AOSS 321, Winter 2009
Earth System Dynamics

Lecture 11
2/12/2009

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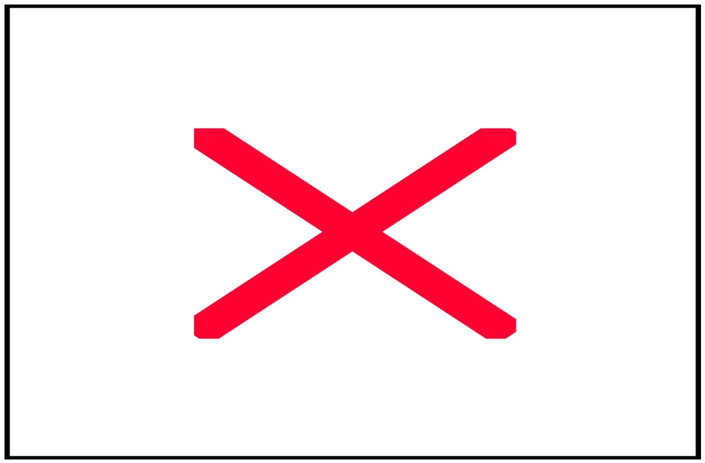
Today's lecture

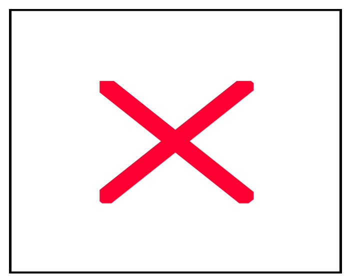
- Derivation of the potential temperature equation (Poisson equation)
- Dry adiabatic lapse rate
- Static stability, buoyancy oscillations
- Derivation of the Brunt-Väisälä frequency

Thermodynamic equation

(Divide by T)

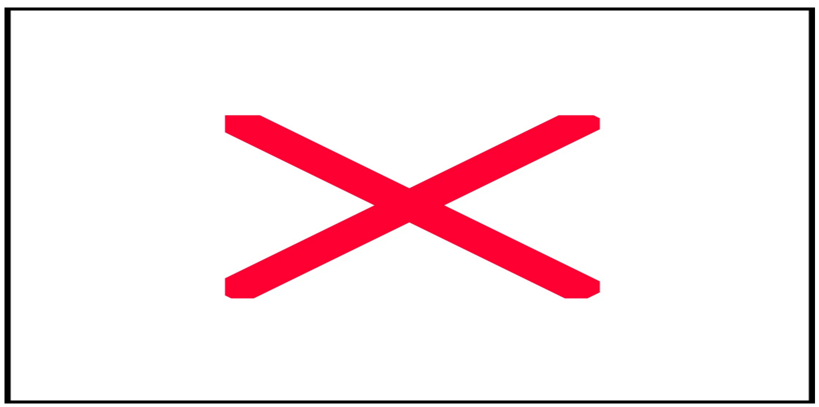


Use equation of state (ideal gas law)



Thermodynamic equation

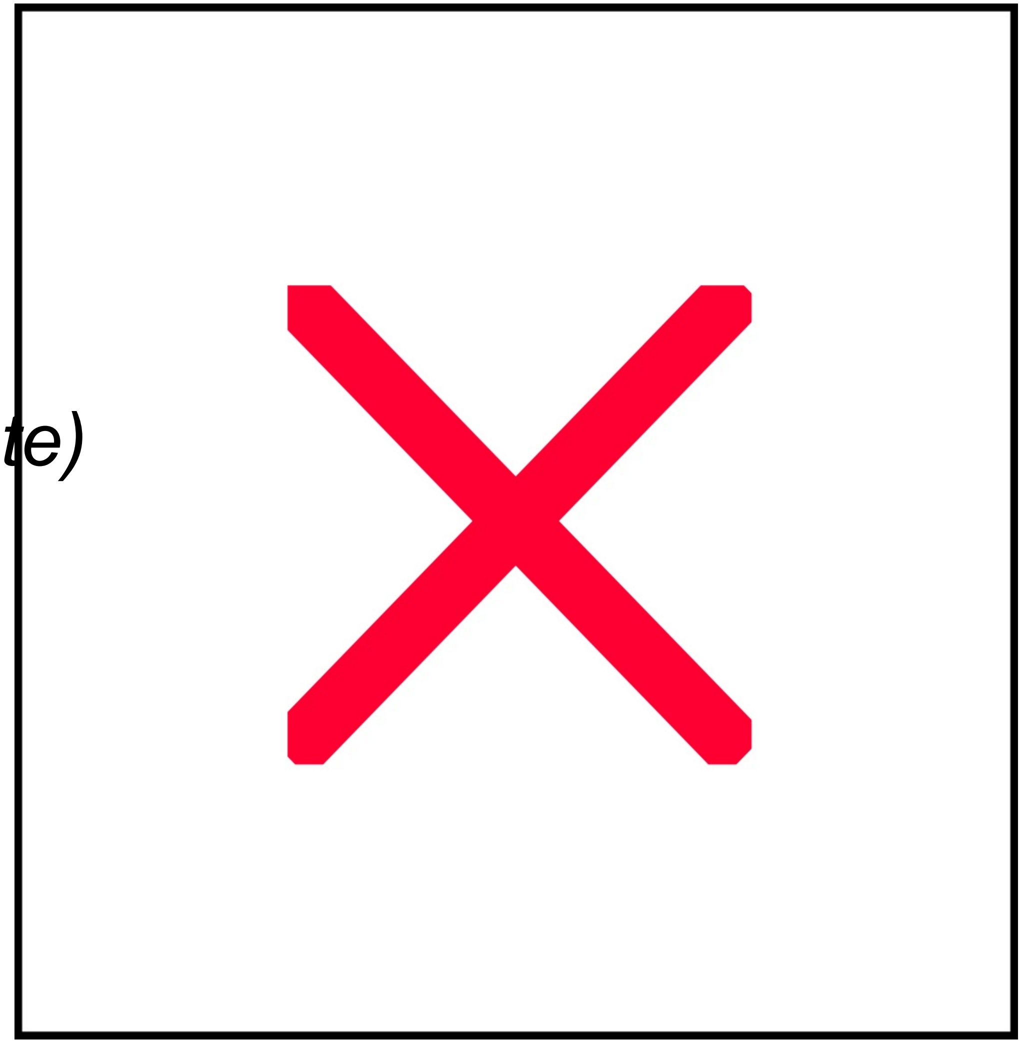
For conservative motions
*(**no heating, dry adiabatic: J = 0**):*



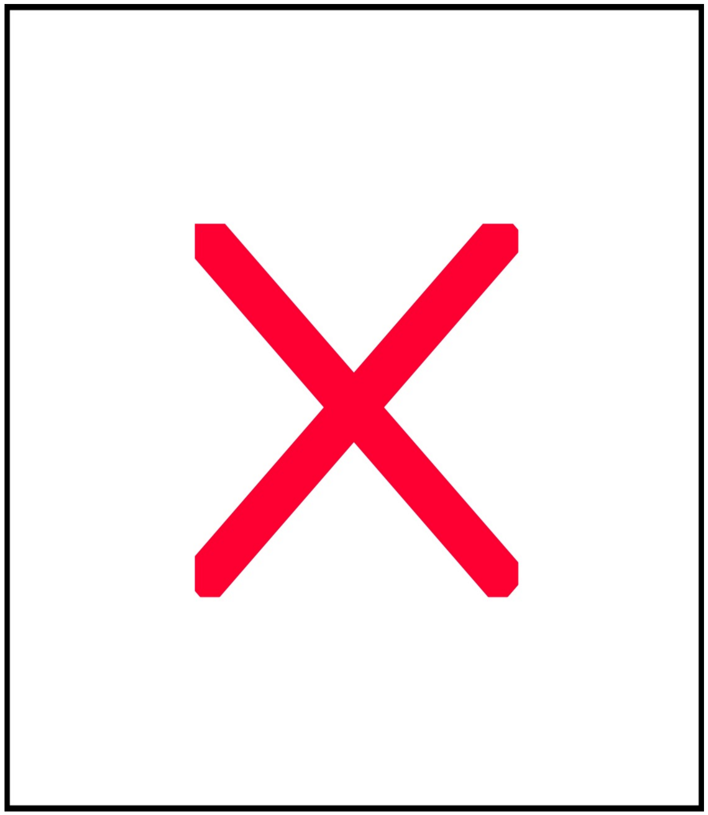
Derivation of Poisson's Equation (1)

(integrate over Dt)

(integrate)



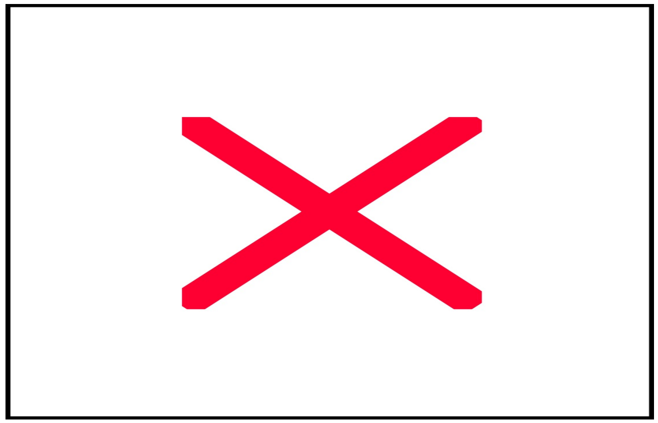
Derivation of Poisson's Equation (2)



Poisson's Equation

θ is called potential temperature!

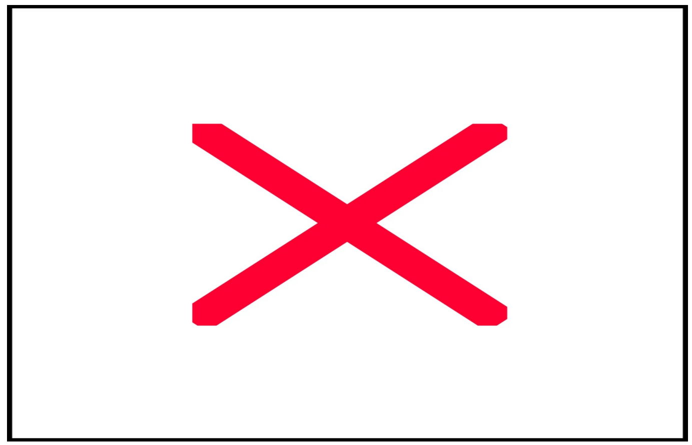
Definition of the potential temperature θ



with p_0 usually taken to be constant with
p_0 = 1000 hPa

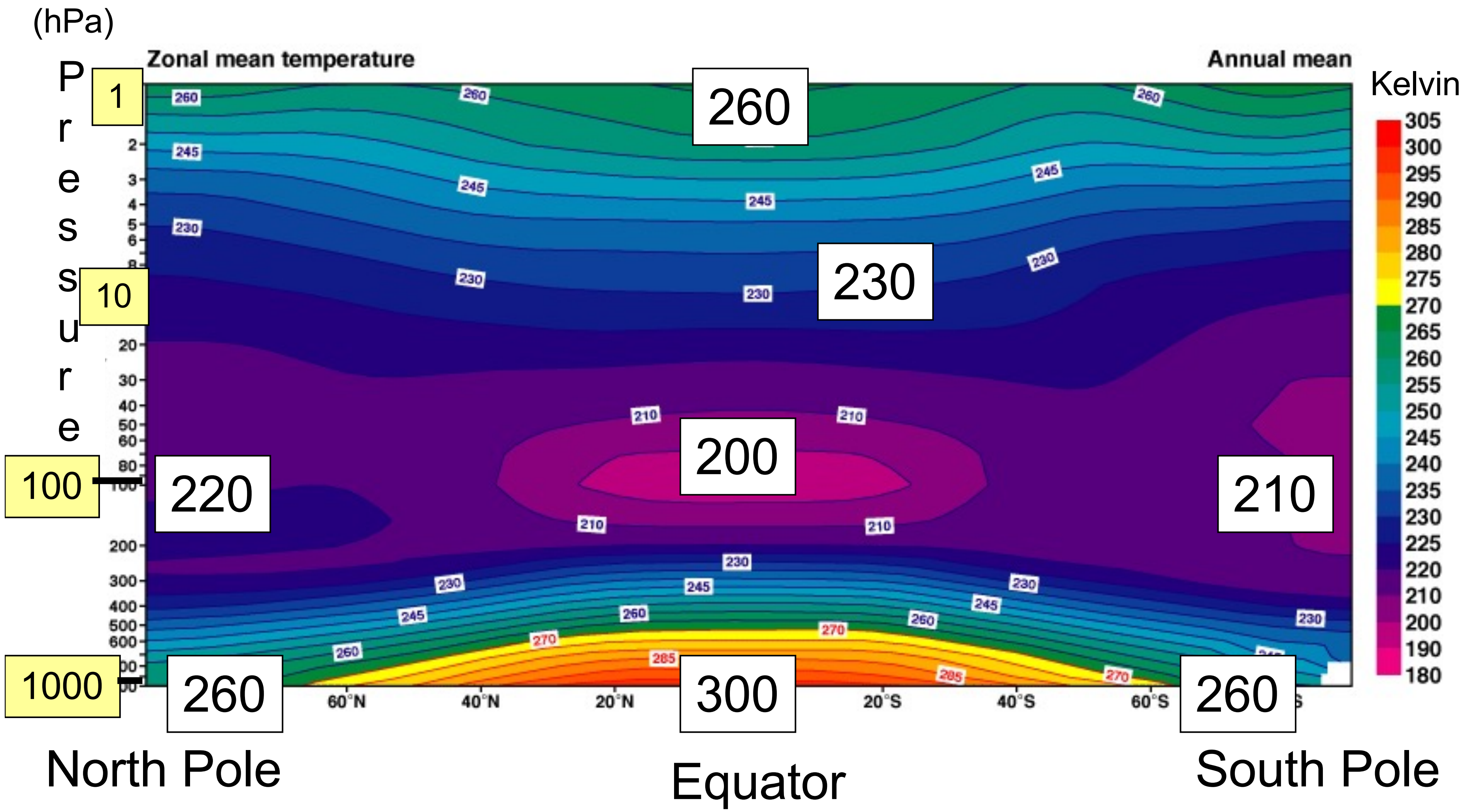
The potential temperature is the temperature a parcel would have if it was moved from some pressure level and temperature down to the surface.

Definition of potential temperature



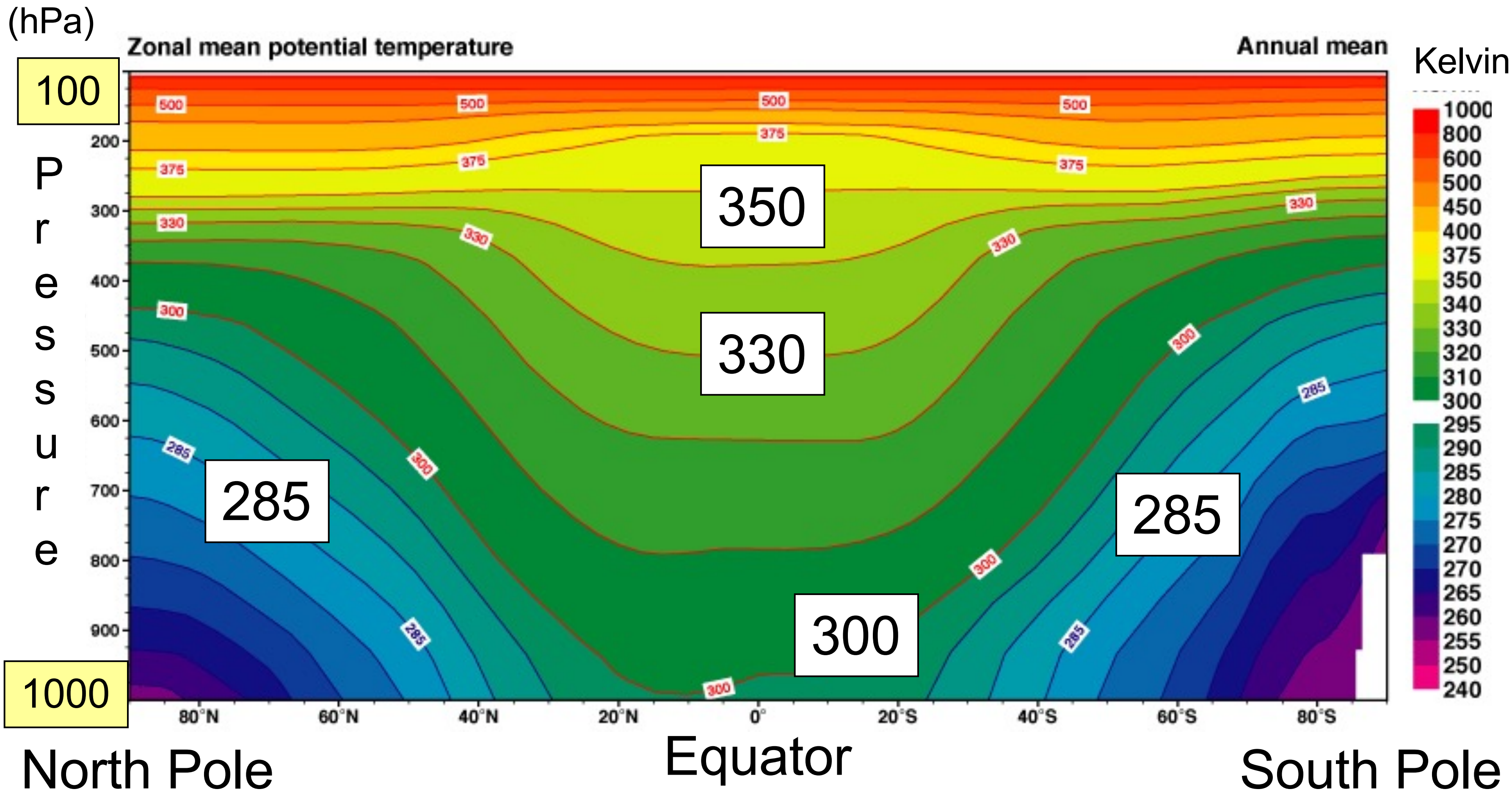
Does it makes sense that the temperature T would change in this problem? We did it **adiabatically.** There was **no source and sink of energy.**

Annual mean zonal mean temperature T



Source: ECMWF, ERA40

Annual mean zonal mean potential temperature θ

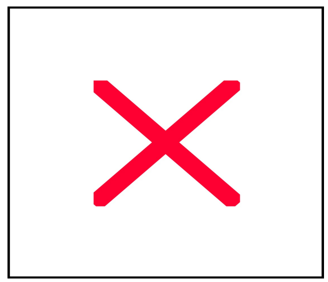


How does the temperature field look?

Source: ECMWF, ERA40

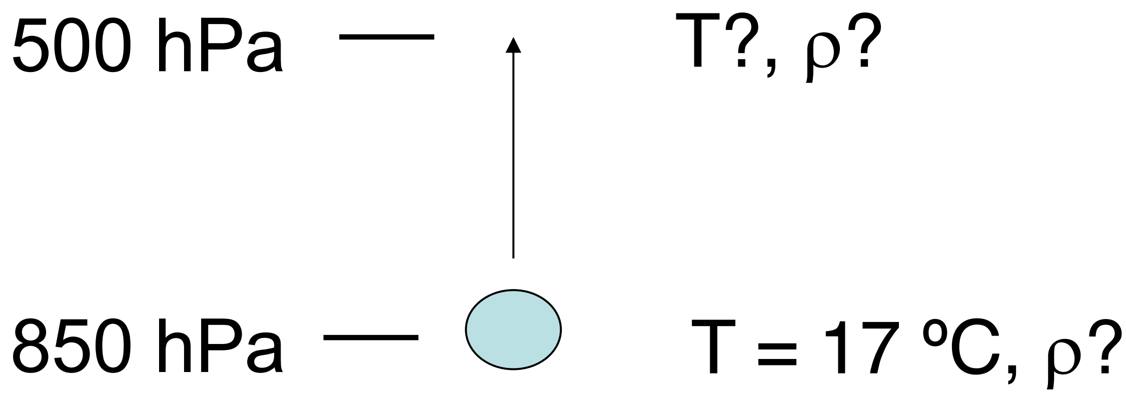
Dry adiabatic lapse rate

- For a dry adiabatic, hydrostatic atmosphere the potential temperature θ does not vary in the vertical direction:



- In a dry adiabatic, hydrostatic atmosphere the temperature T must therefore decrease with height.

Class exercise



- An air parcel that has a temperature of 17 ºC at the 850 hPa pressure level is lifted dry adiabatically.
  What is the temperature and density of the parcel when it reaches the 500 hPa level?

Dry adiabatic lapse rate: Derivation

Start with Poisson equation:

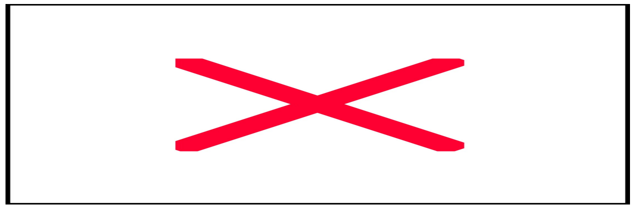
Take the logarithm of θ:

Differentiate with respect to height

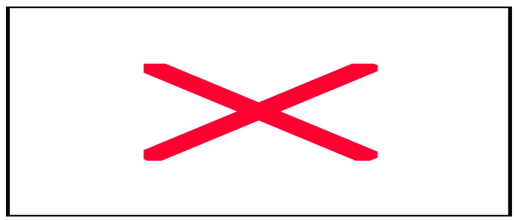
= 0 (p_0 constant)

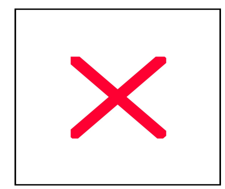
Dry adiabatic lapse rate

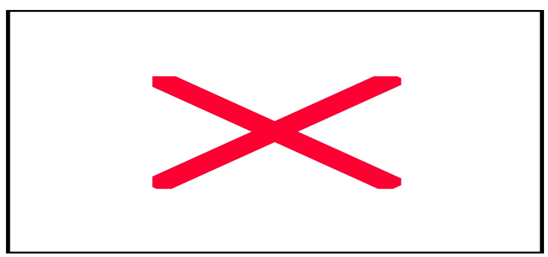
Use hydrostatic equation



Plug in ideal gas law for p, then multiply by T:



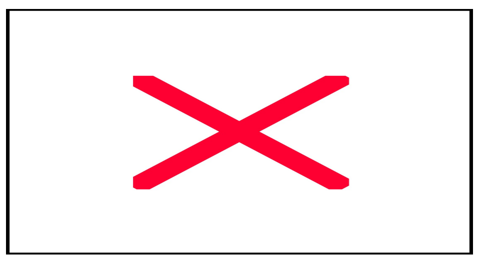
For dry adiabatic, hydrostatic atmosphere with 



Γ_d: **dry adiabatic lapse rate** (approx. 9.8 K/km)

Static stability

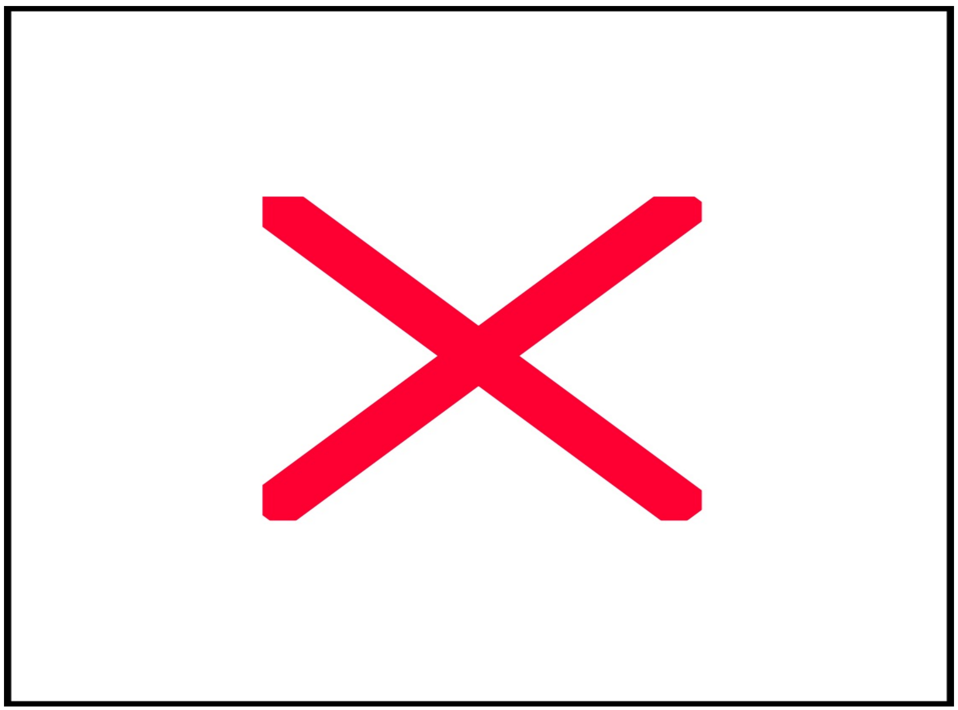
- We will now assess the static stability characteristics of the atmosphere.
- Static stability of the environment can be measured with the buoyancy frequency N.
- N is also called Brunt-Väisälä frequency.
  The square of this buoyancy frequency is defined as



We will derive this equation momentarily, but first let's discuss some static stability/instability conditions.

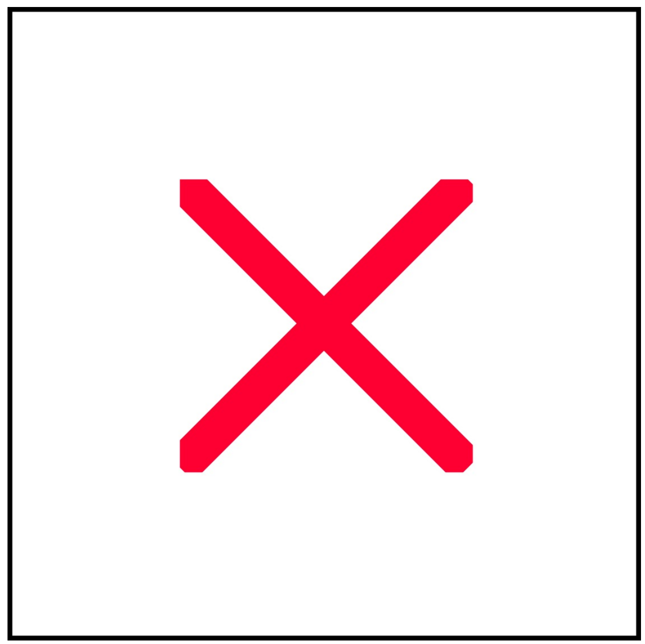
Static stability

We will now assess the static stability characteristics of the atmosphere.



Stable and unstable situations

Check out this marble in a bowl:
http://eo.ucar.edu/webweather/stablebowl.html



Stable and unstable air masses

Stable air:
A rising parcel that is cooler than the surrounding atmosphere will tend to sink back to its original position (why?).

Unstable air:
A rising parcel that is warmer than the surrounding atmosphere will continue to rise (why?).

Neutral air:
The parcel remains at the new location after being displaced, its temperature varies exactly as the temperature of the surrounding atmosphere.

Unstable air

Unstable air: makes thunderstorms possible.
Here: visible since clouds rise to high elevations!

Stable air

Stable air: makes oscillations (waves) in the atmosphere possible, visible due to the clouds!

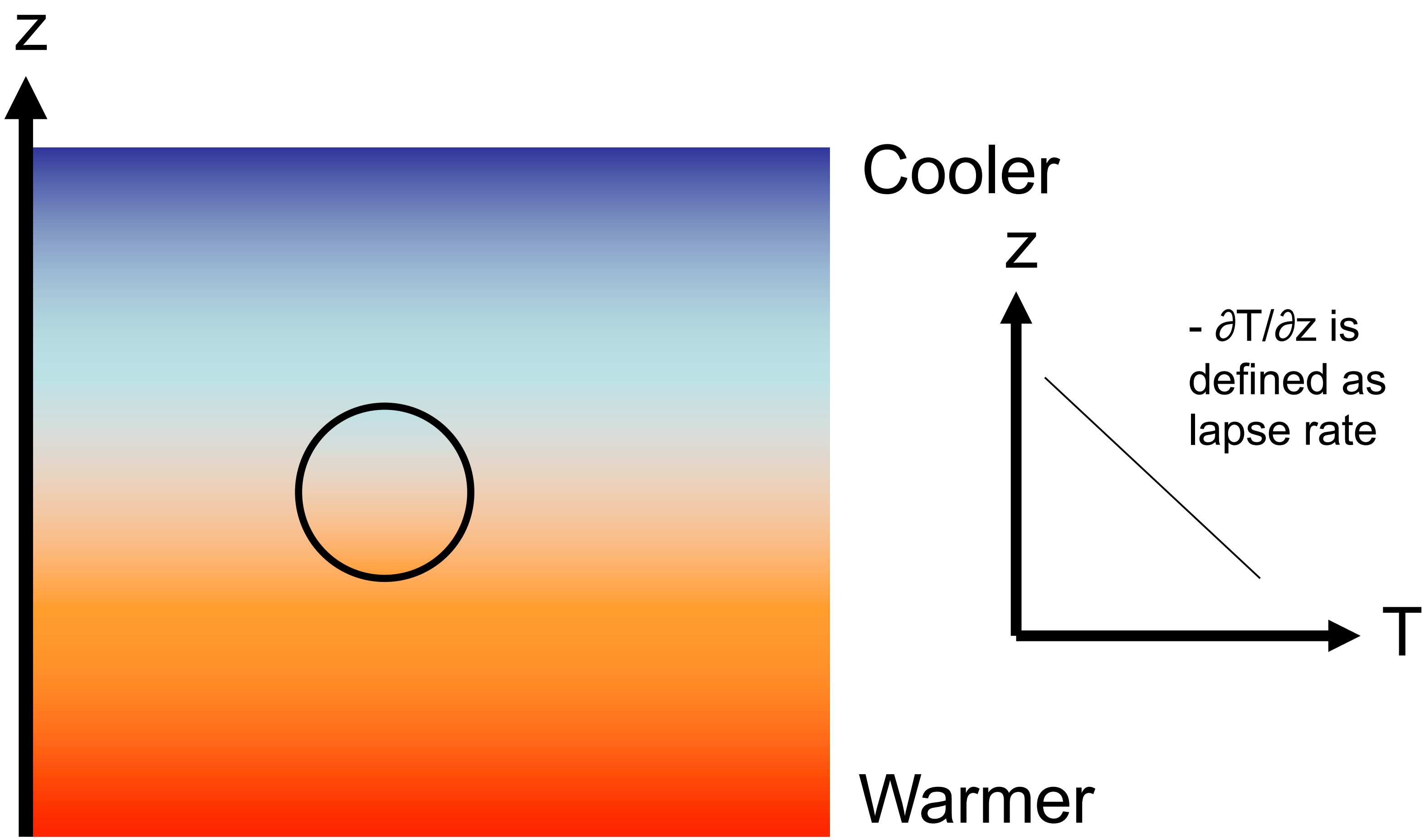
Stable air

Stable air: makes oscillations (waves) in the atmosphere possible, what is the wave length?

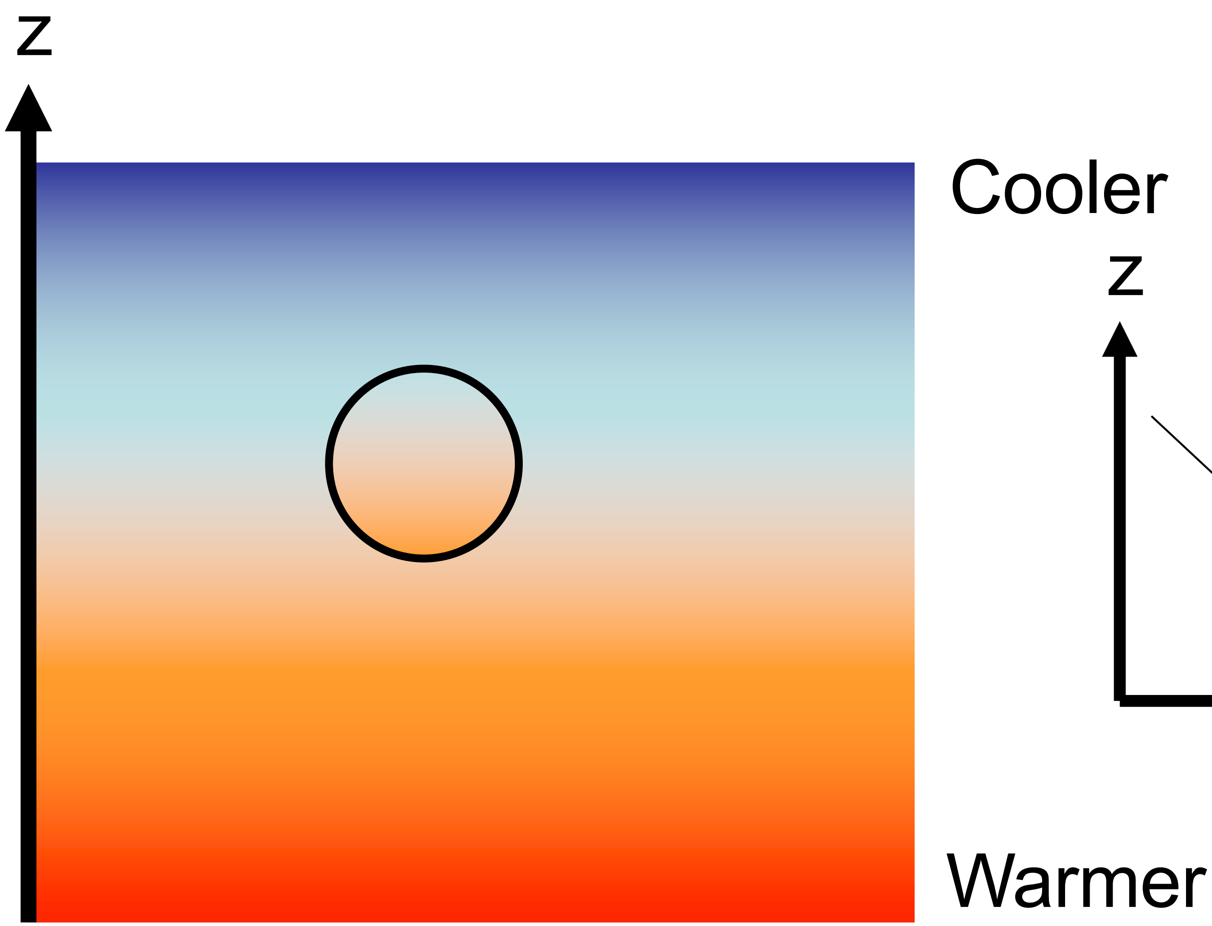
Stable air

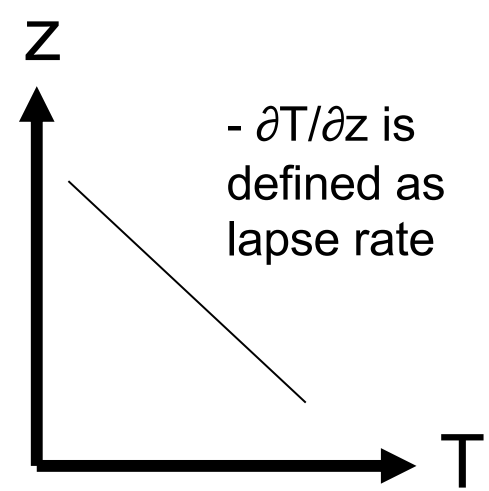
Stable air: temperature inversions suppress rising motions. Here: stratiform clouds have formed.

Let's take a closer look: Temperature as function of height



Let's take a closer look: Temperature as function of height





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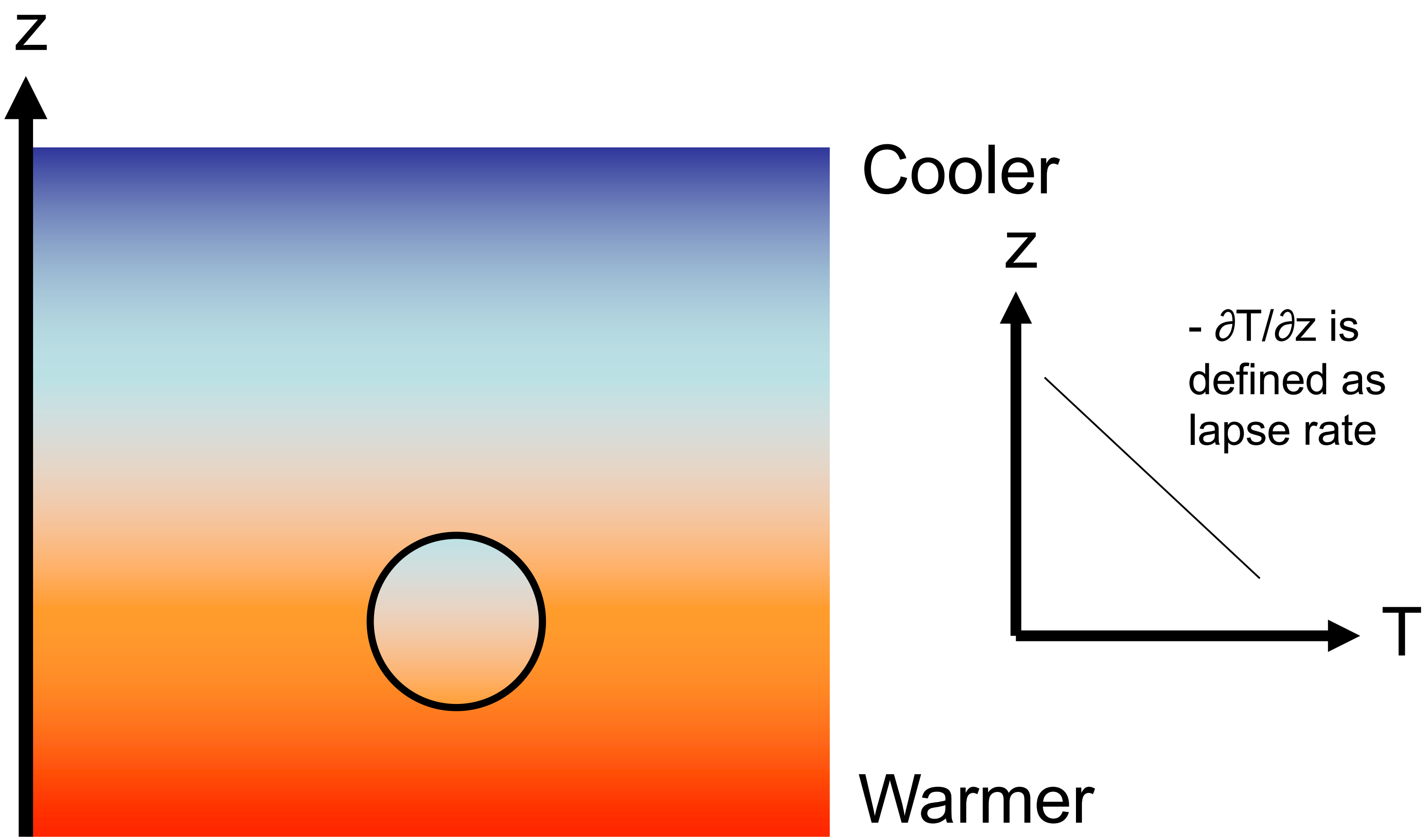
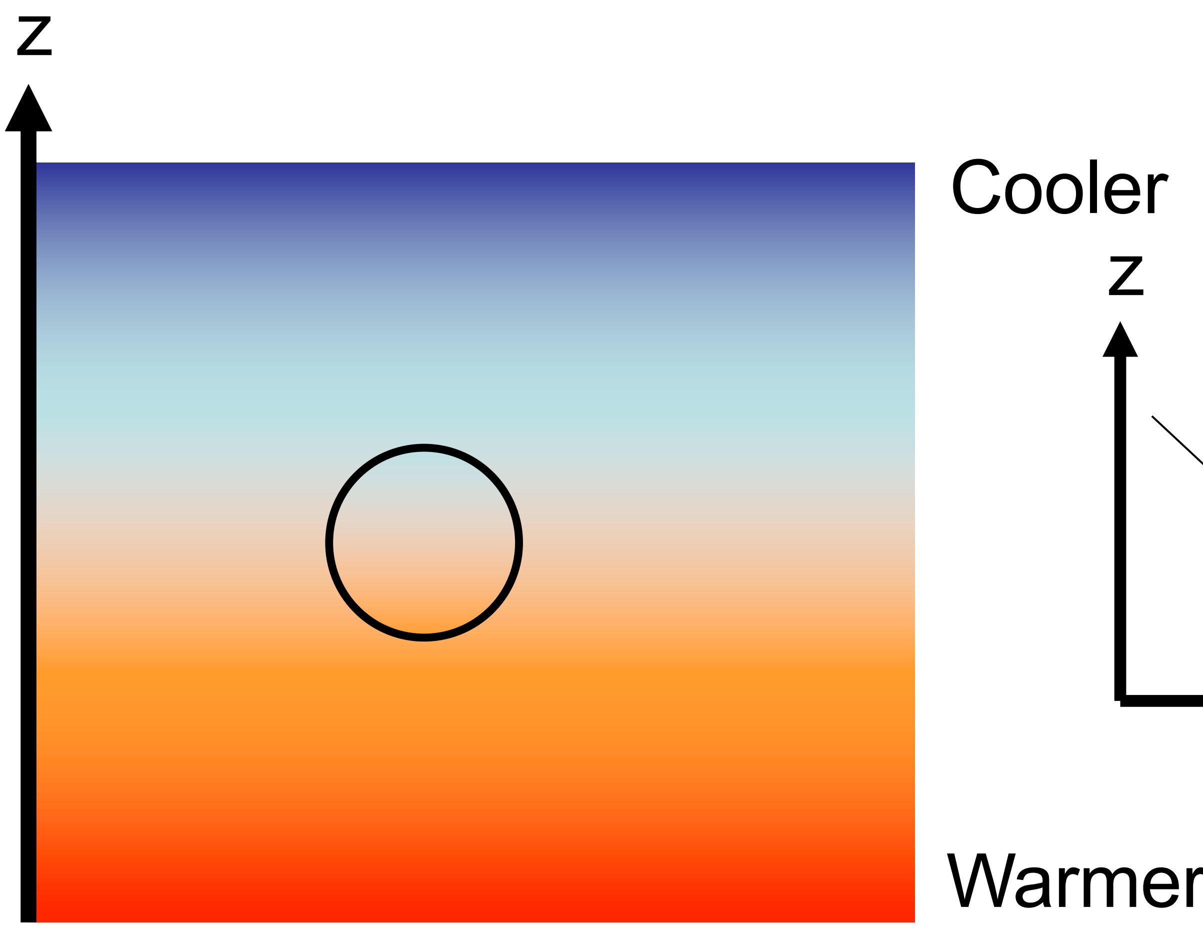
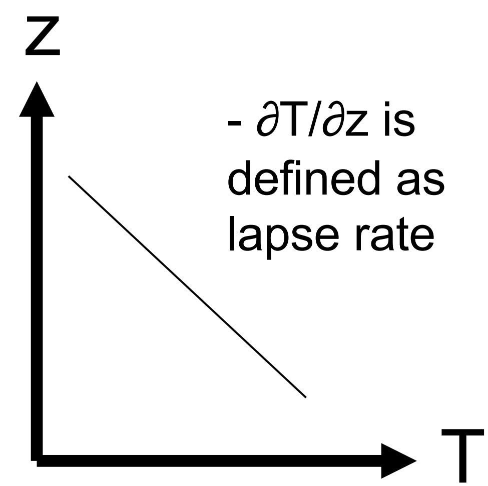


- ∂T/∂z is defined as lapse rate

Let's take a closer look: Temperature as function of height

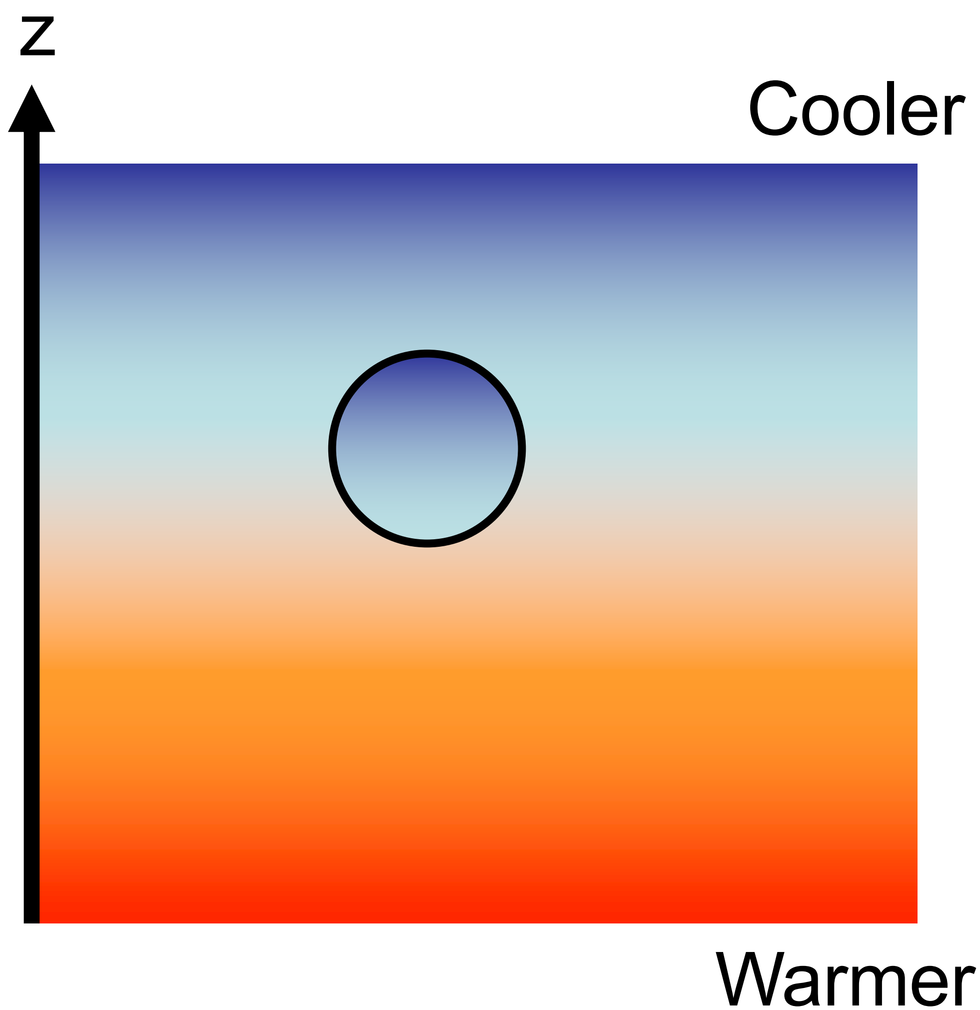




The parcel method

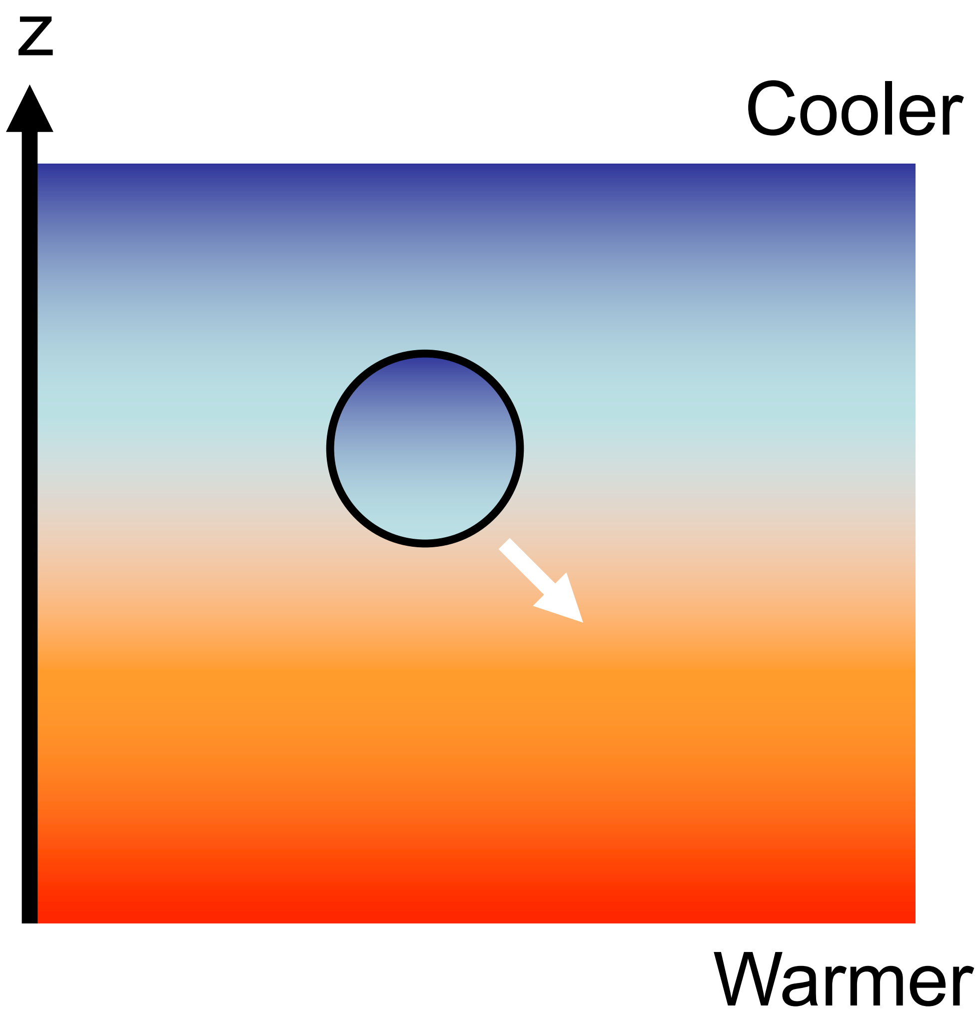
- We are going displace this parcel – move it up and down.
  - We are going to assume that the pressure adjusts instantaneously; that is, the parcel assumes the pressure of altitude to which it is displaced.
  - As the parcel is moved its temperature will change according to the adiabatic lapse rate. That is, the motion is without the addition or subtraction of energy. J is zero in the thermodynamic equation.

Parcel cooler than environment



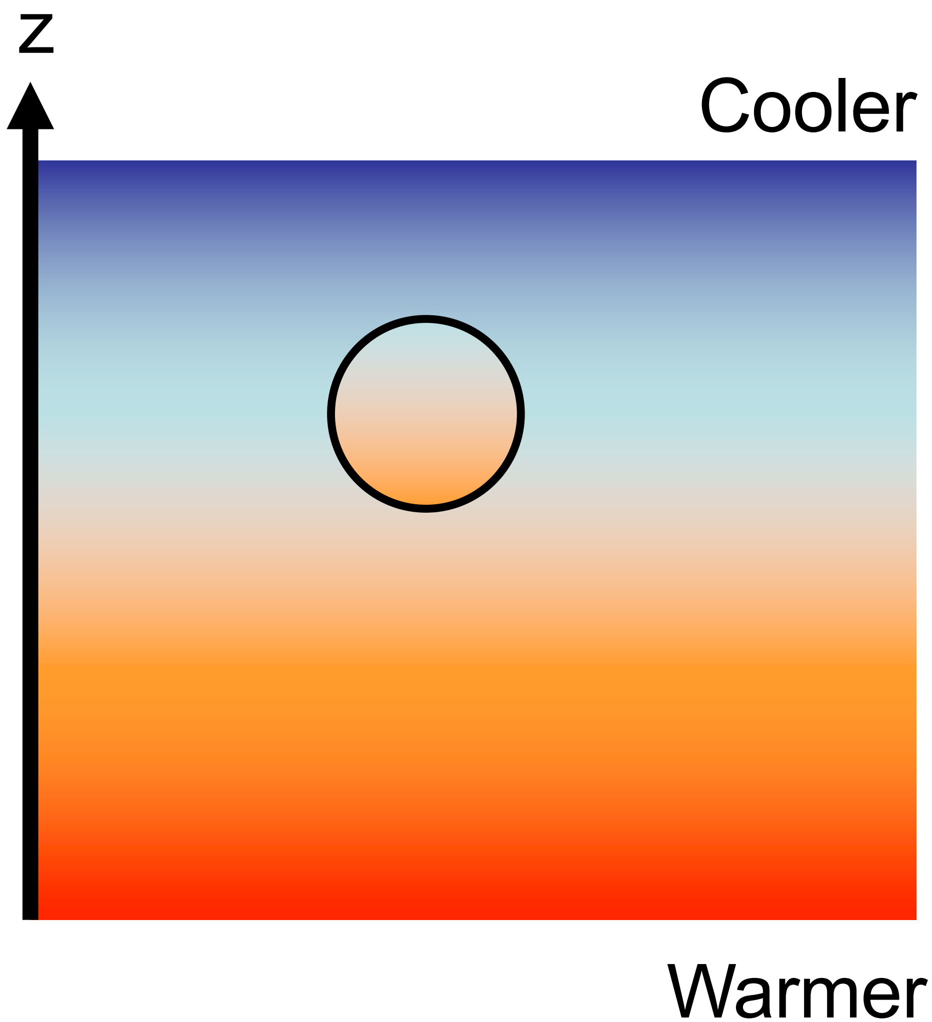
If the parcel moves up and finds itself cooler than the environment then it will sink. (What is its density? larger or smaller?)

Parcel cooler than environment



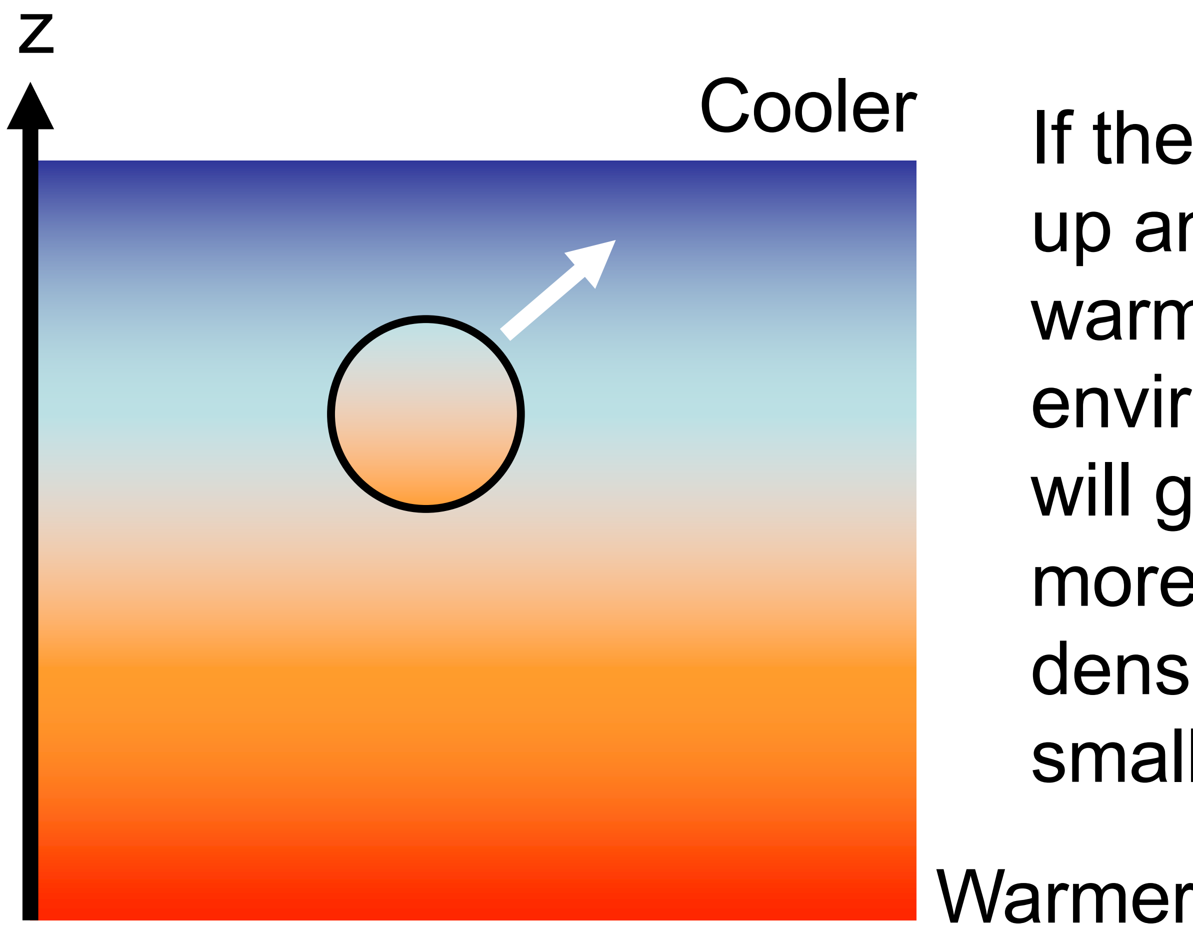
If the parcel moves up and finds itself cooler than the environment then it will sink. (What is its density? larger or smaller?)

Parcel warmer than environment



If the parcel moves up and finds itself warmer than the environment then it will go up some more. (What is its density? larger or smaller?)

Parcel warmer than environment

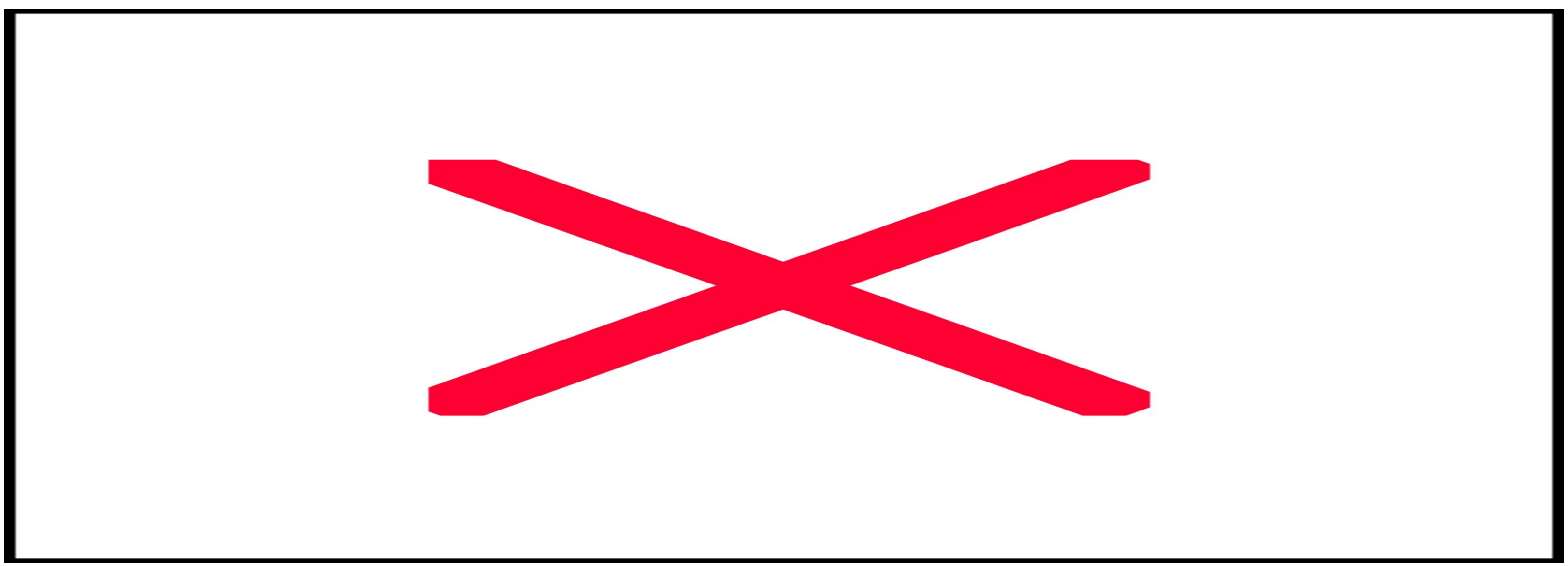


If the parcel moves up and finds itself warmer than the environment then it will go up some more. (What is its density? larger or smaller?)

This is our first example of “instability” – a perturbation that grows.

Let's quantify this:
Characteristics of the environment

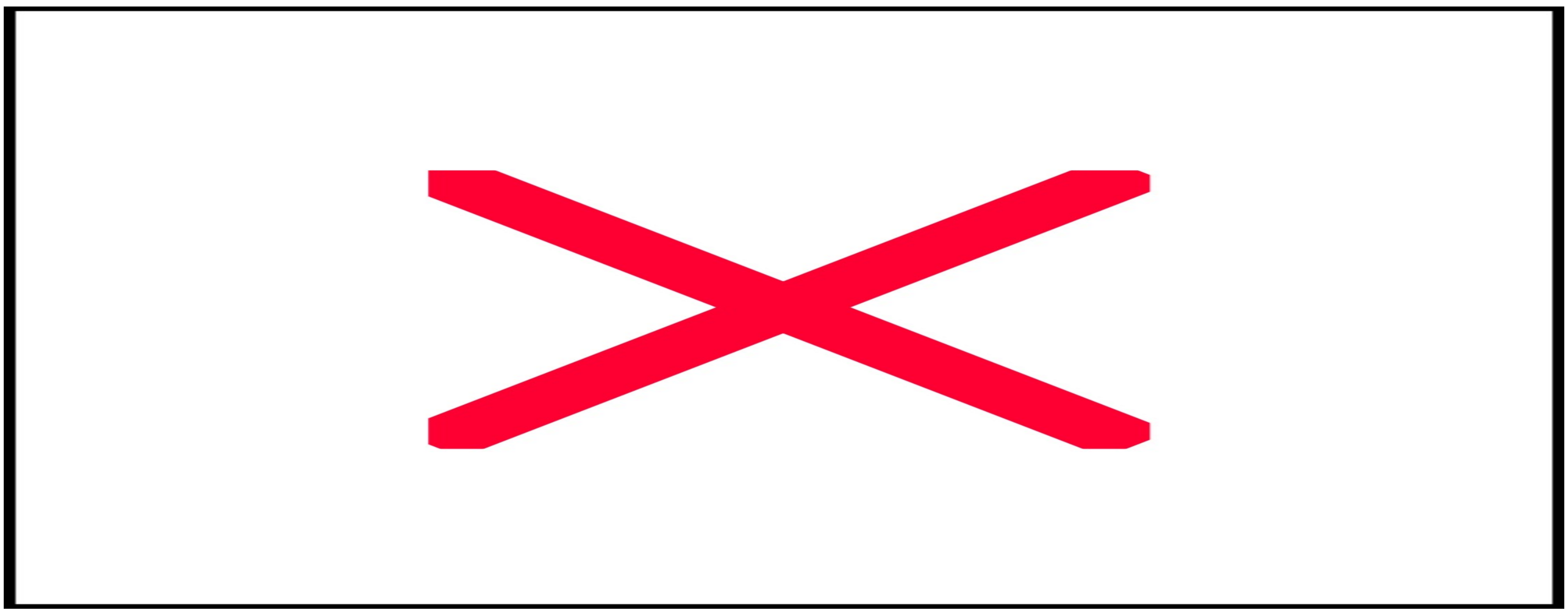
We assume that the temperature T_{env} of the environment changes with a constant linear slope (or lapse rate Γ) in the vertical direction.



T_{sfc}: temperature at the surface

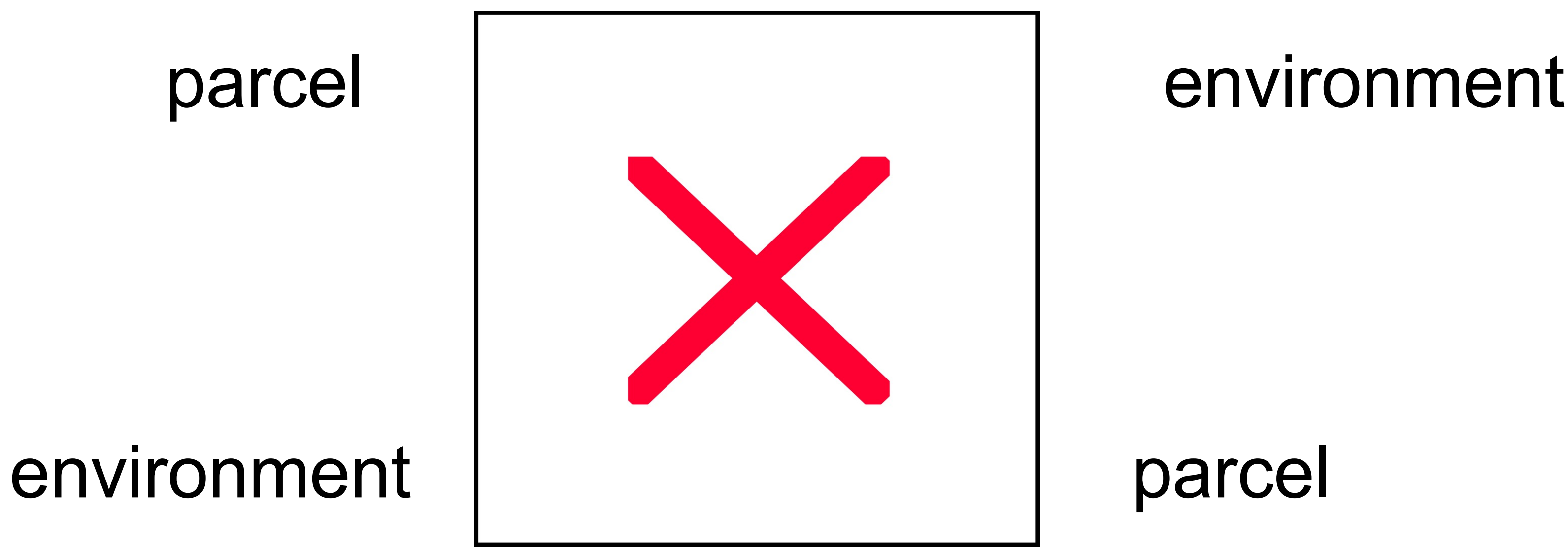
Let's quantify this:
Characteristics of the parcel

We assume that the temperature T_{parcel} of the parcel changes with the dry adiabatic lapse rate Γ_d.



Stable:

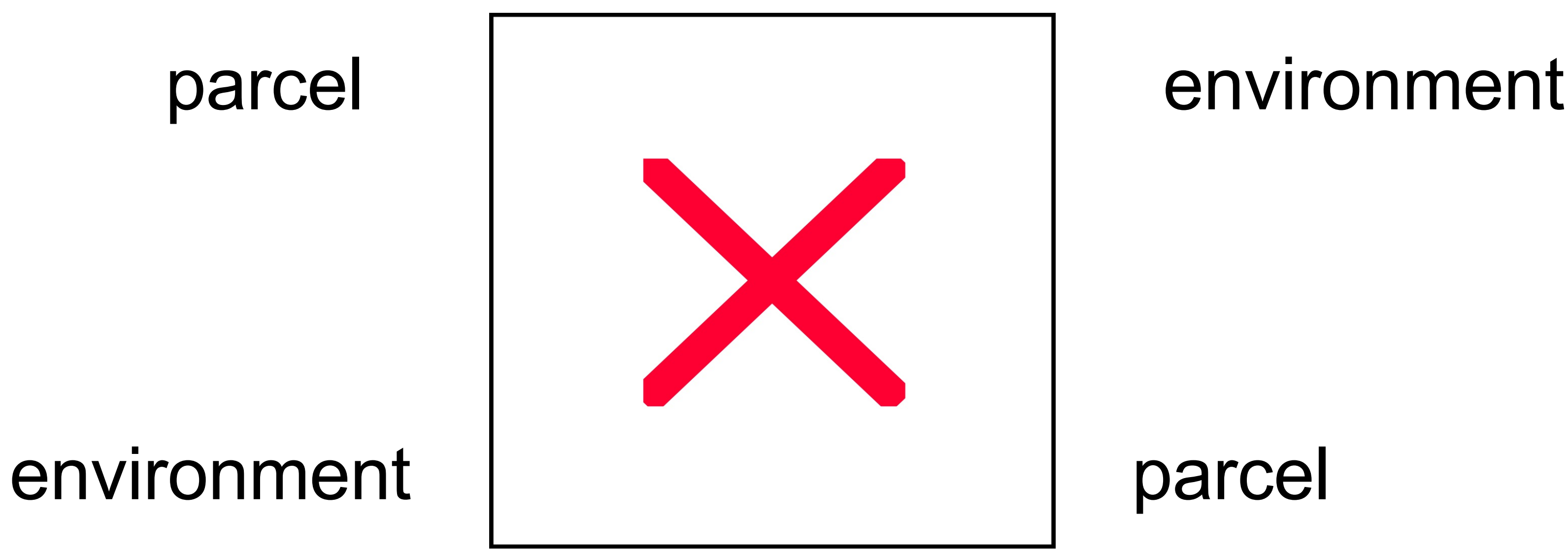
Temperature of parcel cooler than environment



compare the lapse rates

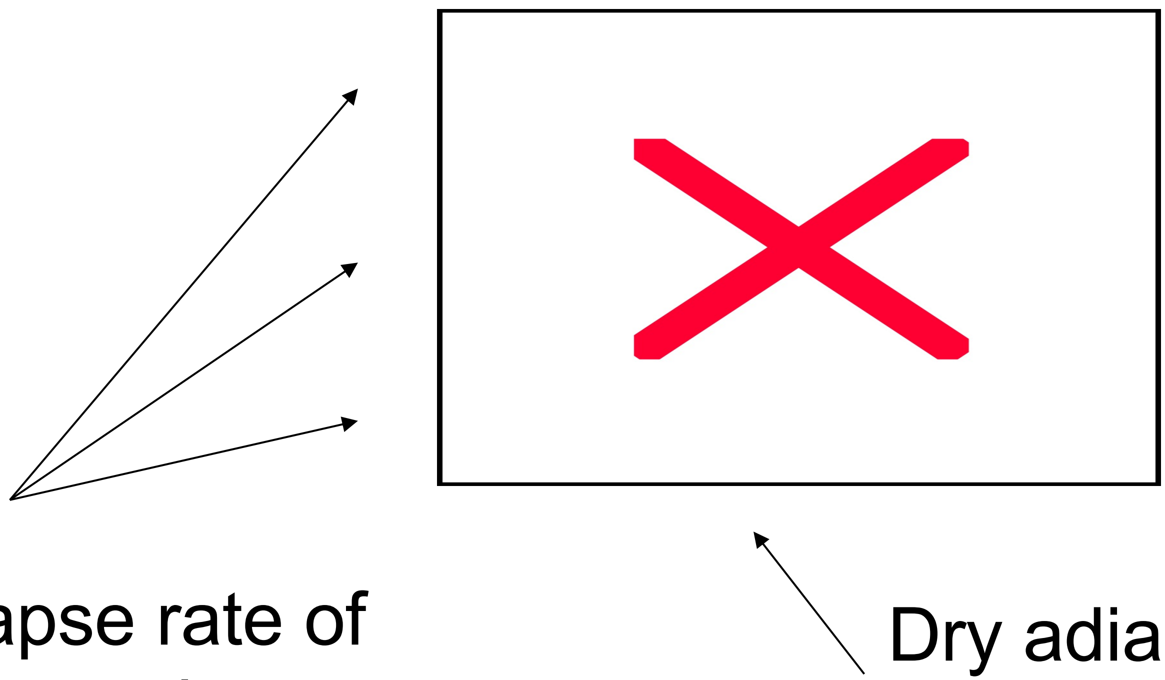
Unstable:

Temperature of parcel greater than environment.

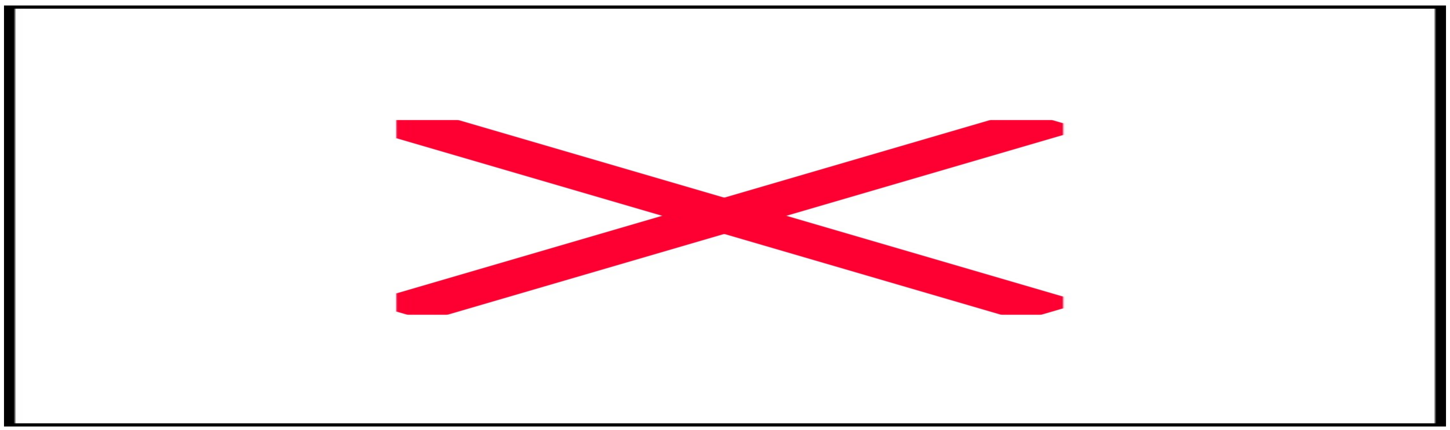


compare the lapse rates

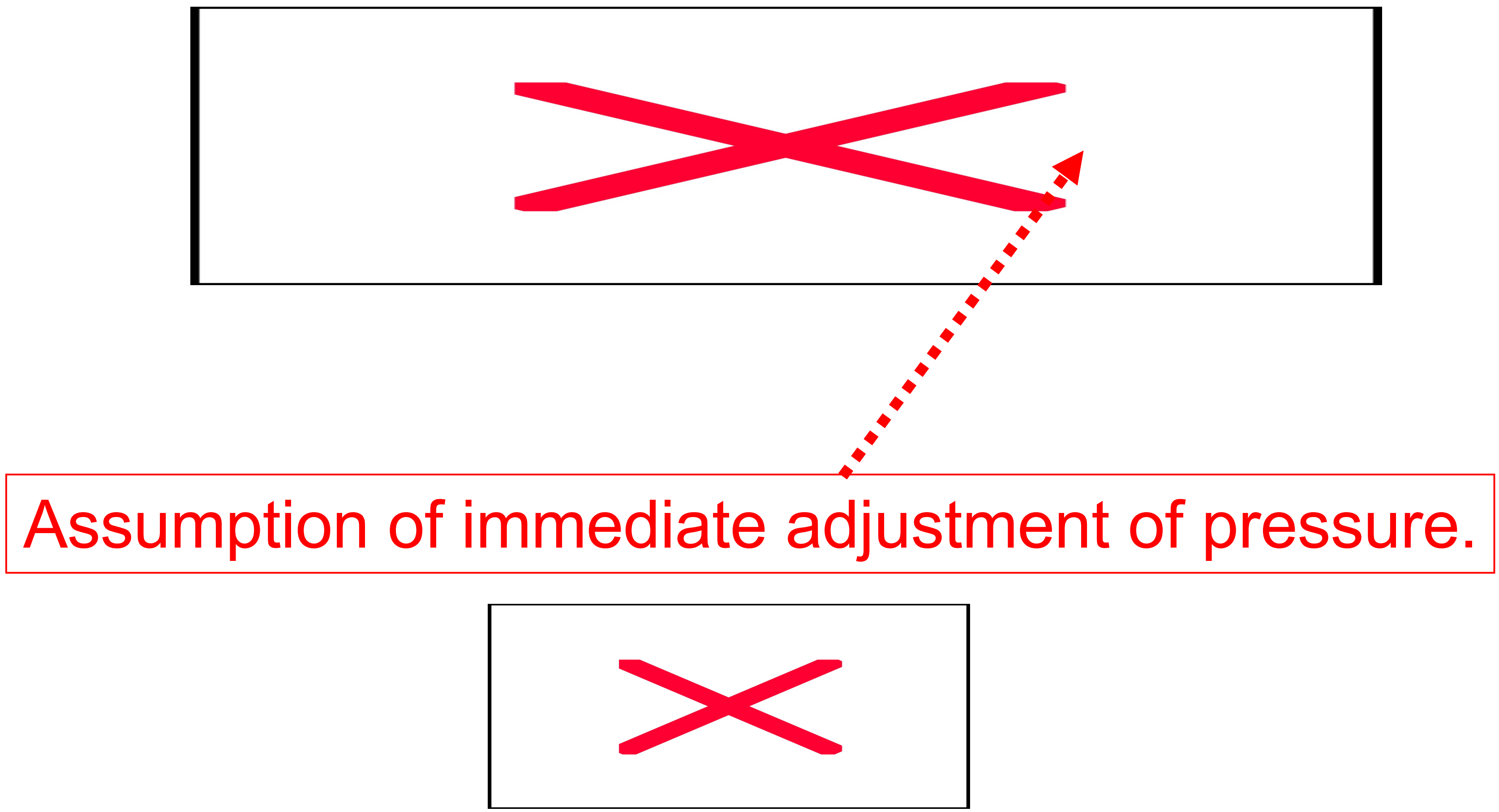
Stability criteria from physical argument



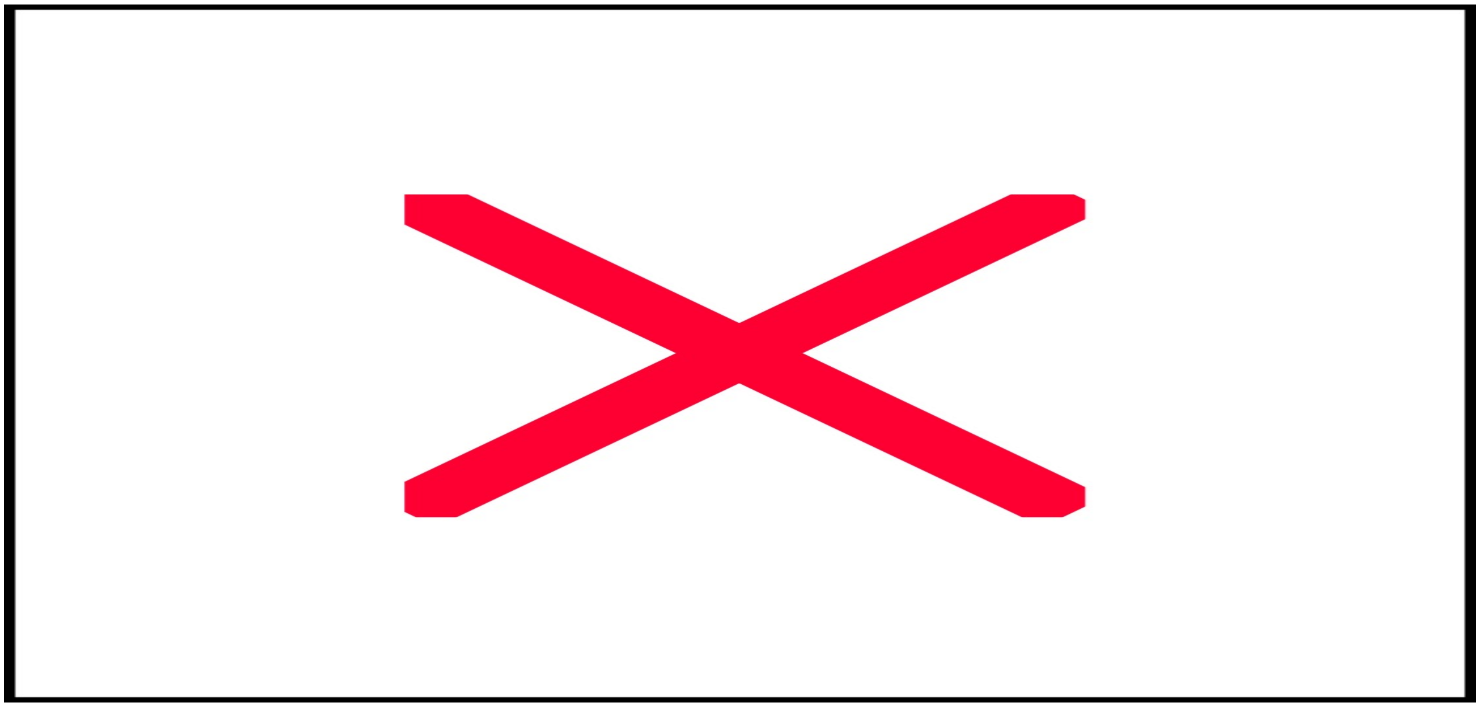
Hydrostatic balance



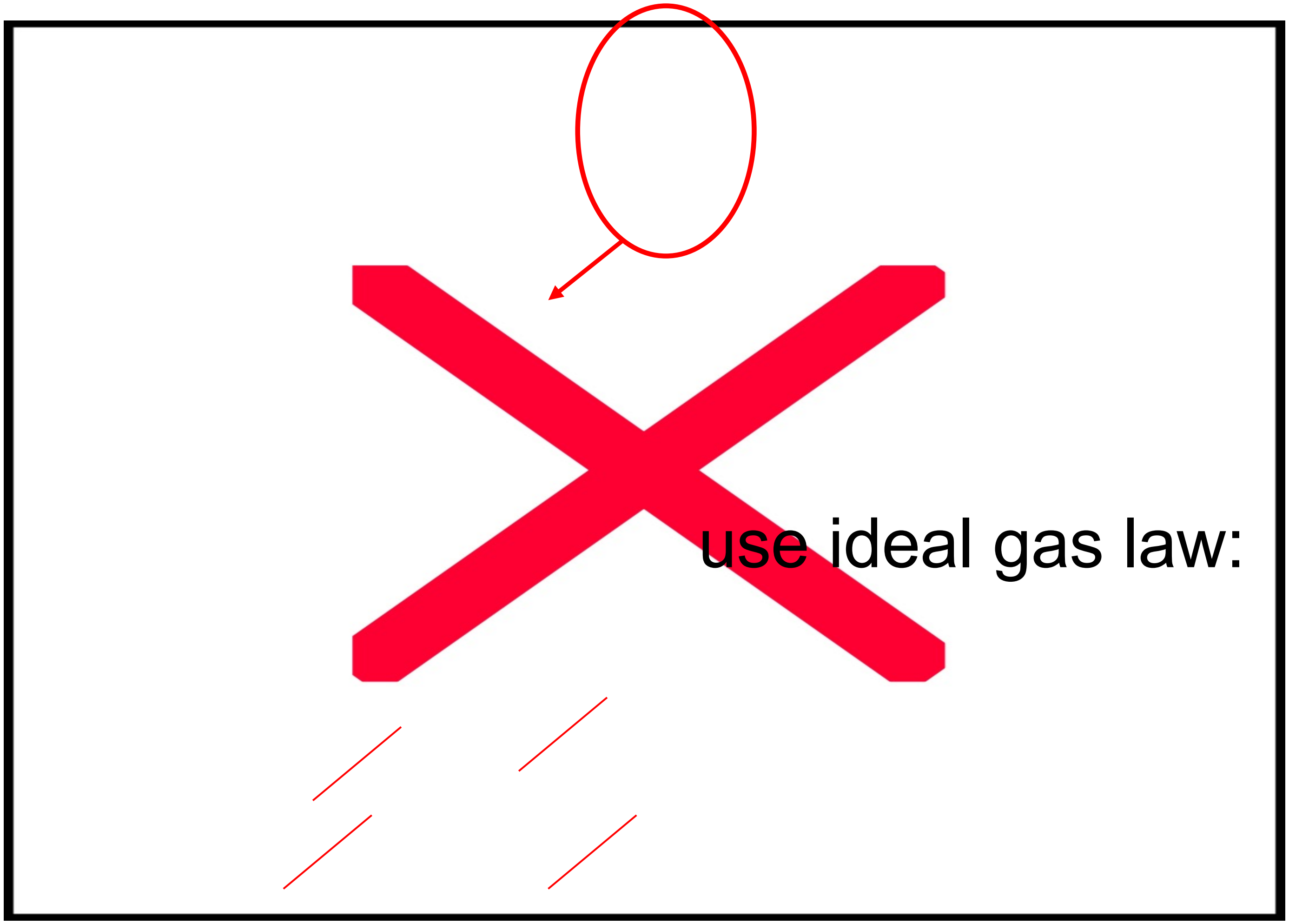
*But our **parcel** experiences an acceleration, small displacement δz*

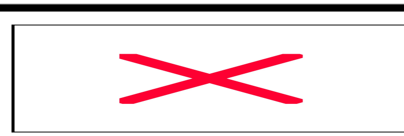


Solve for pressure gradient

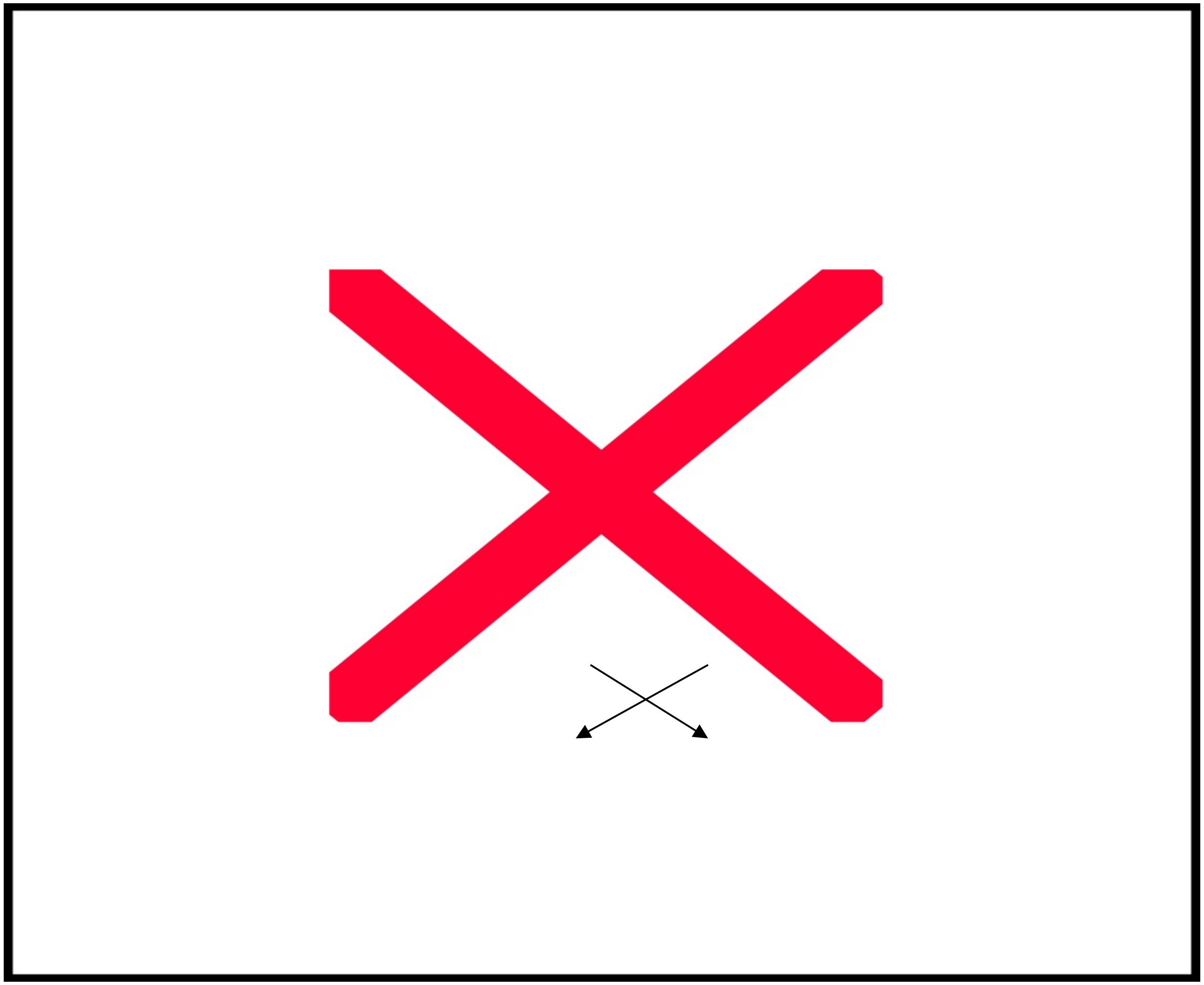


But our parcel experiences an acceleration

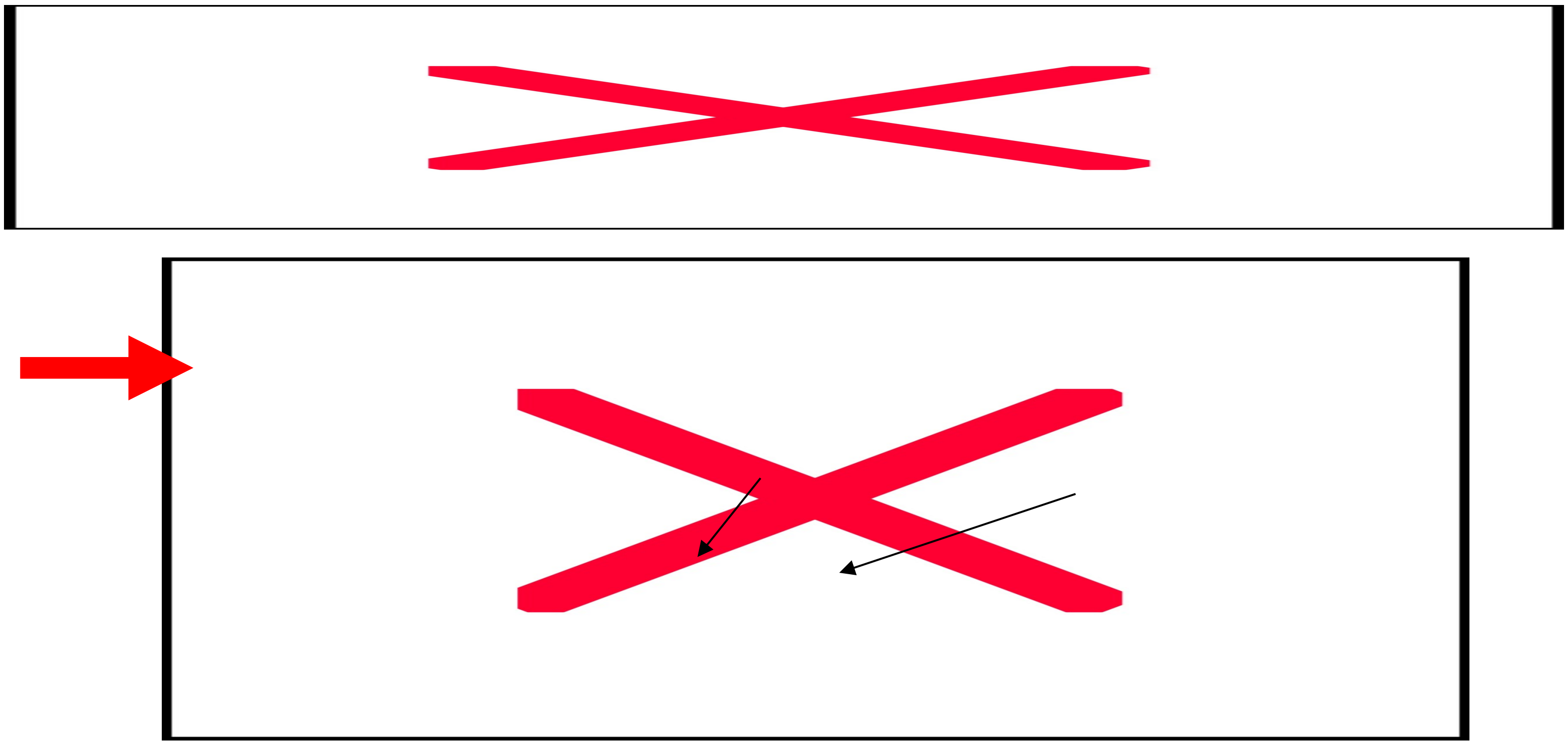


Recall: 

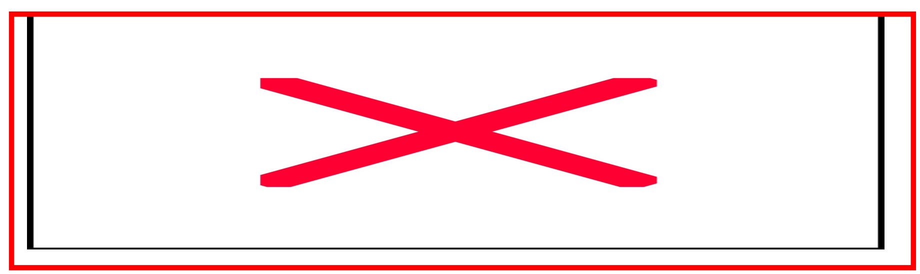
Rearrange:



Back to our definitions of temperature change

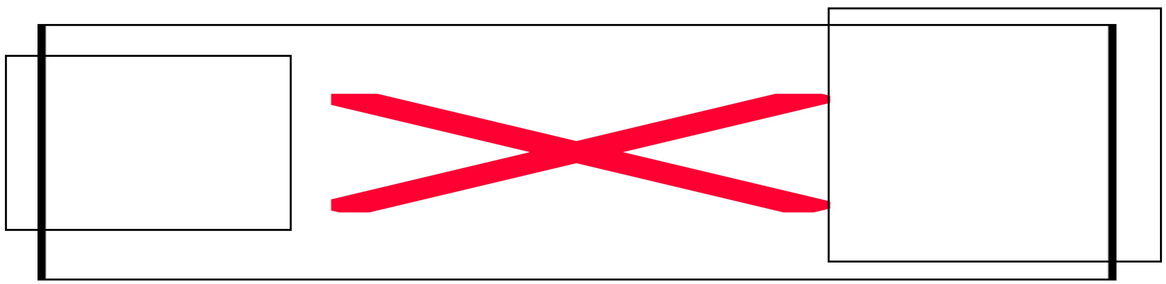


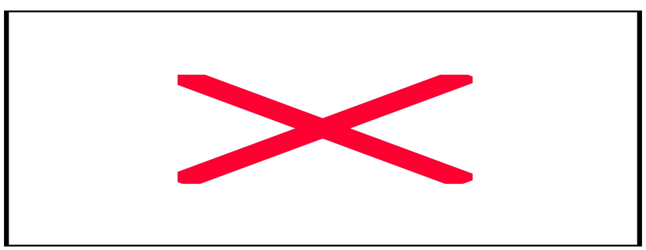
Second-order, ordinary differential equation:



Recall: Dry adiabatic lapse rate

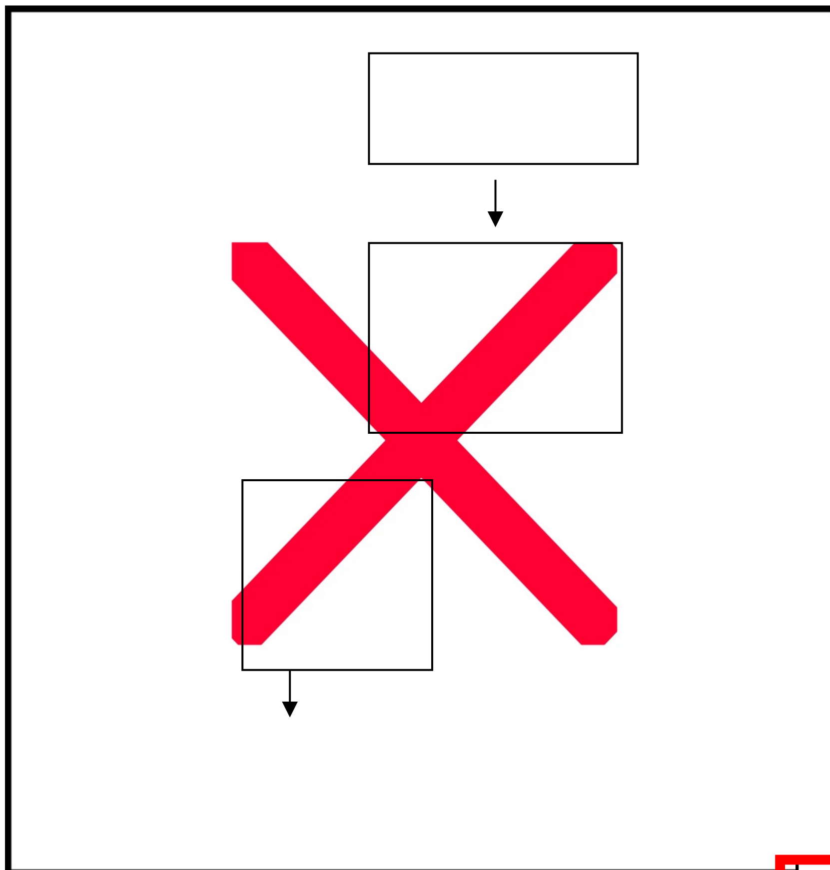
Taking the logarithm of θ, differentiating with respect to height, using the ideal gas law and hydrostatic equation gives:

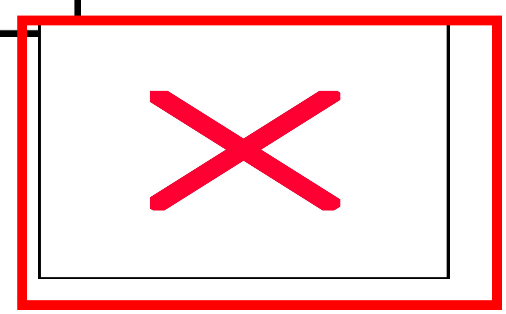




dry adiabatic lapse rate (approx. 9.8 K/km)

Rearrange:



With the **Brunt-Väisälä frequency** 

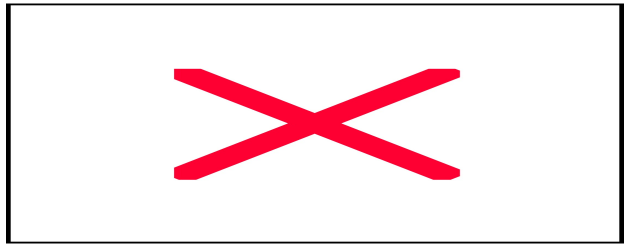
Buoyancy oscillations

1) stable, the solution to this equation describes a buoyancy oscillation with period $2\pi/N$

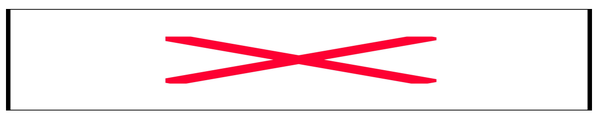
2) unstable, corresponds to growing perturbation, this is an instability

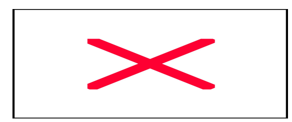
3) neutral

Solution to the differential equation



The general solution can be expressed via the exponential function with a **complex** argument:



with A: amplitude, N: buoyancy frequency, 

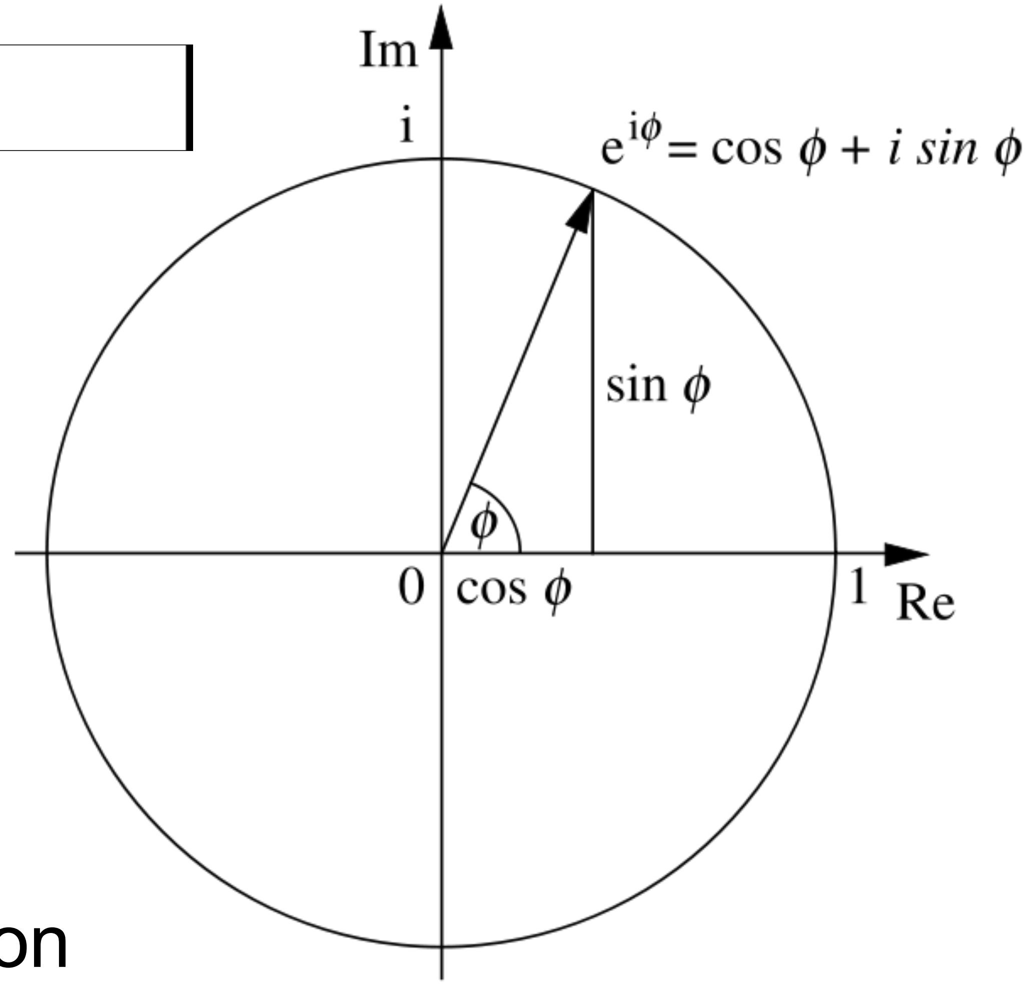
If $N^2 > 0$ the parcel will oscillate about its initial level with a **period $\tau = 2\pi/N$**.
Average N in the troposphere $N \approx 0.01\ s^{-1}$

Remember Euler's formula

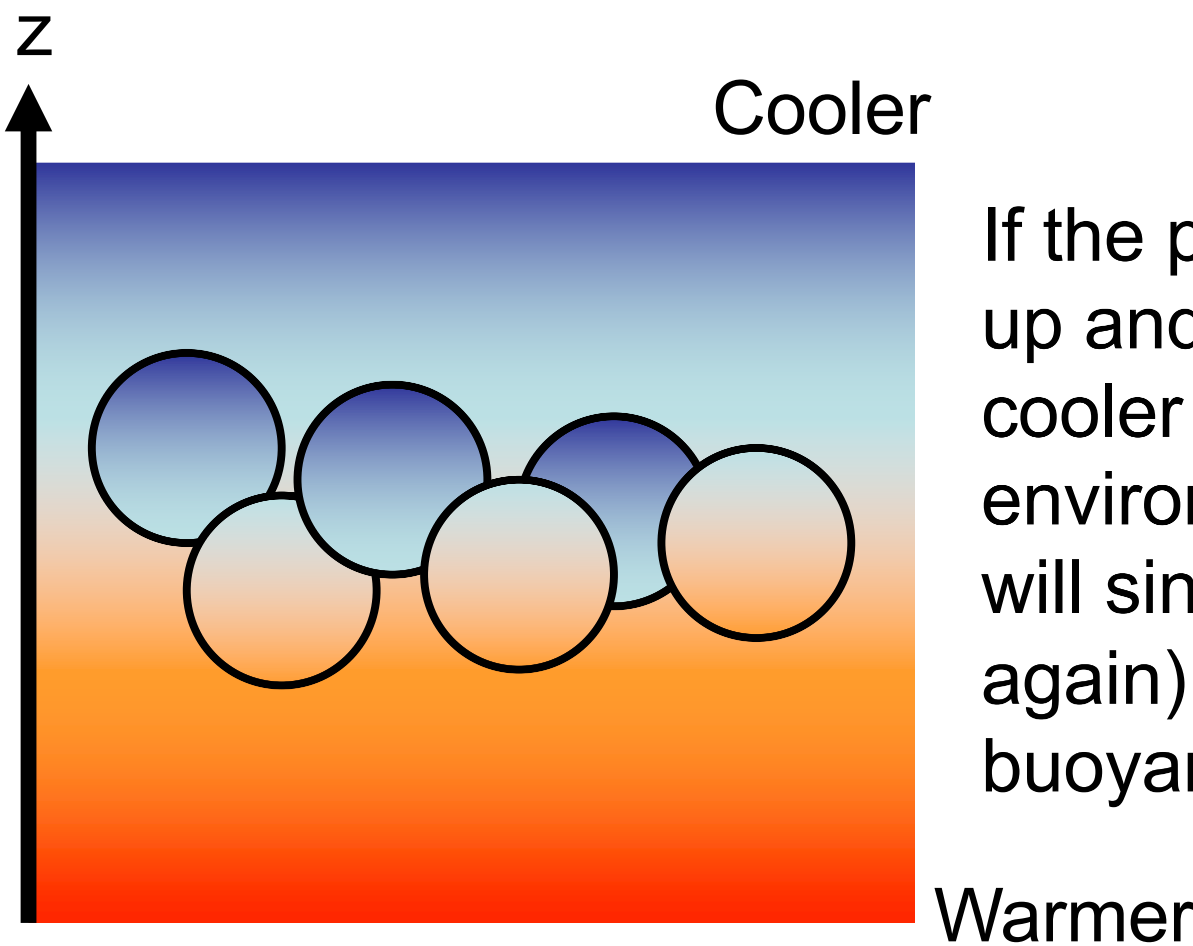
with x: real number

Physical solution:

If $N^2 > 0$ (real) the solution is a wave with period $\tau = 2\pi/N$ (more on waves in AOSS 401)



Stable solution: Parcel cooler than environment

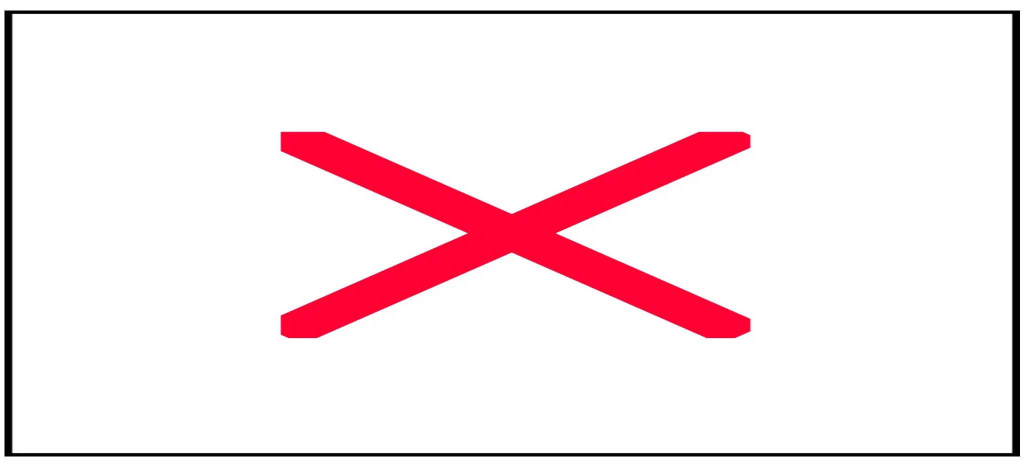


If the parcel moves up and finds itself cooler than the environment then it will sink (and rise again). This is a buoyancy oscillation.

Stable and unstable air masses

Picture an invisible box of air (an air parcel). If we compare the temperature of this air parcel to the temperature of air surrounding it, we can tell if it is stable (likely to remain in place) or unstable (likely to move).

http://eo.ucar.edu/webweather/stable.html



Temperature soundings