AOSS 321, Winter 2009
Earth Systems Dynamics

Lecture 13
2/19/2009

Christiane Jablonowski
cjablono@umich.edu
734-763-6238

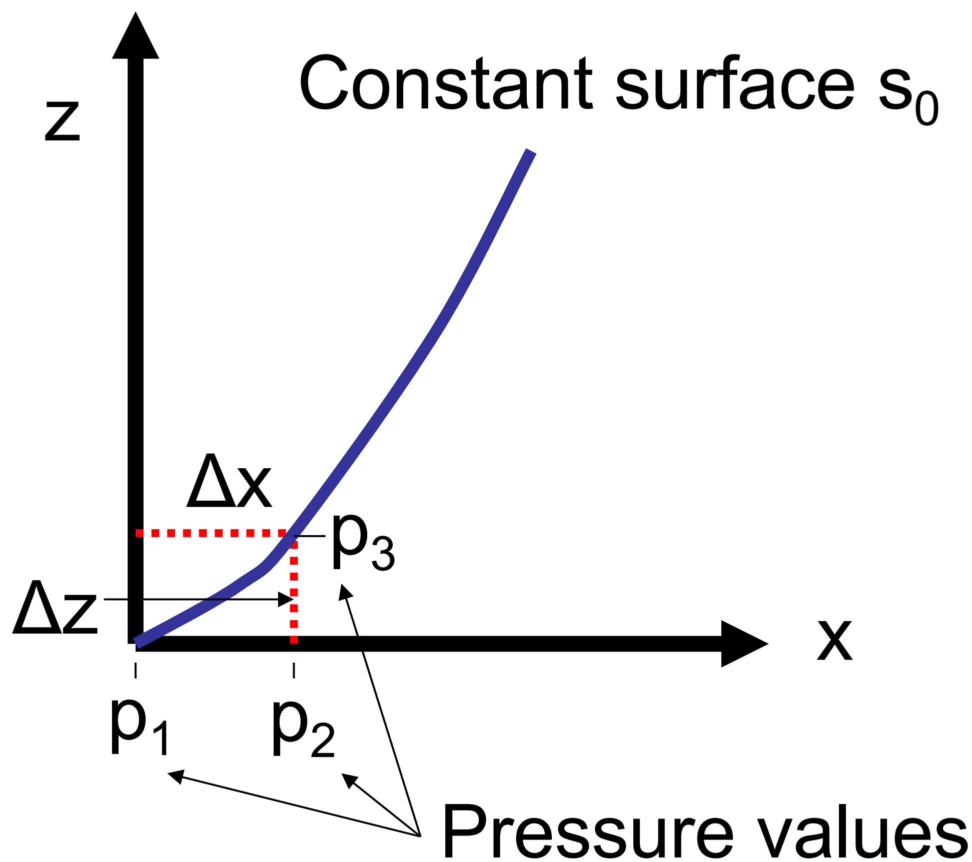
Eric Hetland
ehetland@umich.edu
734-615-3177

Today's class

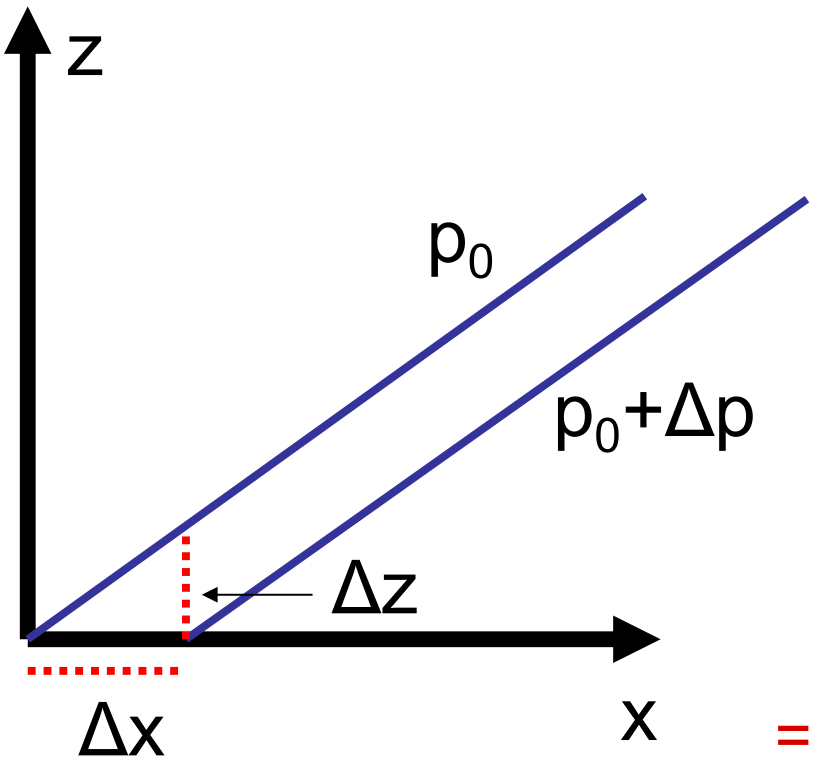
- Equations of motion in pressure coordinates
- Pressure tendency equation
- Evolution of an idealized low pressure system
- Vertical velocity

Generalized vertical coordinate

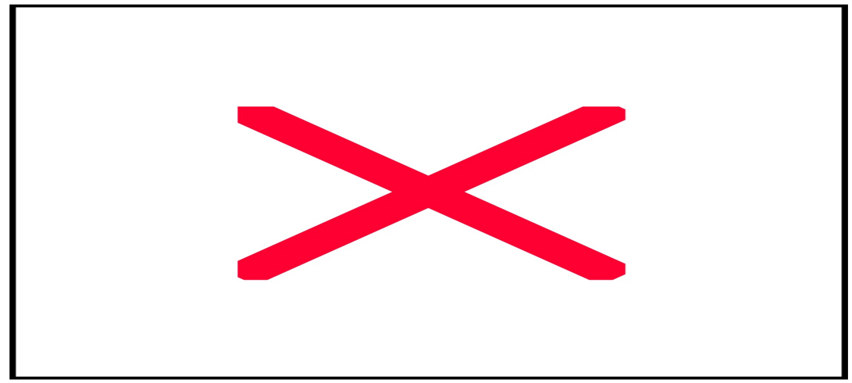
The derivation can be further generalized for **any** vertical coordinate ‘s’ that is a **single-valued monotonic** function of height with $\partial s / \partial z \neq 0$.



Vertical coordinate transformations

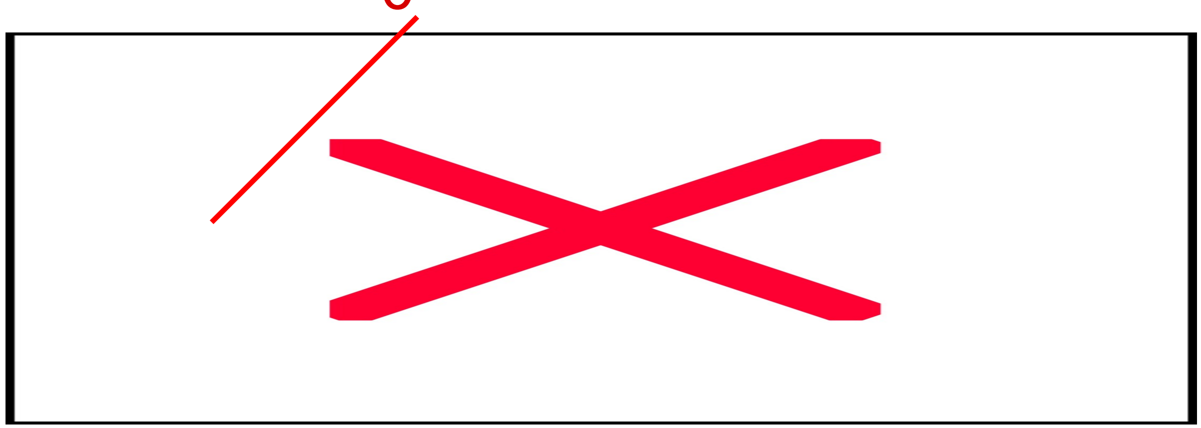


Pressure gradient along 's':



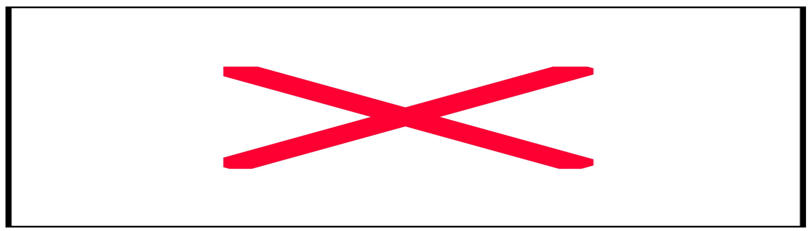
For s = p:
(pressure
levels)

= 0

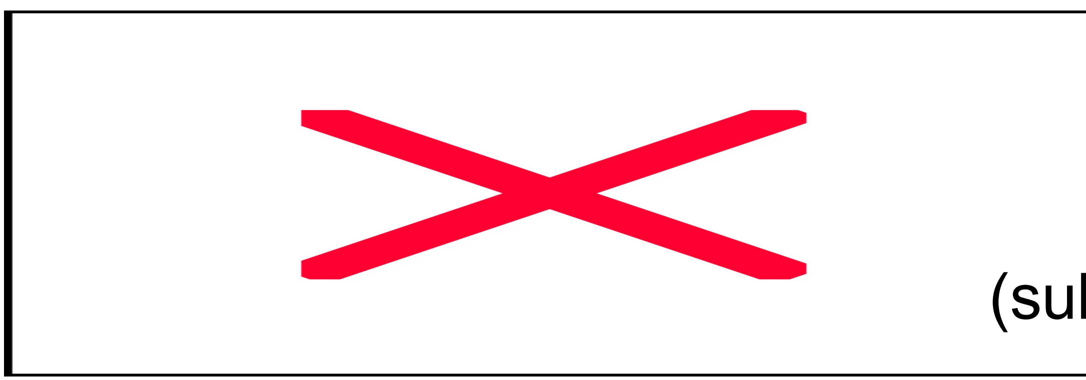


What do we do with the material derivative when using p in the vertical?

By definition:



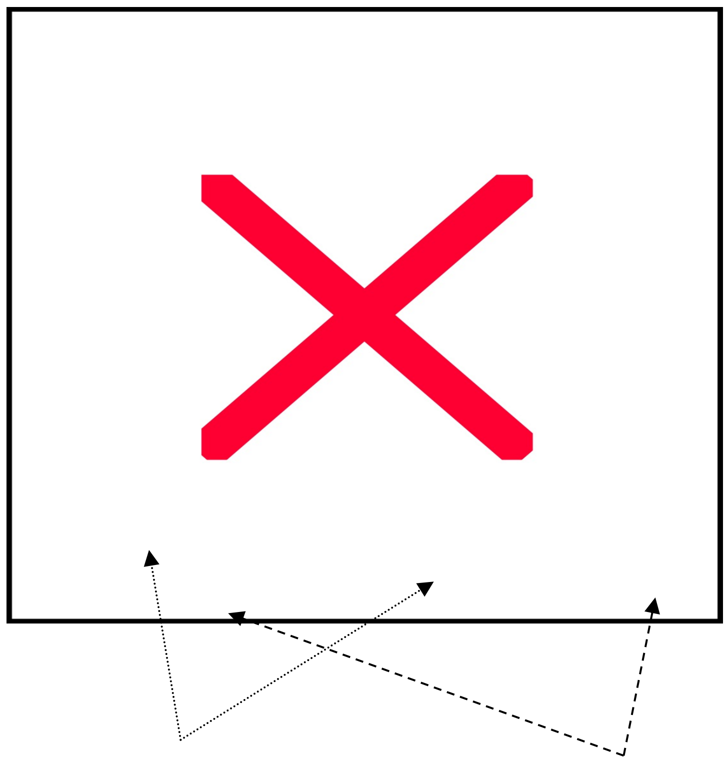
Total derivative DT/Dt on constant pressure surfaces:



(subscript omitted)

Our approximated horizontal momentum equations (in p coordinates)

No viscosity, no metric terms, no cos-Coriolis terms



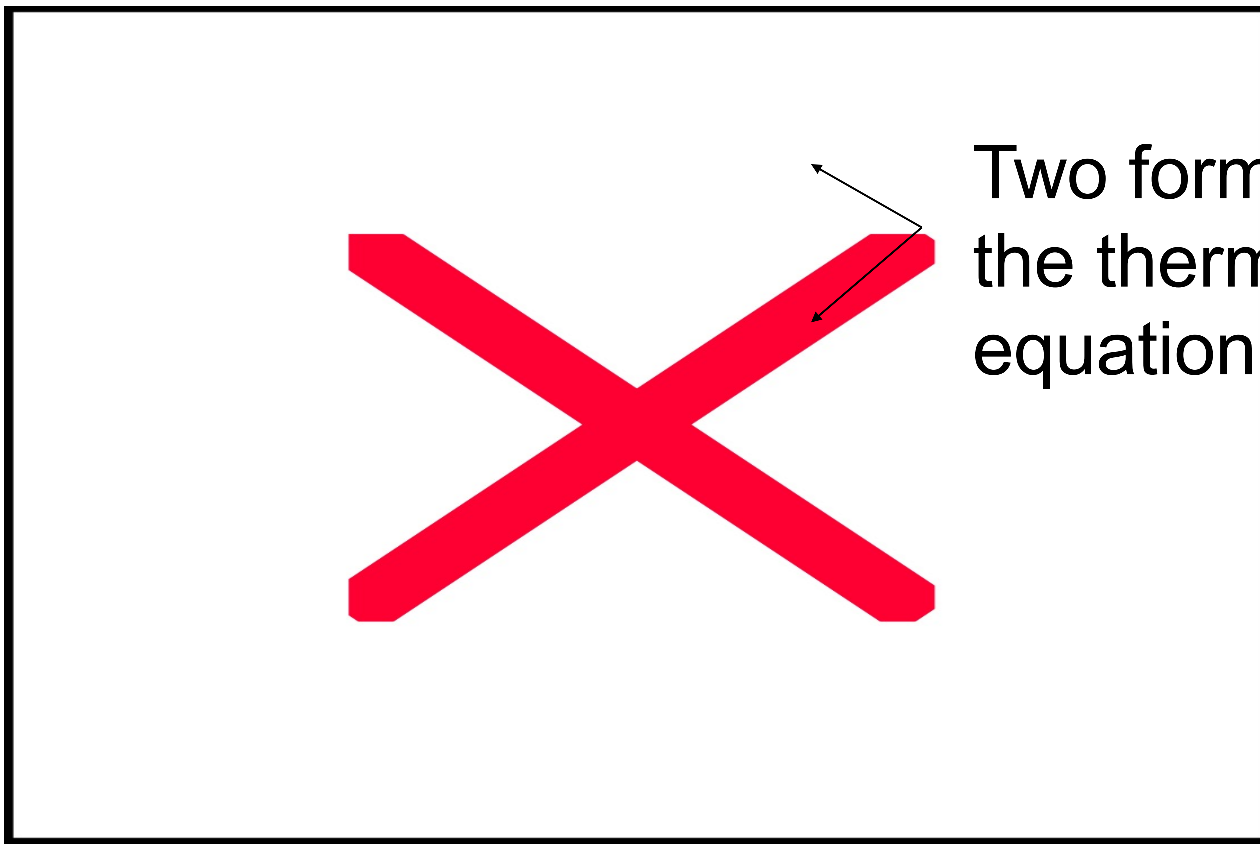
Subscript h: horizontal

Subscript p: constant p surfaces!
Sometimes subscript is omitted,
Φ tells you that this is on p surfaces

Thermodynamic equation

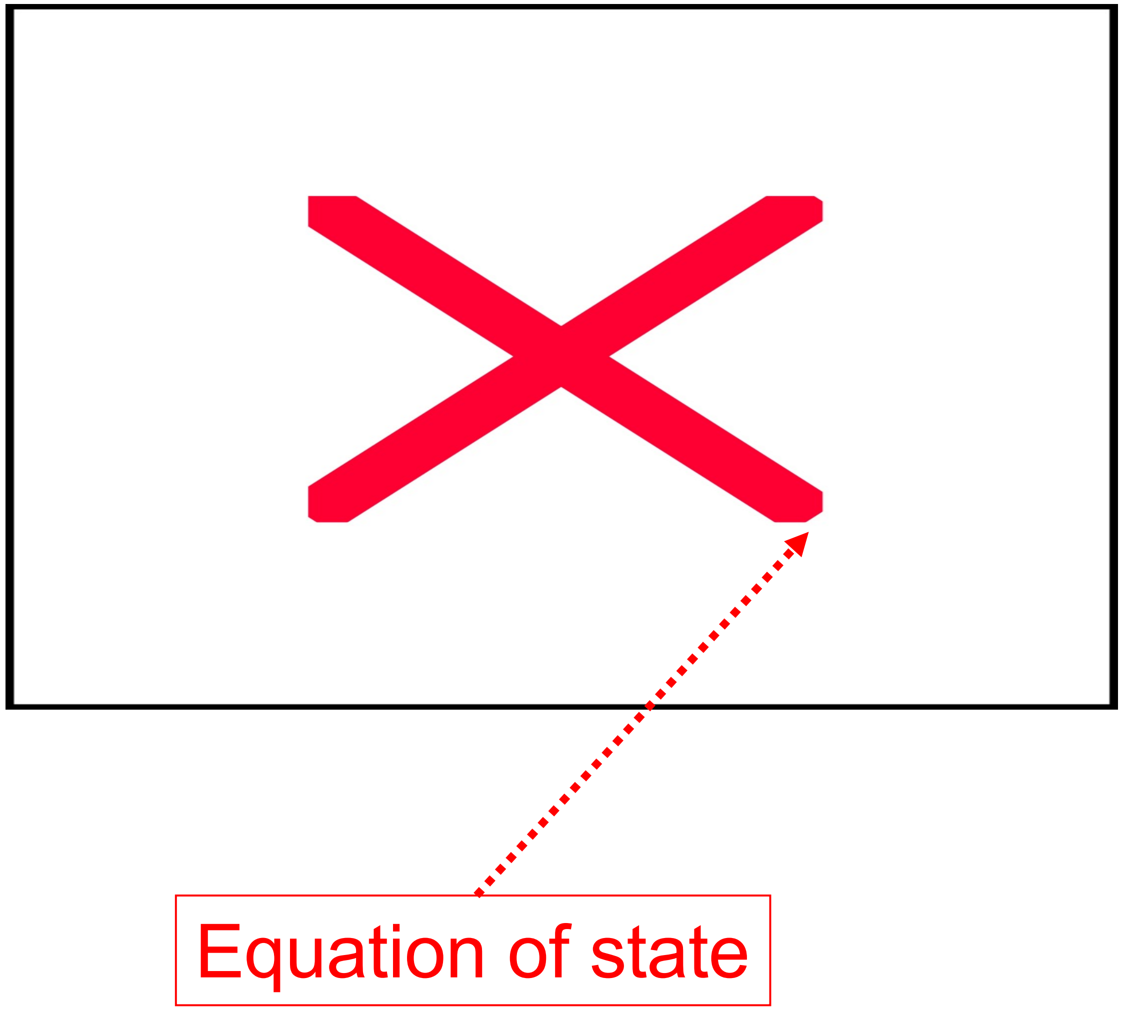
(in p coordinates)

use

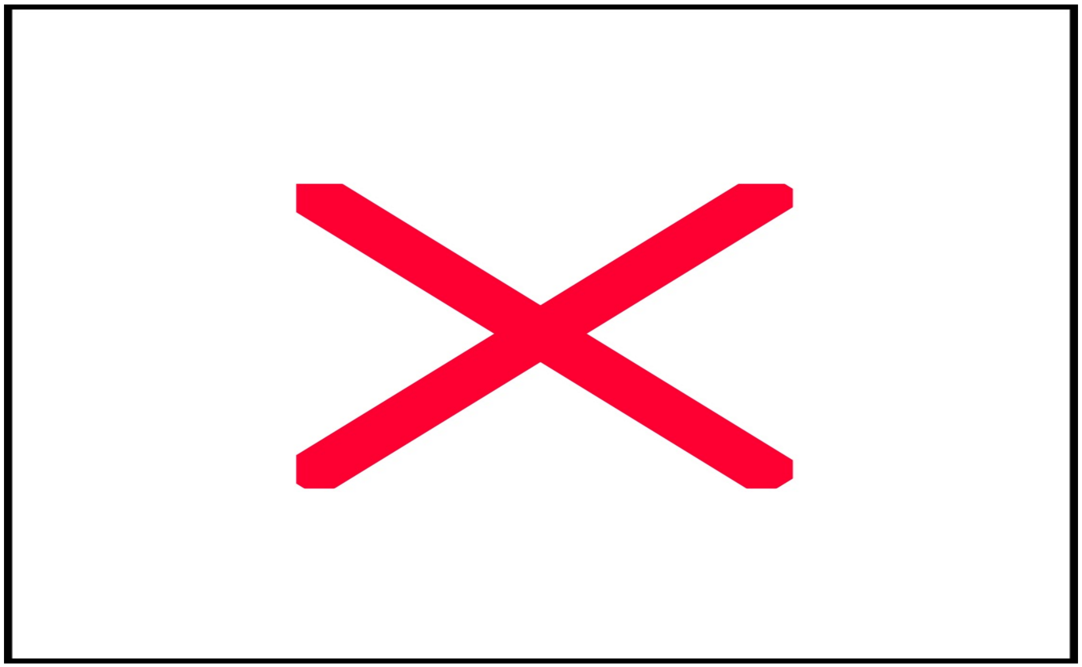


Two forms of
the thermodynamic
equation

Thermodynamic equation
(in p coordinates)



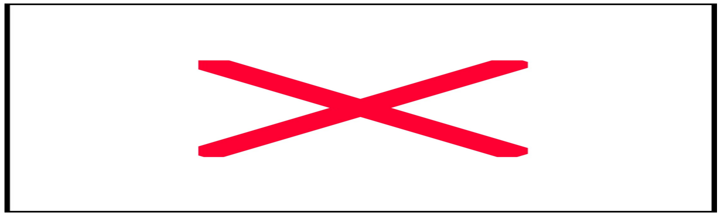
Thermodynamic equation (in p coordinates)

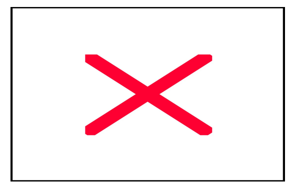
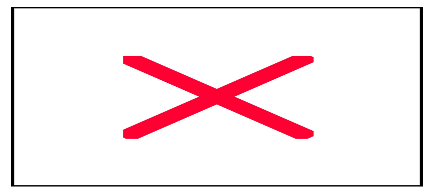


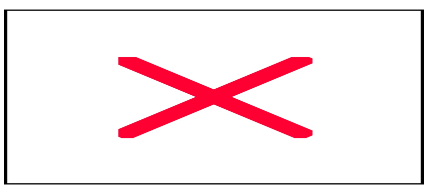
S_p is the static stability parameter.

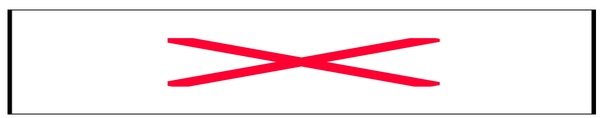
Static stability parameter

With the aid of the Poisson equation we get:



Plug in hydrostatic equation  : 

Using  (lecture 11, slide 14) we get

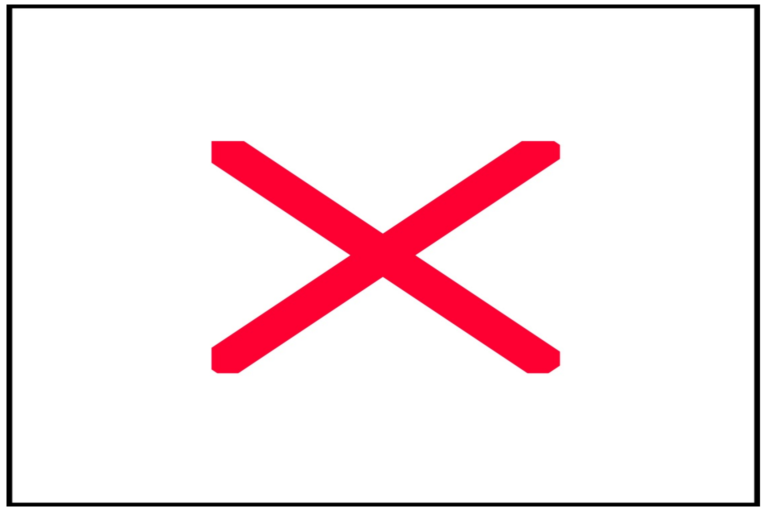
→ 

The static stability parameter S_p is positive (statically stable atmosphere) provided that the lapse rate Γ of the air is less than the dry adiabatic lapse rate Γ_d.

Continuity equation

in z coordinates:

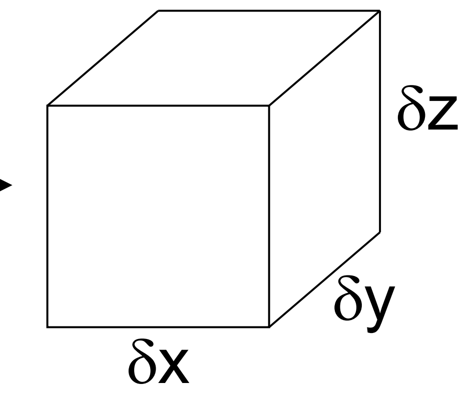
in p coordinates:



Let's think about this derivation!

Continuity equation: Derivation in p coordinates

Consider a Lagrangian volume: $\delta V = \delta x\ \delta y\ \delta z$



Apply the hydrostatic equation $\delta p = -\rho g \delta z$ to express the volume element as $\delta V = -\ \delta x\ \delta y\ \delta p/(\rho g)$

The mass of this fluid element is:
$\delta M = \rho \delta V = -\ \rho\ \delta x\ \delta y\ \delta p/(\rho g) = -\ \delta x\ \delta y\ \delta p/g$

Recall: the mass of this fluid element is conserved following the motion (in the Lagrangian sense): $D(\delta M)/Dt = 0$

Continuity equation: Derivation in p coordinates

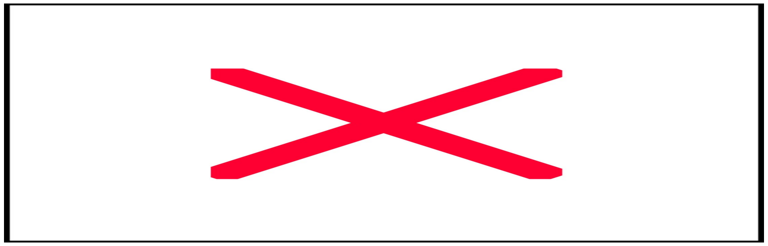
Thus:

Product rule!

Take the limit
δx,δy,δp → 0

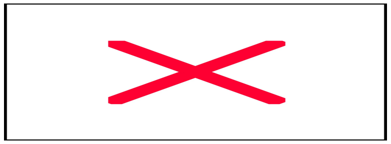
Continuity equation in
p coordinates

Continuity equation (in p coordinates)



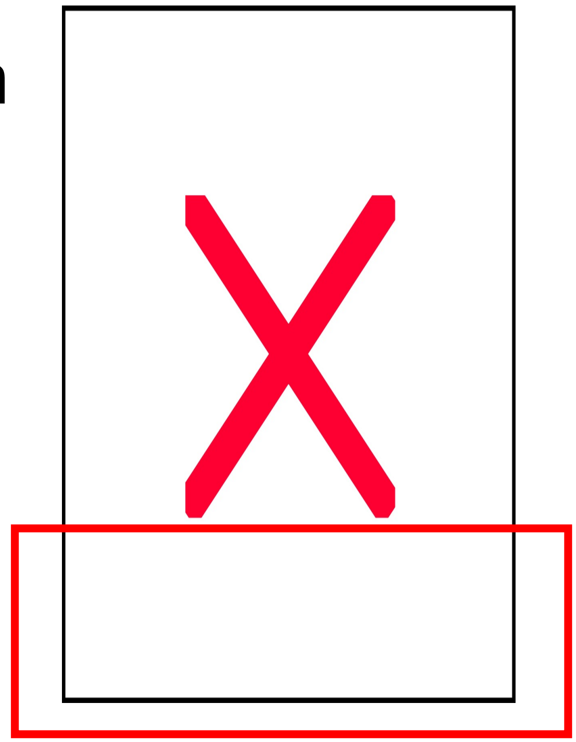
This form of the continuity equation contains no reference to the density field and does not involve time derivatives.

The **simplicity** of this equation is one of the chief advantages of the isobaric system.



Hydrostatic equation (in p coordinates)

Start with

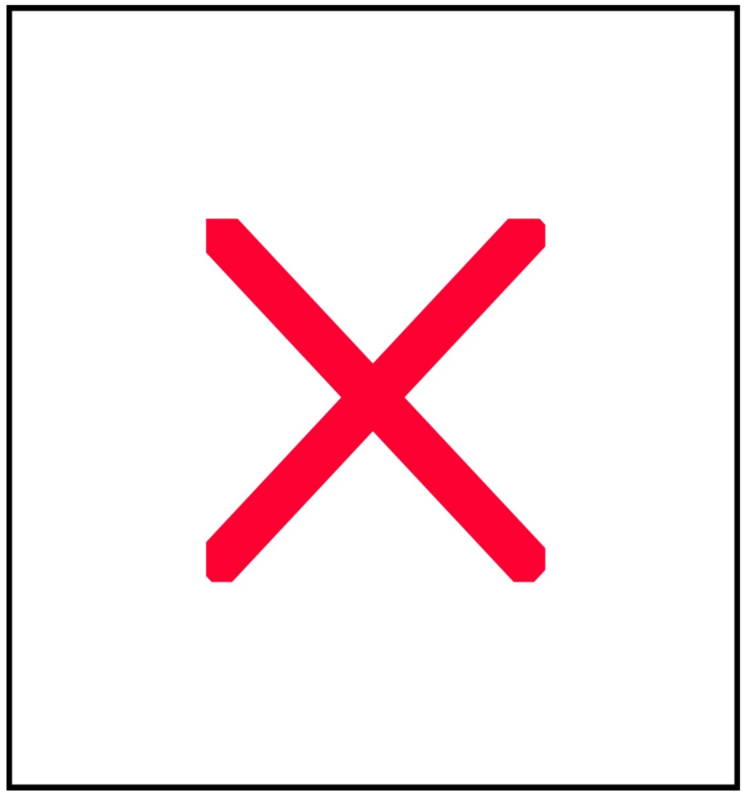


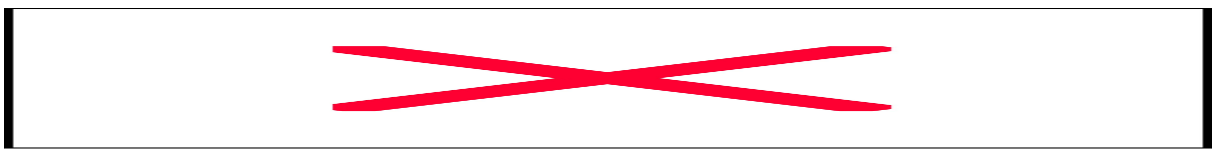
Hydrostatic equation in p coordinates

The hydrostatic equation replaces/approximates the 3rd momentum equation (in the vertical direction).

Approximated equations of motion in pressure coordinates

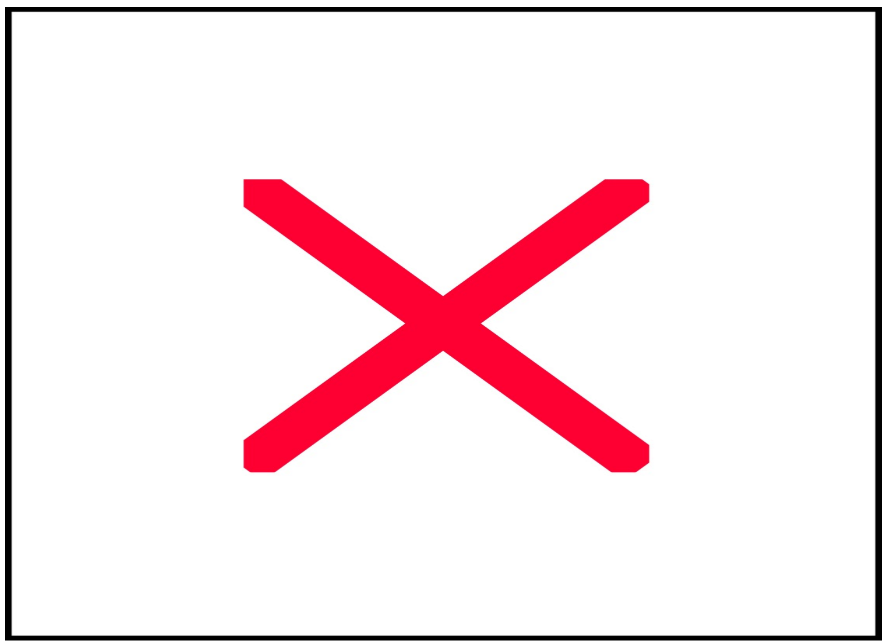
(without friction, metric terms, cos-Coriolis terms, with hydrostatic approximation)





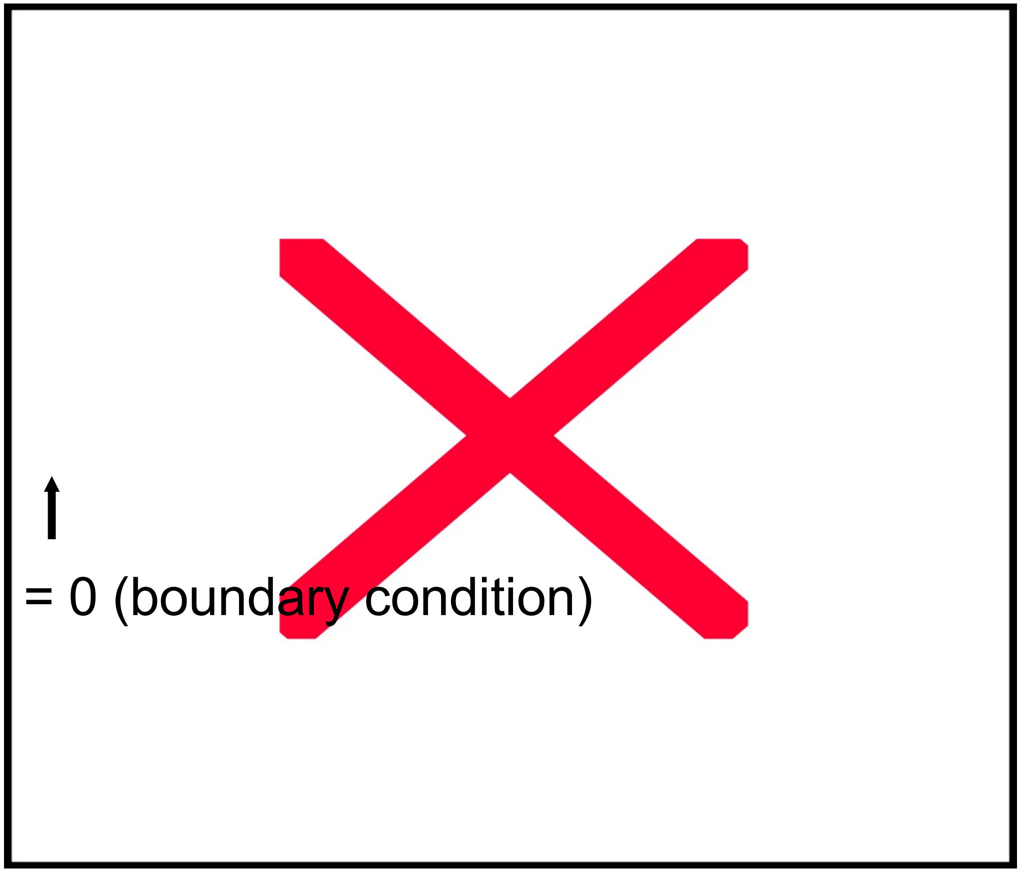
Let's think about growing and decaying disturbances.

Mass continuity equation in pressure coordinates:



Let's think about growing and decaying disturbances

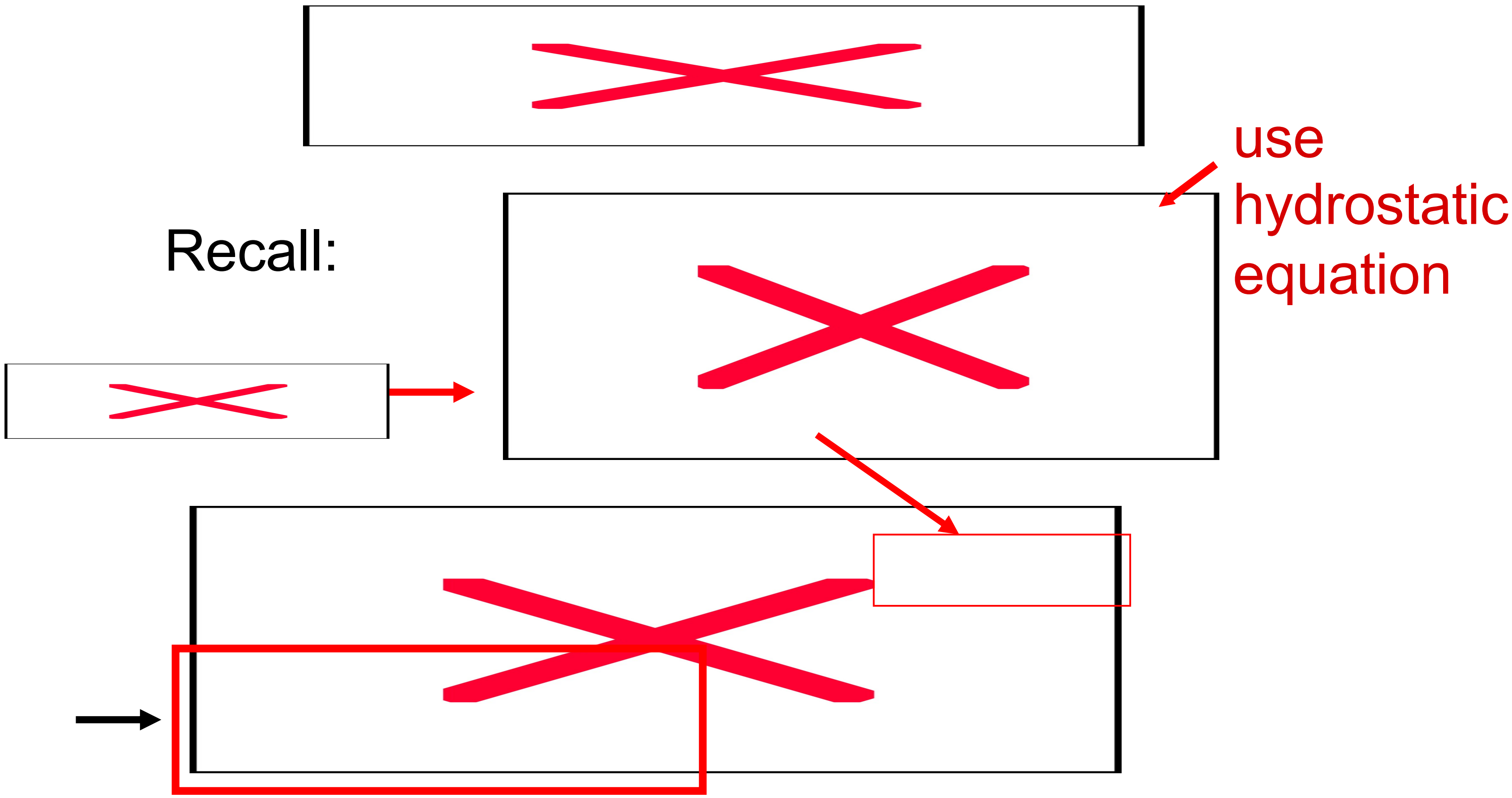
Integrate:



Formally links vertical wind and divergence.

Let's think about growing and decaying disturbances.

Recall:



Both [ ] and w are nearly zero at the surface (note: not true in the free atmosphere away from the surface)

Surface pressure tendency equation

At the surface:

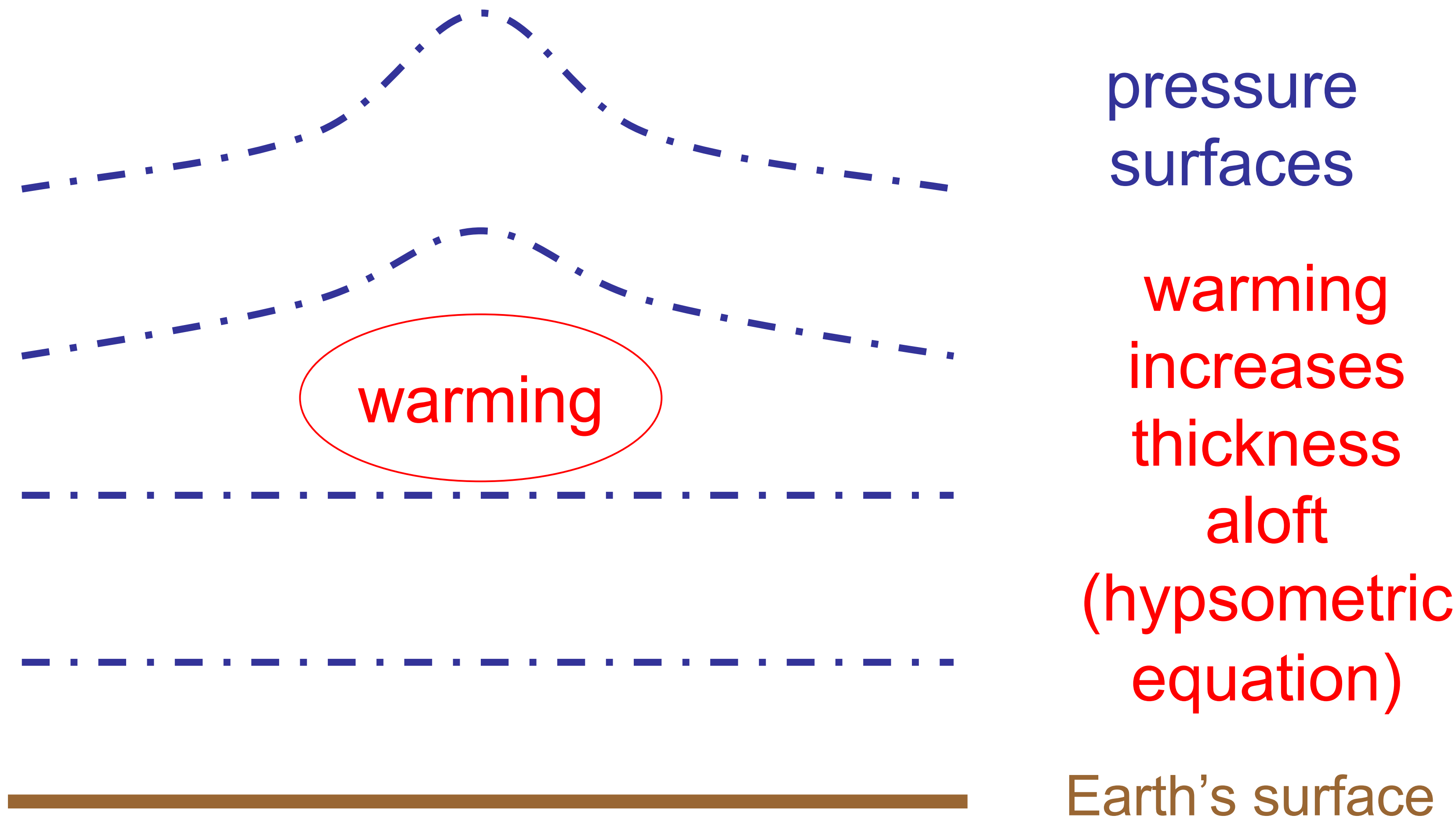
It follows:

Convergence (divergence) of mass into (from) column above the surface will increase (decrease) surface pressure.

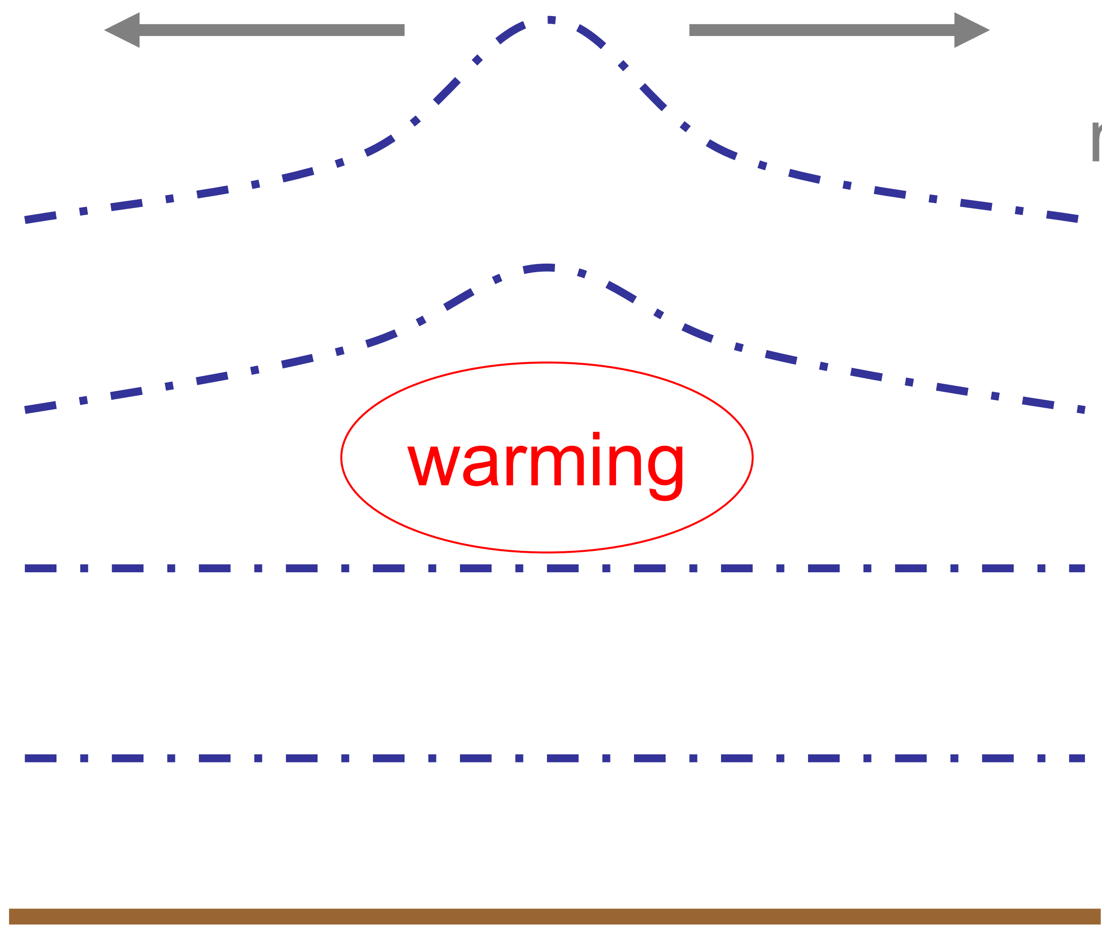
Possible development of a surface low.

Possible development of a surface low.

Possible development of a surface low.



Possible development of a surface low.

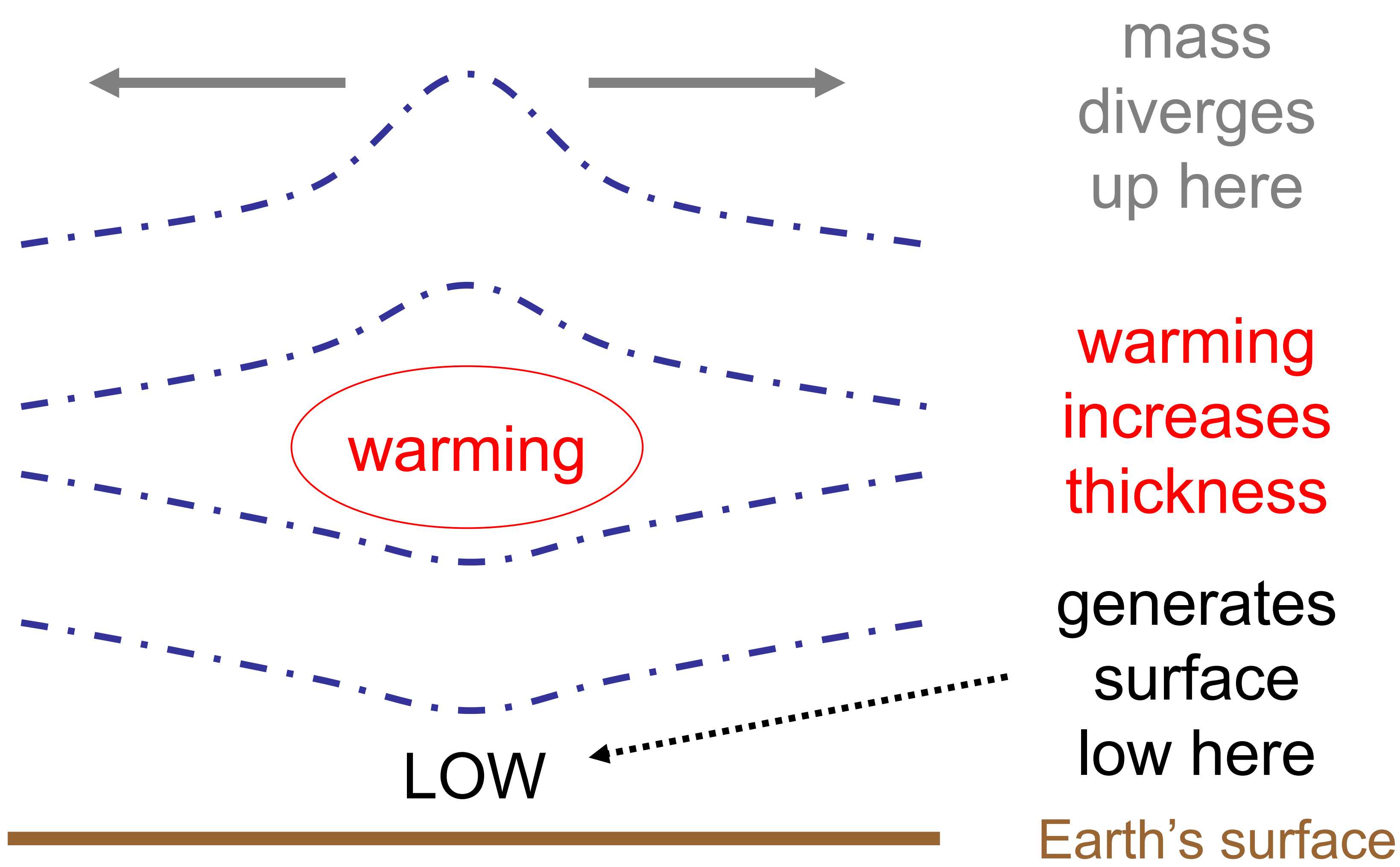


Pressure gradient force is created, mass diverges up here, causes surface pressure to fall

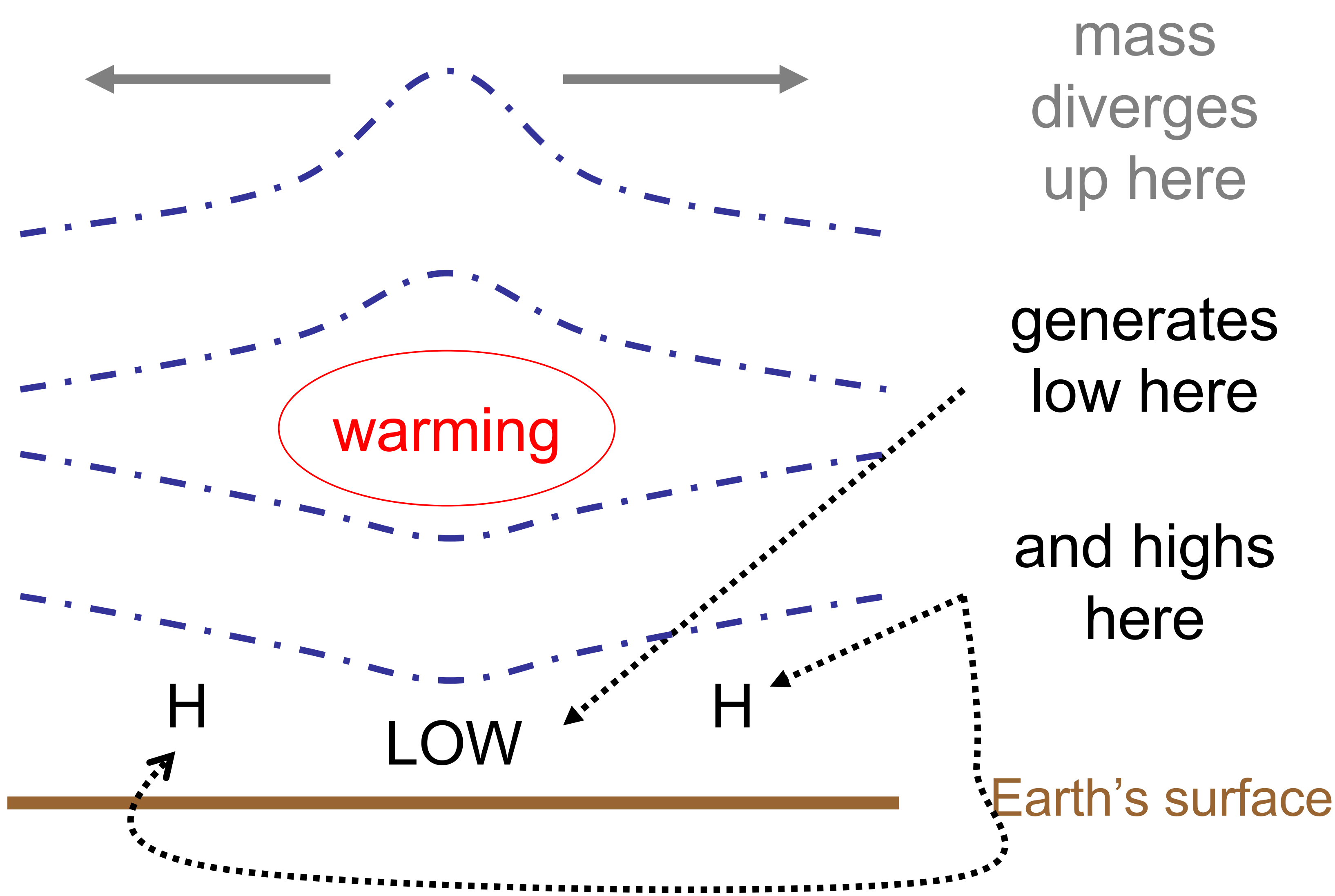
Warming increases thickness aloft

Earth’s surface

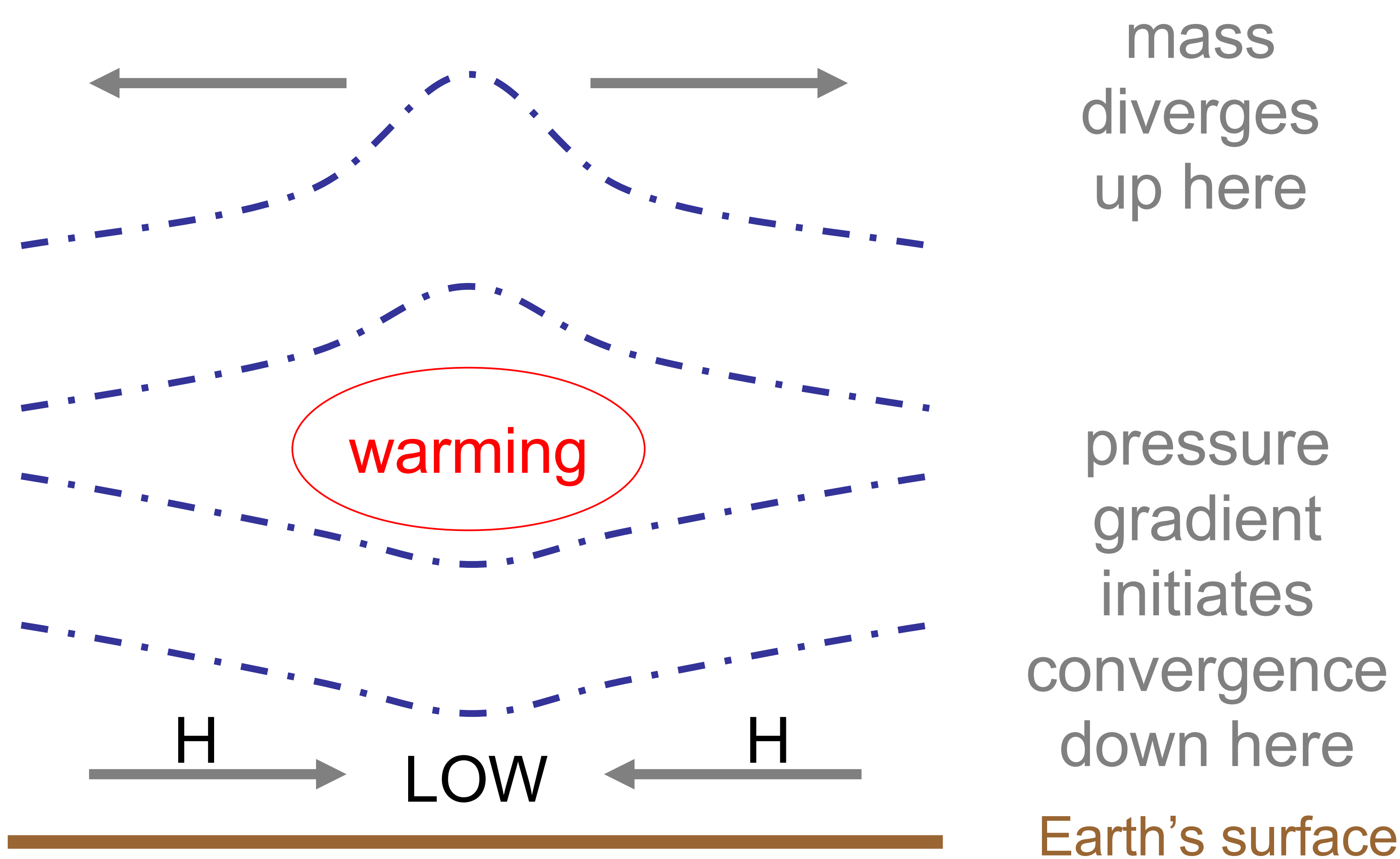
Possible development of a surface low.



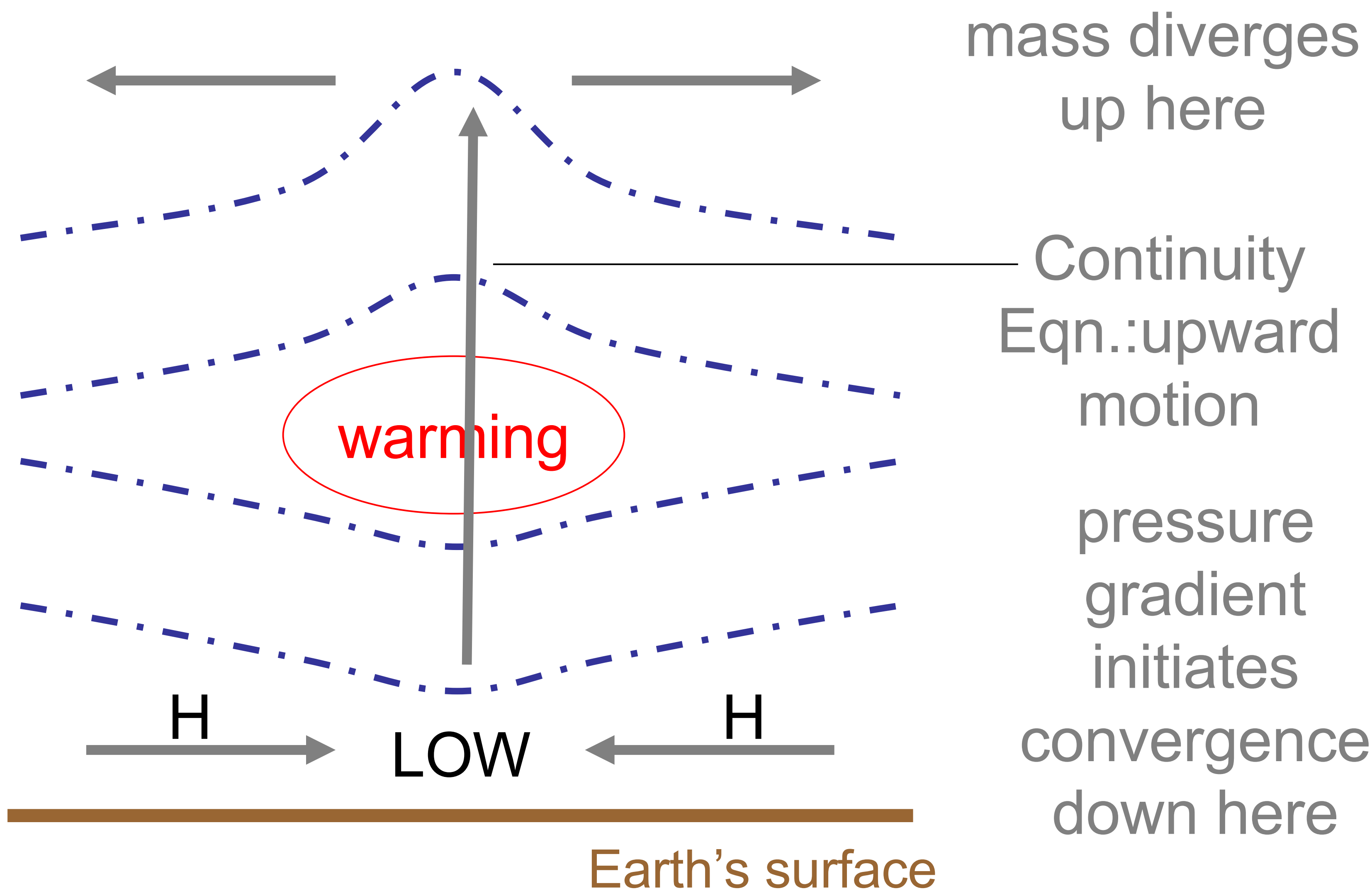
Possible development of a surface low.



Possible development of a surface low.



Possible development of a surface low.



A simple conceptual model of a low pressure system

- Increase in thickness from heating at mid levels.
- Pushes air up, creates pressure gradient force
- Air diverges at upper levels to maintain mass conservation.
- This reduces mass of column, reducing the surface pressure. Approximated by pressure tendency equation.
- This is countered by mass moving into the column, again conservation of mass. Convergence at lower levels.

A simple conceptual model of a low pressure system

- Pattern of convergence at low levels and divergence at upper levels triggers rising vertical motions.
- Rising motions might form clouds and precipitation.
- The exact growth (or decay) of the surface low will depend on characteristics of the atmosphere.
- Link of upper and lower atmosphere.

Vertical motions in the free atmosphere: The relationship between w and ω