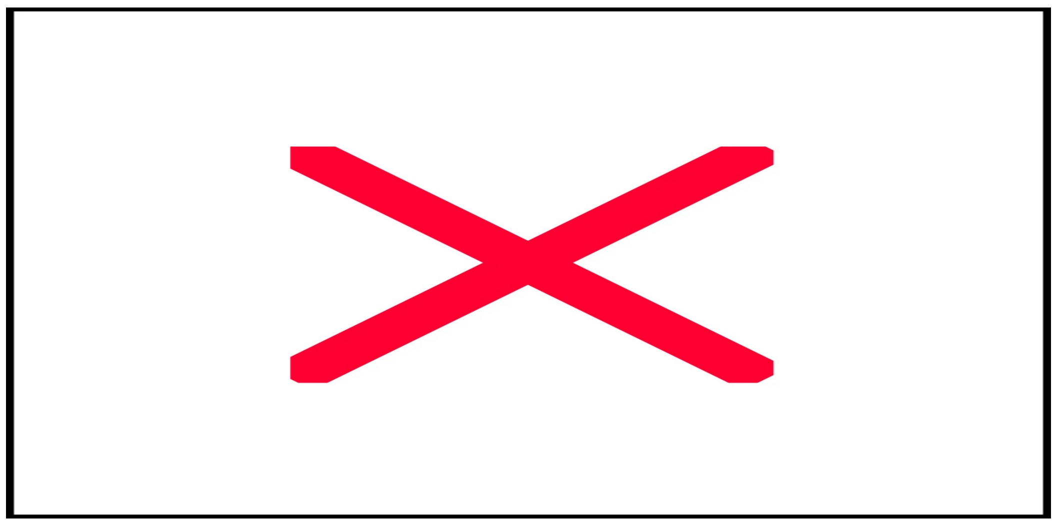

Meridional (N-S) component

# *Displace parcel east (8)*
# *(Conservation of angular momentum)*



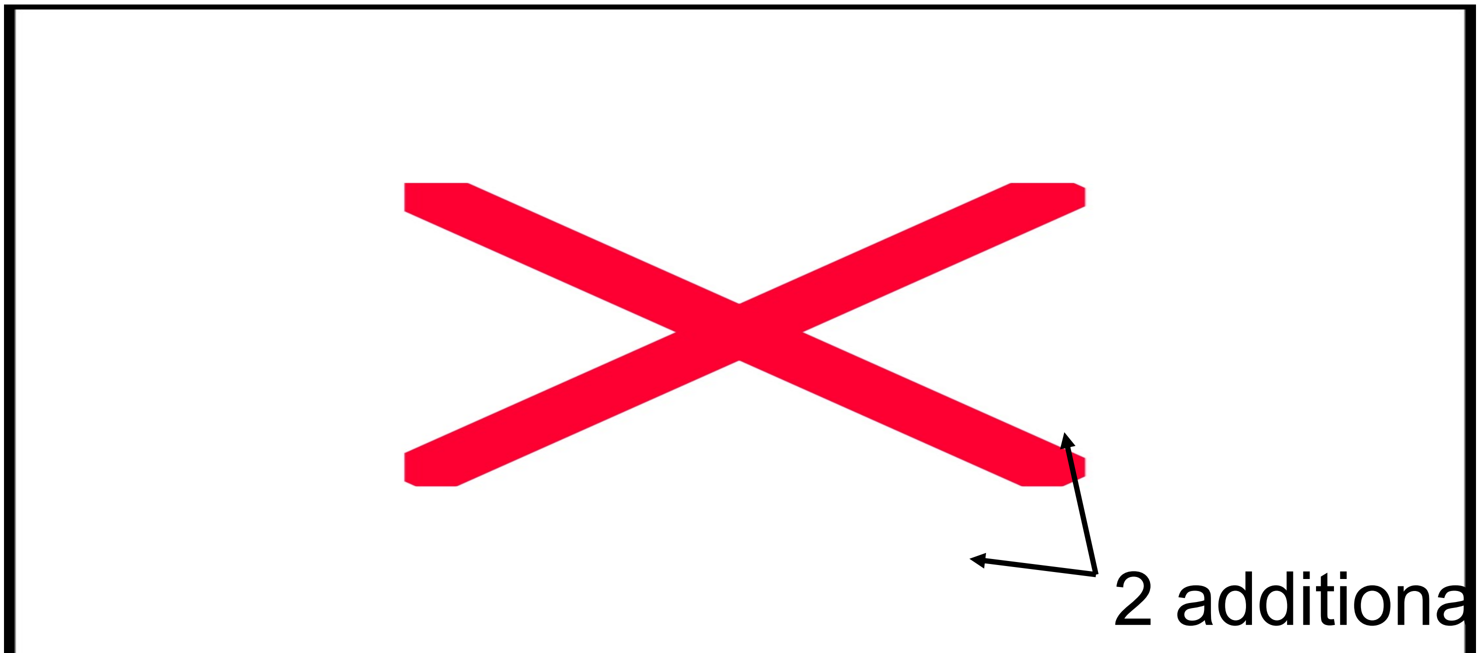
# *Displace parcel east (9)*
# *(Conservation of angular momentum)*

These forces in their appropriate component directions are



# *Coriolis force and metric terms in 3-D*

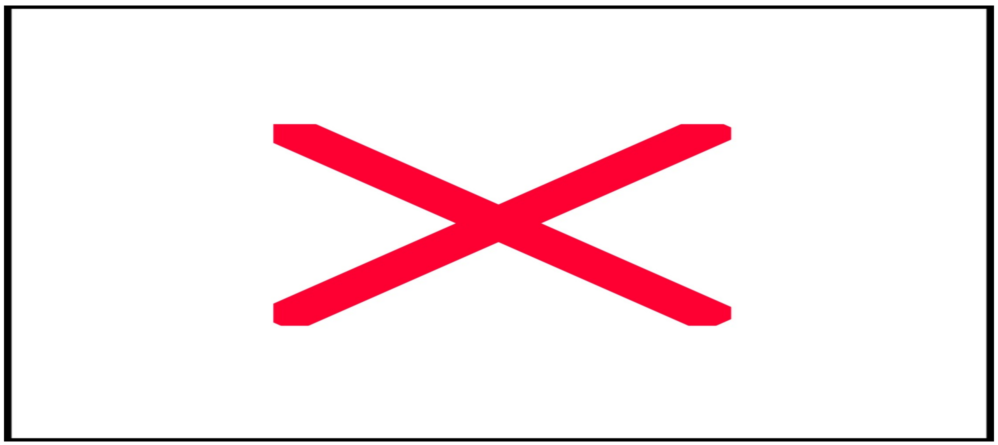
So let's collect together all Coriolis forces and metric terms:

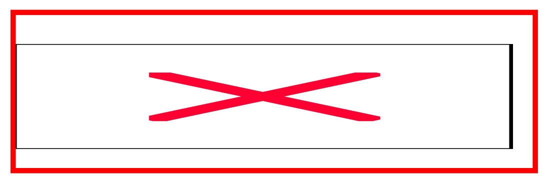


2 additional metric terms (due to more rigorous derivation, Holton 2.2, 2.3)

*Coriolis force and metric terms*

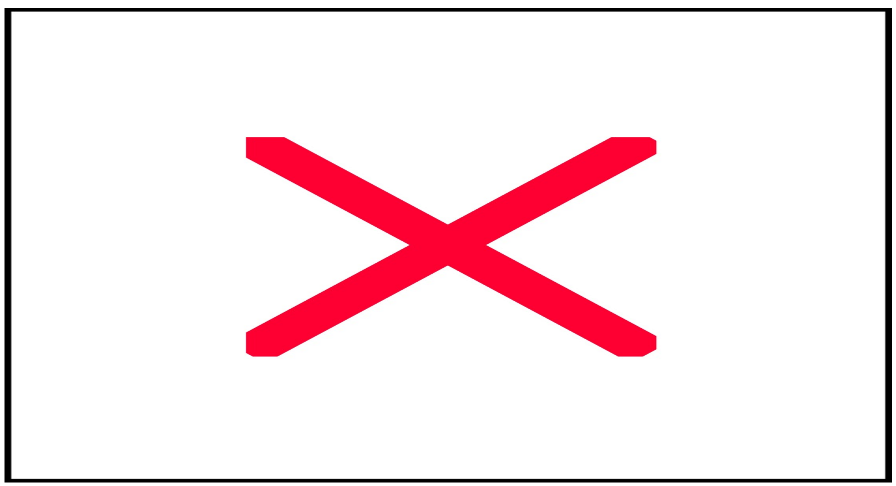
If the vertical velocity w is small (w close to 0 m/s), we can make the following **approximation**:



Define the **Coriolis parameter f**: 

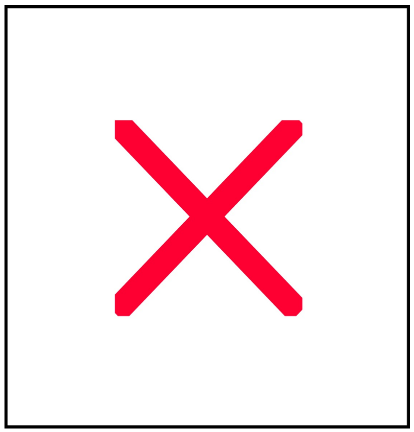
# *Coriolis force and metric terms*

For synoptic scale (large-scale) motions $|u| << \Omega R$ .
Then the metric terms (last terms on previous slide) are small in comparison to the Coriolis terms. We discuss this in more detail in our next class (scale analysis).
This leads to the **approximation** of the Coriolis force:

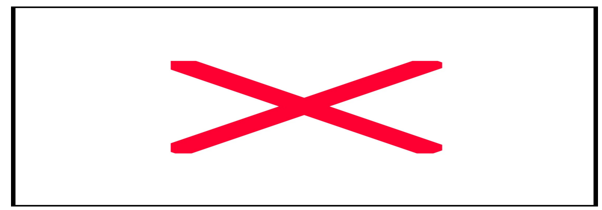


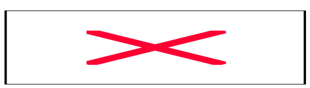
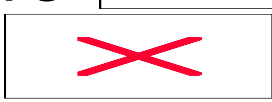
# *Coriolis force*

For synoptic scale (large-scale) motions:



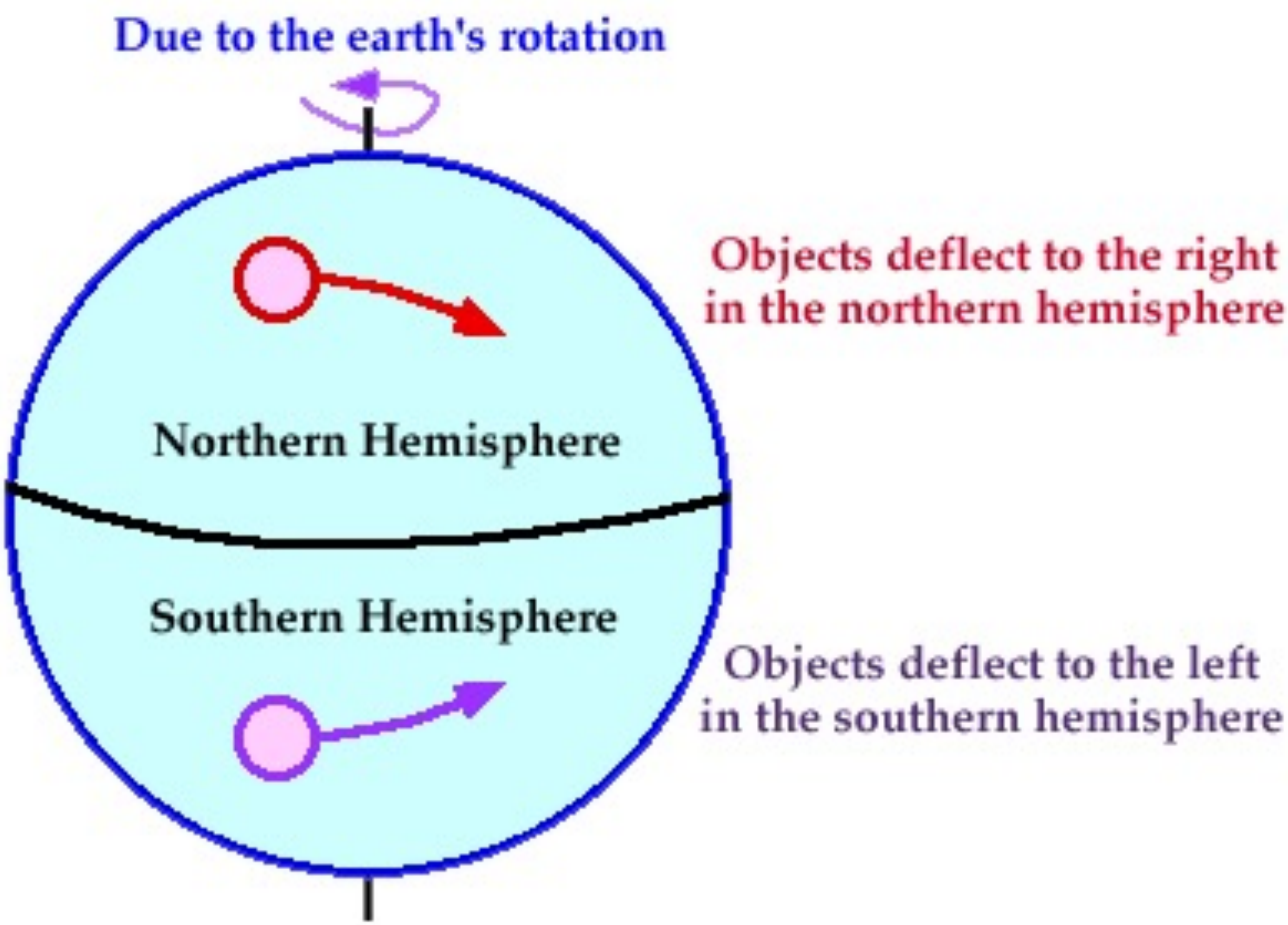
Vector notation:



where  is the horizontal velocity vector and  is the vertical unit vector.

Since -**k** × **v** is a vector rotated 90° to the right of **v** it shows that the Coriolis force **deflects** and changes only the direction of motion, not the speed.

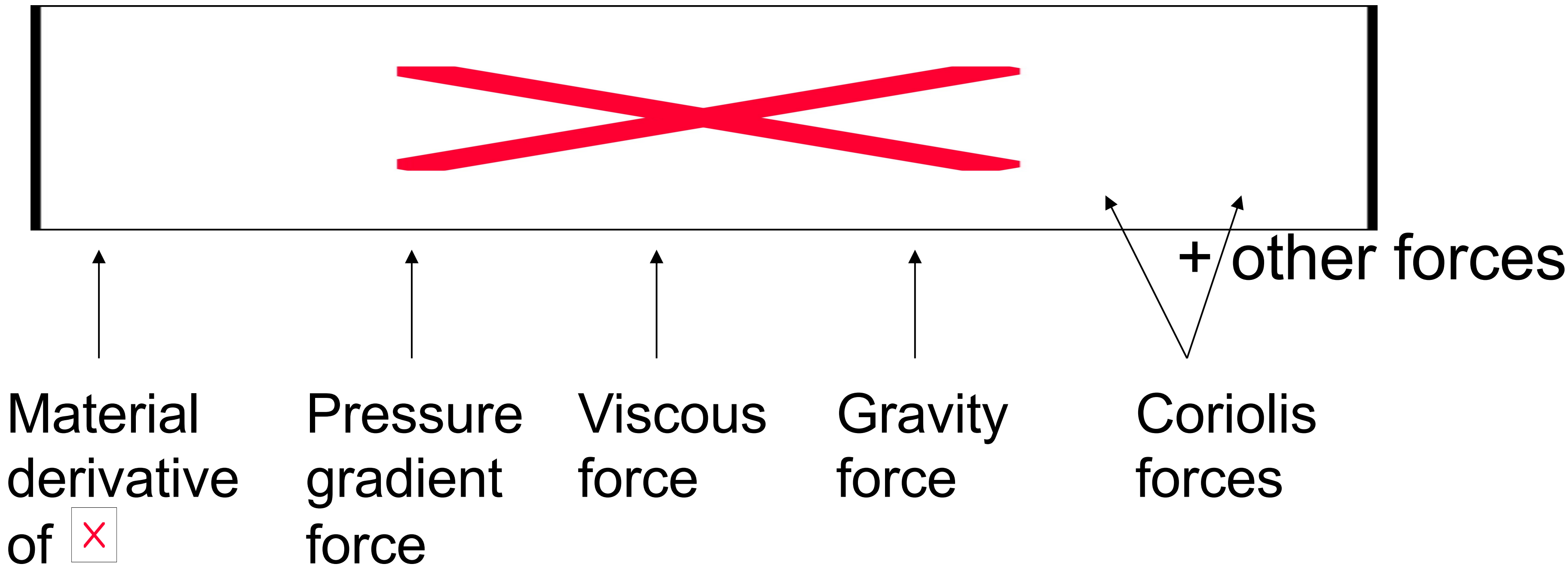
# *Effects of the Coriolis force on motions on Earth*



# *Summary: Coriolis force*

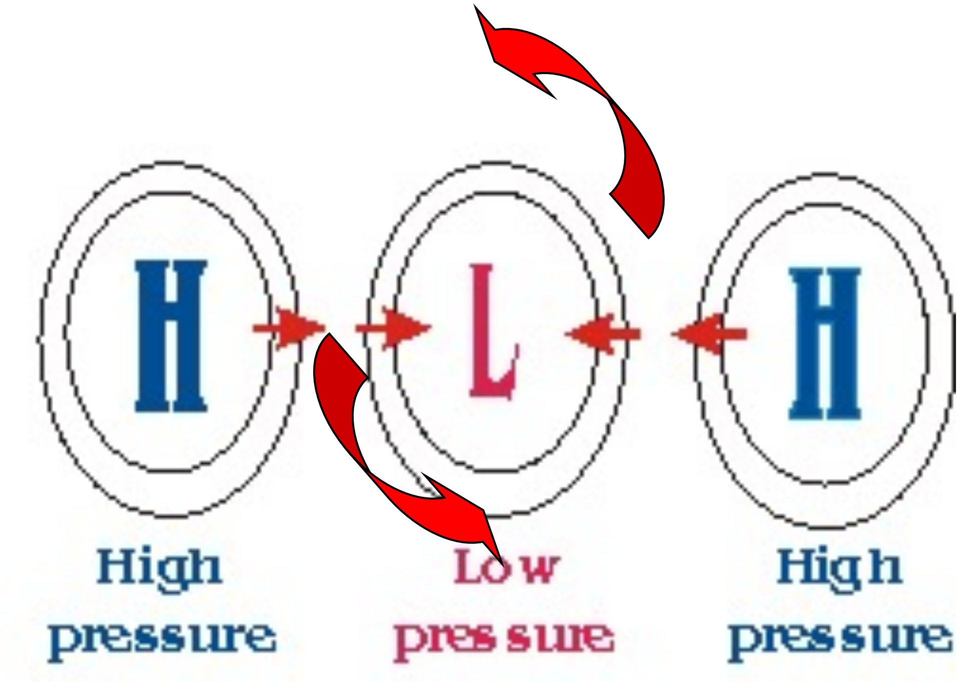
- Fictitious force only arising in a rotating frame of reference
- Is directed 90º to the right (left) of the wind in the northern (southern) hemisphere
- Increases in proportion to the wind speed
- Increases with latitude, is zero at the equator.
- Does not change the wind speed, only the wind direction. Why?

# *Our approximated momentum equation so far*



# *Highs and Lows*

In Northern Hemisphere velocity is deflected to the right by the Coriolis force




Motion initiated by pressure gradient

← →

Opposed by viscosity

# *Class exercise: Coriolis force*

Suppose a ballistic missile is fired due eastward at 43° N latitude (assume $f \approx 10^{-4}$ $s^{-1}$ at 43° N ). If the missile travels 1000 km at a horizontal speed $u_0$ = 1000 m/s, by how much is the missile deflected from its eastward path by the Coriolis force?

Coriolis forces: